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I. EWING, D.D.

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A PLAIN
ELEMENTARY AND PRACTICAL
SYSTEM OF
Natural Experimental Philosophy;
INCLUDING
ASTRONOMY AND CHRONOLOGY.

BY THE LATE

REV. JOHN EWING, D. D.
Provost of the University of Pennsylvania.

Being the Course of Lectures delivered by him in that Seminary, (Enlarged and
Improved from time to time,) for upwards of twenty years.

ILLUSTRATED BY COPPERPLATE ENGRAVINGS.

The whole carefully Revised and prepared for the Press, with sundry
Explanatory and Supplementary Notes;

BY ROBERT PATTERSON,
Professor of Mathematics in the said University.

With a Biographical Sketch of the Author.

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D. CALDWELL,
Clerk of the District of Pennsylvania.

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LIFE OF THE AUTHOR.

When it was resolved to publish the following Lectures, it was intended to annex to them a life of the author, not only for the immediate gratification of his surviving friends, but with a view also to furnish materials to assist the future biographers of our countrymen. The lives of men whose virtues or talents have honoured their native land, have been, in other countries, recorded with diligence, and perused with general eagerness. In Europe, at the present day, every literary character has his faithful *Boswell* to publish his virtues, and to extend and to perpetuate his fame; and although ages pass away without producing an object of biography, so illustrious as *Johnson*, yet almost every year takes from us some one whose example *might give ardour to virtue and confidence to truth.*

In the hasty and indiscriminate biographical compilations which avarice or friendship hourly issues from the press, many names are announced *to the public* for the first time; names, which had previously been scarcely known beyond a limited circle of partial friends. Yet, so pleasing is the study of biography to the learned as well as to the desultory reader; so much more attractive are fire-side anecdotes than the annals of nations, that every work of this description meets a ready purchaser, who finds in it something to amuse and something to instruct.

Biography, indeed, *comes home to the business and bosoms of men.* Man is naturally social, and if he be not driven into foreign wars nor deluded into civil dissensions by the authority of power, or by the seductions of interested individuals, he will be generally found engaged in that interchange of kindly affections which produces his temporal felicity and is best calculated for his eternal welfare. It is natural, therefore, that he should be more interested in the lives of great or good men than in the tumults of nations; and accordingly, the observing inquirer will discover that the pages of Gibbon and Hume which record the intrigues of statesmen and the triumphs of ambition, are neglected for the perusal of those in which they appear as their own biographers.

But the amusement which the study of biography affords, is of trivial importance when its utility is considered. Life is a scene of ceaseless toils and difficulties, which as they are generally of our own creation, might also be avoided by our own precautions, or subdued by our own exertions. There is little variety in the history of man. In all ages he has been the victim to his own passions, or the builder of his own prosperity. He cannot indeed reasonably hope to elude the attacks of artifice or accident, but he can generally be taught to diminish their force. Experience though a skilful, is but a slow teacher, and, unfortunately, it too often happens that when the lesson has been acquired, the power to profit by it has passed away. Biography is therefore a valuable study, since it shows us men, like ourselves, with similar feelings, passions, desires and wants; encountering the same obstacles which hourly cross *our* path; in some instances surmounting, and, in others, subdued by them, but in all directed to the means of success.

The following life is an eminent example of the beneficial results of persevering industry, and the object of recording it will have been attained, if it shall teach the timid, a proper confidence in his own efforts, and the presumptuous, an humble confidence in his God.

Dr. JOHN EWING was born on the twenty-second day of June, 1732, in the township of Nottingham, in Cœcil county, Maryland, near to the line which separates that state from Pennsylvania. Of his ancestors little is known. They emigrated from Scotland at an early period of the settlement of our country, and fixed themselves on the banks of the Susquehanna, near to the spot where he was born. They were farmers, who, if they did not extend their names beyond their immediate neighbourhood, yet maintained within it that degree of reputation which their descendants can speak of without a blush.

His father was enabled by his industry to support his family* from the produce of his farm, and to give to his children that degree of education which country schools at that time had to offer. This indeed was little, but it was all that was necessary to such a mind as Dr. Ewing's. It was sufficient to furnish the rudiments of science, which, however early they are lost by ordinary minds in the distractions of a life of business, only serve to fan the fire of ambition in stronger intellects, and to direct and guide their possessors to fame.

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* There were five brothers: William, George, Alexander, John, and his twin brother, James, who is the only one now living.

The school-house at which Dr. Ewing was taught the elements of his native language and the first rules in arithmetic, was at a considerable distance from his father's residence. The daily exercise of walking thither in his youth, tended to invigorate a constitution naturally strong, and enabled him to acquire a stock of health which carried him through sixty years without sickness.* At this school it cannot be supposed that he learned much, but he was soon removed from it and placed under the superintendance of Dr. Alison, a clergyman eminent for his erudition and piety, who then directed a school at New London cross roads, in the state of Pennsylvania. After having finished those studies usually taught in his school, he remained with him three years as a tutor. To this he was led, not merely by inclination, but by necessity. His father died about this time, and left his small property to be distributed according to the laws of the state of Maryland, in which that of primogeniture prevailed. The eldest son inherited the patrimonial estate, and left

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* Dr. Ewing has been heard to state a fact which he witnessed at this period of his life, and which I cannot resist relating, since, established by his character for veracity, it may shed some light on a question in natural history, hitherto involved in some obscurity. As he went to his school one morning at an early hour, he observed a bird in extreme agitation, flying repeatedly across the road, but never going beyond the fence on either side, on which it constantly alighted. It would rest there for a moment and then return to the opposite fence, always descending in its flight, until it nearly touched the ground. Its agitation arrested his attention, and he stood to observe the cause. On the spot where it seemed disposed to alight in its flight, he observed a snake, which had evidently fixed on its victim, and fascinated it beyond the power of escaping by its own efforts. He frightened the snake away by throwing at it a stone, when the bird instantly flew off with evident symptoms of joy.

Dr. Ewing and his remaining brothers, to struggle in the world with twenty pounds each. At this distribution of his father's property he did not repine, for he then felt a confidence in his own powers which did not deceive him, which, poverty could not diminish, and which enabled him subsequently to attain that honourable elevation which he adorned by his virtues as well as his talents.

Under the kind care of Dr. Alison he made considerable progress in his favourite pursuit, the study of mathematics. Books of science were not at that time easily obtained in America, especially in places remote from cities; but such was his thirst for knowledge that he frequently rode thirty or forty miles to obtain the loan of a book which might afford him some information on the subject of his favourite speculations. Those authors who were safe guides could not always be obtained. Incorrect writings sometimes fell into his hands, the errours of which did not escape the detection of his penetrating and original genius. It often occurs that difficulties only quicken the eagerness of the mind in its pursuits, and bring into action its latent energies. Such was the result of difficulties on Dr. Ewing at this early period of his life. His mind did not shrink from intellectual conflict, but gathered vigour from hindrance, and bade defiance to difficulty. At this period he certainly learned much from books, and much from the conversation of Dr. Alison, of whom indeed he always spoke with kindness, but he acquired more from the habits of close thinking in which he early indulged. To the two former he was much indebted, but if we allow to those sources of information all that they merit, it will yet not be hazardous to say that in the science of mathematics he was selftaught, and could never have reached that station which he afterwards adorned, struggling as he was with poverty and harassed with difficulties, without receiving from other than human aid the impulse which carried him forward.

In the year 1754 he left the school of Dr. Alison, and removed to Princeton for the purpose of entering the college. Mr. Burr, the father of the late vice-president of the United States, was then president of that institution, and of that great and celebrated man he was a favourite pupil. He joined the senior class, and, impelled by pecuniary embarrassments, engaged at the same time as teacher of the grammar school which was connected with the college. His intention was to graduate, and for this purpose it was necessary that he should study in private some branches of learning to which he had previously been unable to attend. These causes made his labour greater than that of his classmates. His studies were arduous and multiplied; but he brought to the contest a mind which difficulties did not easily subdue. He graduated with his class in the year 1755, and finding that he had still to toil for a subsistence, he immediately accepted the appointment of tutor in the college. At this period he resolved to choose his profession; and feeling the study of theology congenial with his wishes, and calculated to permit him to mingle with it scientific researches, he adopted it with his usual promptitude and his usual zeal.

In pursuance of this design, he returned to Dr. Alison, his former tutor and friend, and, after the usual period of preparatory study, he was licensed to preach the Gospel by the presbytery of Newcastle, in the state of Delaware. At the age of twenty-six, before he undertook the pastoral charge of any congregation, he was selected to instruct the philosophical classes in the college of Philadelphia, during the absence of the provost, the late Dr. William Smith. Whilst he was engaged in the discharge of this honourable office, he received an invitation from the presbyterian congregation of his native place to settle himself among them as their pastor. This was an invitation on which he deliberated, before he declined it. To be selected by the friends of his youth as their spiritual guide; to fix himself with a decent stipend on his native spot among his relations and former associates, was a temptation calculated to win a man who was social in his affections, and who was little troubled with the unquiet spirit of ambition. But he was by this time married, and having early known the value of a liberal education, he wished to give his offspring the opportunity of possessing those instructions which he himself had so long toiled to acquire; which, during his life, he praised as more valuable than wealth, and recommended to the attention of his children by all the persuasions of paternal affection. Whilst, however, he was deliberating, he received, in the year 1759, an unanimous invitation from the first presbyterian congregation in the city of Philadelphia to undertake their pastoral charge. This he did not feel himself at liberty to decline, but accepted it, and fixed himself for his life.

From this period until the year 1773, he continued to discharge his duties with a diligence and zeal seldom surpassed. In the bosom of his congregation he found affection and friendship, and learned that life has few stations to offer to an unambitious heart more valuable than that of a pastor beloved by his flock.

He was now at liberty to pursue his favourite studies without other intrusions on his time than method and diligence could render harmless. During this period his studious researches enabled him to accumulate materials for the compilation of his *Lectures on Natural Philosophy*, and such was the vigour of his understanding, such his habits of constant study, and so ample his stores of knowledge, that the volume which accompanies this sketch of his life is copied from the original manuscript.

New scenes now opened upon his view. In the year 1773 he was commissioned, with the consent of his congregation, in conjunction with Dr. Hugh Williamson, late a member of Congress from the state of North Carolina, to solicit subscriptions in Great Britain for the academy of Newark in the state of Delaware. He took with him letters of recommendation from men of science and respectability to several eminent characters. These, aided by his own reputation for mathematical science, his general information, and his virtues, procured for him the intimacy and friendship of several persons, who at that period and since held the highest stations of literature. Among these were the celebrated historian Dr. Robertson, Dr. Webster, Mr. Balfour, and Dr. Blacklock, the blind poet of Scotland. He visited every place of importance in England, Scotland, and Ireland, and in all of them was received with that attention and respect which are due to the man of science and the minister of God. The cities of Glasgow, Montrose, Dundee and Perth, presented to him their freedom, and, from the university of Edinburgh, of which Dr. Robertson was then the *Princip*
out application, the degree o

Dr. Robertson, on presenting his diploma, declared that he had never before conferred a degree with greater pleasure. The acquaintance, thus commenced with this celebrated personage, ripened into intimacy, and until the death of the latter, in 1793, he made constant and affectionate inquiries about Dr. Ewing from travelling Americans who visited him at Edinburgh. A few days before his death, some young American gentlemen waited upon him, to whom he spoke of his friend "*as a man of great talents for whom he entertained a great personal regard*," and his last words at parting were, "*Do not forget to present my kind regards to Dr. Ewing.*"

Such a testimonial from such a man as the historian of Charles the Fifth, the descendants of Dr. Ewing may be permitted to remember and to speak of to the world.

When he first visited England, the approaching contest with his native land was a topic of conversation in every society. He was warmly and uniformly the friend of his country, and although he had frequent offers of reward from men, high in power, if he would remain in England, yet his knowledge of the causes of the revolution; his acquaintance with the spirit and resources of his countrymen, and his integrity forbade him to listen to them. He held frequent conversations with the minister, Lord North, to whom, with that frankness and independence of sentiment which characterized him, he communicated all his information respecting the resources and power of the people of the united colonies. To the minister he predicted the issue of the contest, and urged him to pause before he alienated irretrievably from the mother country the affections of loyal subjects. These conversations he was in the habit of repeating to his friends on his return from England, not without some degree of surprise that the minister should have involved his country in a war with a people, of whose character, numbers, spirit and resources he was utterly ignorant.

Among the eminent literary characters whom Dr. Ewing met at the hospitable table of Mr. Dilly, the London bookseller, was the truly great Dr. Johnson. He loved to speak to his friends of this interview, which serves to illustrate the character of a man, of whom every one, who has read, knows something. When Mr. Dilly invited Dr. Ewing to dine with him, he added, " You will meet the great Dr. Johnson, but you must not contradict him; we never contradict him." The day arrived, and Dr. Ewing, on entering the parlour of Mr. Dilly, found several eminent literary characters engaged in easy conversation, which, however, was instantly suspended when Dr. Johnson entered the room. There was a general silence. He scarcely noticed any one, but, seizing a book which lay on the table, read in it attentively until dinner was announced. Here, every one seemed to forget himself, and anxious to please him by the most assiduous attentions. He attended however to nothing but his plate. He did not seem to know that any one was present, until, having eaten voraciously without exhibiting many of those *graces* which constituted so great a portion of Chesterfield's morality, he raised his head slowly, and, looking around the table, surveyed the guests for the first time. They were by then engaged in a discussion of the expected controversy with America, and, as Dr. Ewing had lately left his native country, he, with his usual frankness, and without adverting to, or regarding the prejudices of Dr. Johnson, began to defend the cause of the colonies. Johnson looked at him with sternness, and said, "*What do* "*you know, Sir, on that subject?*" Mr. Dilly's caution was forgotten, and Dr. Ewing calmly replied, that having resided in America during his life, he thought himself qualified to deliver his opinions on the subject under discussion. This produced an animated conversation. Johnson's prejudices against the Americans were strong; he considered them, as he always termed them, rebels and scoundrels, and these epithets were now by no means sparingly used. It is difficult to say how far he might have been provoked, by opposition in argument, if a fortunate turn had not been given to the dispute. Johnson had rudely said, "Sir, what do *you* know in America. You never read. You have no books there." "Pardon me, Sir," replied Dr. Ewing, "we have read the *Rambler*." This civility instantly pacified him, and, after the rest of the company had retired, he sat with Dr. Ewing until midnight, speaking amicably and eloquently, and uttering such wisdom as seldom falls from the lips of man.

In the summer of 1775, Dr. Ewing returned to his native land, with a mind highly improved by his travels. He had directed his inquiries to the study of man, in all the varieties which Great Britain and Ireland afforded. He had collected much information and many anecdotes, which on his return were reserved for the amusement and instruction of that social circle, which he loved to collect at his own fireside. His parlour was always the scene of cheerfulness and hospitality. His finances indeed were never more than moderate, but he was always able to furnish for his guests something more valuable than the delicacies of the season, or the wines of France.

War had now commenced between the United States and Great Britain, and he adhered to the cause of his country with steadiness and zeal. When the British army was expected in Philadelphia in 1777, he removed his family to his native place, where he continued to reside until the city was evacuated by *Clinton*, immediately before his retreat through the state of New Jersey to New York. He then returned to his congregation, and in 1779 was elected to the provostship of the University of Pennsylvania, which station he filled until his death.

To this station he was fully competent. In all the branches of learning and science usually taught in colleges, he was uncommonly accurate, and in his mode of instruction and of communicating information he was probably never surpassed. On his appointment he prepared the Lectures which are now published, and which he delivered to his pupils during a period of twenty years. They will be found to contain all that is necessary for the mere student; written in a plain and simple style, and arranged with great method and perspicuity. As a teacher perhaps no one was ever more beloved. His authority over his pupils was that of a parent, and while he maintained that discipline without which genius will be wasted, and diligence useless, he won their affection by the mildest manners.

All his hours were now occupied. He attended at the university during the mornings and afternoons of every day, and devoted his remaining time to the duties of his pastoral charge, and a necessary attention to his private affairs. These were arduous and multiplied. Visiting the sick, and interchanging with his parishioners the visits of friendship, occupied much of his time. And when from the performance of these duties, he retired to his closet, he was obliged to prepare, usually two, but always at least one discourse for the approaching sabbath. But these difficulties yielded to his love of method and untiring diligence. He rose with the sun, and retired to rest at a late hour in the night; yet his constitution was naturally so robust, and the care of his health so judicious, that, during a period of forty years, he was never prevented by sickness from attending to his pastoral duties.

But these were not his only employments. His mathematical reputation attracted the attention of his fellow citizens, and on various occasions he was appointed to perform public duties. He was one of those gentlemen who were commissioned to run the boundary line of the state of Delaware, and to settle the boundary line between the states of Massachusetts and Connecticut and between Pennsylvania and Virginia. He was also appointed in conjunction with the late David Rittenhouse, by the state of Pennsylvania, to survey the most practicable ground for a turnpike road between Philadelphia and Lancaster. He was a distinguished member, and for some time one of the vice-presidents of the American Philosophical Society, to which he made several valuable communications, which are recorded in the volumes of their transactions.* He also made several valuable additions to the astronomical articles in the American edition of the British Encyclopædia, published by Mr. Thomas

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* The following were the communications which he made to the A. P. S. "An Account of the Transit of Venus over the "sun, June 3d, 1769, and of the Transit of Mercury, November "9th, 1769, both as observed in the statehouse, Philadelphia." "An improvement in the construction of Godfrey's Quadrant."

Dobson. About the year 1795, he commenced the compilation of a course of Lectures on Natural History, for the use of the pupils in the university, and made some progress in the work, but his health did not permit him to complete his plan.

From the year 1779 to the time of his death, his life had little variety. He continued to discharge the various duties of pastor, preceptor, husband, parent, and friend, without making, as it is believed, one good man his foe. The compensation which he received from the university and from the church, although not large, enabled him, with economy, to raise a numerous family, and to acquire a moderate property. But he was not versed in the artifices of business. He was a friend, and he trusted. He was himself free from guile, and therefore easily duped, and thus, in his old age, he had the mortification to see his little property swept from him by those to whom he had formerly loved to render acts of kindness. Yet he did not speak harshly of those who had injured him. Some of them indeed he forgave, though he could not forget. But for the conduct of the rest, he was always desirous to find excuses, and he continued during his life to defend those who could find no other apologist.

In the summer of 1796 he was attacked with a violent disorder, which it required a long time to subdue. He never however recovered from its effects; but although it left him so feeble as to be unable to walk without aid, he still persevered in performing his public duties. His remaining strength began to fail him during the early part of the year 1802, and in the month of August, he removed his family on account of the yellow fever to the house [illegible] Montgomery county, in Pennsylvania on the 8th of September of that year, in the 71st year of his age.

The following sketch is extracted from a funeral sermon preached by his pastoral successor, the rev. Dr. John Blair Linn, on the 21st of November, 1802, in the first Presbyterian Church, in the city of Philadelphia.

“ The unembellished incidents which have now
“ been narrated of Dr. Ewing's life, his religious and
“ scientifical writings; his observations and deport-
“ ment in the different relations of society, declare
“ that his mind was uncommonly strong and penetra-
“ ting, and that he had a mild and correct taste. Were
“ we to distinguish between his powers, we would
“ say that his understanding predominated over his
“ imagination. He had more the mind of Locke than
“ of Milton. He looked through nature more with the
“ eye of the philosopher than of the poet. The sub-
“ limer and minuter forms of matter were objects of
“ his investigation; and we cannot but suppose him
“ to have been gifted with diversified talents, who
“ could scan the illuminated glories of the heavens,
“ and inspect the insect which is only visible to the
“ microscopic eye: We cannot but suppose that his
“ researches were extensive, who looked into the mind
“ of man, analyzed his faculties and affections; who
“ unfolded to him the great truths of his God, who
“ looked through the howling wilds and taught the
“ properties of the brutal tribes, who looked through
“ the fields of air and described the race which travel
“ on the wing. In the science of mathematics, Dr.
“ Ewing, if not unrivalled, was unsurpassed by any
“ character in this country. His knowledge of the
“ learned languages was very considerable. The He-

" brew language, which is too often neglected by the " ministers of God in the present day, was one of his " favourite studies. In the mornings of his latter days, " he always read a portion of the scriptures in their " original tongue; and you could seldom enter his " room without seeing on his couch beside him his " Hebrew bible. His qualifications as a minister of " the gospel were many and eminent. Science was to " him a powerful assistant in the labours of his sacred " office. She was with him a handmaid to religion, " and, aided by her, he was an able champion of the " cross, both in the advocation of its cause, and in " the repulsion of the attacks of impiety and error. " He was mighty in the scriptures. To the fountain " of all religious knowledge he went for instruction. " His religious opinions were not so much founded " upon the systems written by fallible men, as upon " the scriptures of infallibility. He adopted not Calvin or Arminius, or Socinus, but the word of God " as his guide. He read, he examined, he decided for " himself. With the works of commentators and systematical writers he was familiar; he considered " them as indispensable assistants to the student, but " his veneration for these did not impress upon him " a blind obedience to their dictates: He was first " convinced by his own researches that they corresponded with the sacred volume, before he acknowledged their authority. His own investigation confirmed him in his belief of the doctrines of grace. " These were the doctrines which he preached and " which he endeavoured to impress upon the hearts " of his people.* His discourses were written with

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* " Among the practical writers he thought that Doddridge " was the best: and he thought that the method which he fol-

" accuracy; the truths which they contained were well " examined and digested before he ventured to offer " them to the public. He thought it a duty which he " owed to his God and his hearers, to think before " he spoke, to study and to ponder in private, before " he arose in the presence of an audience as the messenger from heaven. To God he looked for aid and " support; but he looked for assistance in his study, " before he trusted to divine impulse in the sacred " desk. Perhaps it may be said with truth that no " minister in this country has adopted a better method " of instruction than that which distinguished his discourses; and perhaps it may be said that none more " fully illustrated and confirmed by plain and decisive " reasoning, the passage which he chose for discussion. The style in which he embodied his conceptions was always perspicuous and occasionally ornamental. Ornament however he did not often employ. " He sometimes poured forth 'thoughts that breathed " and words that burned,' but his most usual manner was sober and temperate, such as was adopted " before him by Tillotson and Sherlock. Mere declamation was never heard from him; his discourses " were always solid and edifying, and so equal in the " scale of merit; that perhaps to no one which he " wrote in the vigour of his mind could a decided " preference be given."

" His delivery was pleasing and happy. If, in his old " age, from debility, it was not remarkable for animation, yet it was distinguished for correctness, and

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" lowed in his discourses was a good model for the practical and " devout preacher.

" could sometimes touch the finest springs of tenderness and pity.*

" The temper of Dr. Ewing was generous, and not " often ruffled. His manners and deportment were easy " and affable. Free from guile himself, he suspected not " guile in others. He had a freeness of salutation which " sometimes surprised the stranger, but which was admired by those who knew him, as it proceeded from " a heart open and honest. His talents for conversation " were remarkably entertaining. From severer studies " he could unbend, and become the companion of innocent mirth and happy gaiety. In the house of bidden " joy his religion did not wear the frown; it covered not " itself with the mantle of sorrow, but it taught him to " rejoice with those that rejoice, as well as to weep " with those that weep. He was perfectly free from " pedantry, and from every thing that bore its resemblance. In the company of philosophers, he was in " his conversation the philosopher, and with the unlettered, the man of ease and accommodation. His " talent of narration was universally admired. His observation of men and manners in this country and " abroad furnished him with many scenes and facts " which as painted and related by him were extremely " entertaining. In domestic life he was amiable. He " had all the heart of the husband; he had all the heart " of the parent; he had the full heart of a friend; surrounded by a large family, he had care and tender-

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* The discourse which he, not long before his death, delivered from the answer of Jacob to Pharaoh, bore witness to this truth. The remarks which he offered on life, and the conclusions which he drew for its improvement, were conceived and delivered with eloquence. He then spoke from the heart, and impressed the hearts of his people who saw in him the venerable patriarch bowing beneath infirmity and years.

" ness for them all. His affection for his children was " such that, even in his moments of severest study, he " received them with smiles, and laid aside his books " to partake of their infantile sports.

" Dr. Ewing was tall in his person, and, while in " younger life, was handsome and graceful. His constitution was remarkably sound and strong. He was " settled with his congregation forty years without being prevented more than once or twice by sickness " from discharging the duty of his pastoral charge. The " only serious disorder which he had, was the one which " proved fatal, and which first seized him (in 1796) six " years before his death. After his first attack he frequently preached, but never regained his strength of " body, or vigour of mind. In his sickness he discovered patience, fortitude and resignation to the will of " his heavenly Father. No murmur escaped his lips, " and his last moments were closed apparently without " a pang and without a struggle. In a good old age, in " his seventy-first year, he fell to the ground *like as a* " *shock of corn cometh in his season.* A short time before his death he buried the last of those members " of his congregation who signed his call."

November 1, 1809.

INTRODUCTION.

GENTLEMEN,

HISTORY informs us, that the Egyptian priests, the Persian magi, and Chaldeans, had acquired great knowledge in astronomy and natural philosophy long before the commencement of the christian era. They certainly carried geometry and mathematics to very great perfection. The Greeks were only scholars of the Egyptians; and possibly never understood the first principles of the Egyptian philosophy. Pythagoras seems to have been the best instructed among them; and from the small remains of his doctrine, that have reached our time, it is probable that the Egyptians were well acquainted with what is now called the Newtonian system of the universe; so that we are only regaining the principles of physics which were formerly known, and lost many ages ago.

When learning began to flourish in Europe, it received a severe and long check from the desolations of war, and the irruptions of barbarous nations.

But nothing so much prevented the propagation of knowledge, as the envious and illiberal craft of the heathen and popish priesthood. To secure their influence over the common people, they confined all learning to their own order; communicating their knowledge only to the *initiated*, i. e. such only, as had given sufficient proofs of their fidelity and taciturnity, after a long and severe trial. They proscribed all books, that might propagate real and useful knowledge, which were accordingly ordered to be brought in and burnt; as it was deemed an unpardonable sin to read them. Whoever attempted to put mankind upon a free and impartial inquiry into the truth of popular opinions was sure to undergo a cruel and unmerciful persecution. Socrates, justly accounted the wisest and best man that ever appeared in the heathen world, was persecuted and condemned to death, as an enemy to the established religion of his country, and a corrupter of the morals of the youth. Copernicus, who revived the ancient system of Pythagoras, which was afterwards demonstrated by sir Isaac Newton, dared not to publish his opinions, till near his death, when he thought himself nearly beyond the reach of their persecution. Galileo, who first applied the telescope to astronomical purposes, thereby confirmed this system. The nobility and gentry of Italy flocked to his house to view the planets; and they saw with their own eyes, that the planets were round globular bodies, similar to our earth, and the satellites of Jupiter, revolving round him in the same manner as the moon revolves round our earth, while both these primary planets regarded the sun as the center of their annual motions. The priests, who had engrossed all the schools of Europe into their hands, and could not bear to be convicted of teaching errors in philosophy, were soon alarmed and clapped this heretical astronomer into the inquisition; where, to free himself from the rack, he was obliged to recant and deny the truth of what he and many thousands had seen with their own eyes.

Knowing the importance of instilling strong prejudices into tender minds, and the force of these early prejudices, during the whole course of life, they engrossed the education of youth, solely into their own hands, in all the schools of Europe. And to divert the inquisitive mind from the search after real knowledge, which would soon overthrow a dominion, which was only supported by the ignorance of the people, they introduced into their schools a kind of learning of things, which existed nowhere but in their own imagination: abstracted notions, a multiplicity of terms and hard words which have no meaning but to cover ignorance, perplexed definitions, distinctions without a real difference, and endless disputes about mysterious trifles, of no real use either to the promotion of knowledge, or the advancement of human happiness.

Thus the schoolmen continued dictators in the republic of literature for many ages. The master, with an air of confidence, asserted the unintelligible doctrine; and the unenlightened pupil received it with all the submission that was due to an irresistible demonstration; until the reformation in religion took place, and many countries shook off the authority of the pope; which unfettered the human mind, and introduced a more free and rational method of investigation. In the year 1640 Rene Descartes published his philosophy, and asserted boldly, that we must receive nothing in philosophy, upon mere authority; but on the contrary must doubt of every proposition, until we have sufficient evidence of its truth. He has the honour of introducing a more safe and liberal method of philosophizing, and thereby of paving the way for all the discoveries, that have since been made in physics. He indeed carried the humour of *doubting* a little too far. The only self-evident proposition, which he admitted, was, "*I think*, *therefore I am.*" But, certainly I can no more doubt of your existence, who sit before me, than I can of my own. He had not the advantage of the accurate observations and numerous experiments, that have been since made, whereby the falsehood of many received errors has been detected, and many phenomena discovered, of which he had no knowledge. It is no wonder, then, that he should have failed in his solution of many phenomena; and that his system of physics should appear, at this day, rather an agreeable romance, than a true history of nature and her operations. However, he had the honour of exciting that passion for new discoveries, and that liberty of philosophizing, which, from his time to the present, has so much advanced the general interests of learning; and he was the first, among the moderns, who made use of geometry in physical researches.

This unusual liberty of investigation, in philosophical matters, alarmed the schoolmen, whose assertions had hitherto passed for demonstrations, on the single authority of their names; and who could not bear to be convicted of teaching errors and trifling fooleries, instead of substantial knowledge. And as they were not able to withstand the force of his demonstrations, these zealous doctors had recourse to an easier method of suppressing the progress of his philosophy, which was, to brand him with the odious name of an infidel and atheist; and thereby to excite the indignation of an enraged populace against him; at the same time that they petitioned the parliament of Paris, to interpose with their authority, to put an effectual stop to the further spreading of his dangerous heresy.

Had this excellent philosopher adhered to his own rules, he would have made much greater progress in the knowledge of nature. But he made too hasty transitions from his first and slightest observations, to general axioms; which he adopted as certain and general principles, without any farther examination: whereas he should have proceeded cautiously, step by step, and have reserved the most general principles, for the last result of his inquiries. Philosophers, after him, proceeded with still greater negligence, in the investigation of natural causes, and therefore contributed but very little to the increase of real and useful knowledge. The custom was too general, to frame hypotheses and conjectures; and if upon comparing them with the phenomena that first occurred, there appeared some kind of agreement, it was held sufficient. Yet nothing less was undertaken than the fabrication of whole systems, and the immediate fathoming of the greatest depths of nature: as if the secret causes of natural effects, contrived by infinite wisdom, could be searched out by the slightest endeavours of our weak and shallow understandings. Whereas the only method, that can afford us any good prospect of success is to make our inquiries with the greatest caution, and by slow degrees. And after all our care and labour, in explaining the works of the incomprehensible architect, we may rationally expect that the greatest part of nature will forever remain beyond our reach.

When false axioms or fundamental principles are thus incautiously adopted, without sufficient examination, and objections against them are evaded by frivolous distinctions, the united endeavours of future ages cannot make this erroneous method of investigation successful, until the original error be corrected. This neglect of the proper means to enlarge our knowledge, and presumption to attempt, what was beyond the reach of our limited faculties, lord Bacon, that accurate philosopher, judiciously observes to be the principal obstruction to the progress of science. In the spacious fields of nature, if once we forsake the true path, we are in the most imminent danger of wandering on in uncertainty and error: and even when we set out in the true way of philosophical inquiry, we are apt to be entangled in various errors, from the prejudices to which the human mind is subject through the very condition of humanity, from the weakness of our senses and faculties, from the authority of great names, from our partiality for ourselves and our own inventions or opinions, and from a lax and indefinite use of words in common conversation: so that even a strict definition of the terms we use can hardly remedy this inconvenience. These are fruitful sources of philosophical errors: and the unphilosophical method, of establishing fundamental principles from a few hasty observations and experiments, is so far from assisting us in overcoming these prejudices, that it is rather calculated to rivet them more firmly in the mind.

What else but mistakes and errors can this bold and presumptuous treatment of nature produce? Have we the wisdom necessary to frame a world, that we should think it so easy a matter to enter into the secret springs of nature, and to discover the original causes of her hidden operations? What chimeras, what monsters, has not this preposterous method produced? What schemes and hypotheses of the subtilest wits, has not a stricter inquiry into nature overthrown and manifested to be ridiculous and absurd? Every new discovery lets us see more and more of the weakness of our faculties, and the absurdity of our guesses. Dr. Harvey, by that one discovery of the circulation of the blood, has dissipated all the speculations and reasonings of many ages, upon the animal economy. Assellius, by detecting the lacteal veins, showed how little grounds physicians and philosophers had for conjecturing that the nutritive part of the aliment was absorbed by the mouths of the veins spread upon the bowels. And Pecquet, by finding out the thoracic duct, as evidently proved the vanity of the opinion, that the alimental juice was immediately conveyed into the liver, and there converted into blood.

As these things show us the great absurdity of proceeding on conjectures in philosophy by informing us how far the operations of nature are above our shallow conceptions; so on the other hand, the instances of success, we meet with in a more judicious method, show us that our bountiful Creator has not left us wholly without the means of delighting ourselves with the contemplation of his wisdom! And what surprising advancements in the knowledge of nature may be made by philosophical inquiries, by pursuing a proper course, when these searches are conducted by a genius equal to so divine a work, will be best understood by considering the discoveries of the incomparable sir Isaac Newton. And that we may have as good an idea as we can, of these discoveries, which have laid open to us more of the secrets of nature, than was known before him, I must beg your attention to the principles, on which he proceeds. For without a clear conception of these, it is impossible to form any true idea of the singular excellence of the inventions of this great philosopher.

The principles of the philosophy, we are about to explain, are, upon no consideration, to indulge conjectures concerning the power and laws of nature, but to use our utmost endeavours to investigate the true laws by which the constitution of things is regulated. The philosopher's first business is to distinguish between what is within his power, and what is beyond his reach; to assume no greater degree of knowledge, than what he already has; but to advance, by slow and cautious steps, in the search of natural causes; and to secure to himself the knowledge of the most immediate cause of any phenomenon, before he extends his views farther to causes more remote. This is the method, in which philosophy ought to be cultivated. Nor has the philosopher any reason to be ashamed that he knows no more than the immediate cause; since he may make very useful deductions from the degree of knowledge he has already acquired; and by proceeding cautiously and slowly he may at length arrive at the extent of his wishes. If he have sufficiently discovered and proved any one cause, he has so far entered into the real constitution of things, and has laid a safe foundation, for supporting and facilitating the labours of others, in their search after more distant causes; while, in the mean time, he may apply his knowledge to useful purposes. It was no unuseful discovery, that the ascent of water in pumps was owing to the pressure of the air by its weight or spring; although the causes, which render the air heavy and elastic, are yet unknown. For although we are ignorant of the original from whence these powers of the air are derived, yet the bare knowledge of these powers will afford us much advantage. When we know with certainty the degree of force, with which they act, we may know the utmost that is to be expected from them, and be prevented from spending much time and labour, in vain attempts, to perfect these machines, beyond the limits prescribed to them by nature, or to raise water to a greater height, than the known weight or spring of the air can sustain. A notable example of this you have, in the fruitless endeavours of philosophers to perfect the refracting telescope, by grinding the glasses into various new figures, until sir Isaac Newton demonstrated that they were limited by another cause, than was suspected before, namely the different refrangibility of the rays of light, and not by the figure of the glasses. This discovery caused every optician and even sir Isaac Newton himself to despair of perfecting the refracting telescope, and set them upon the construction of reflectors, until another discovery, of the different refracting and dispersing powers of different kinds of glass, was made by Mr. Dolland, which has since enabled him to bring them to the wished for perfection.

But farther, it was no trifling discovery, that the contraction of the muscles of animals put their limbs in motion; though the original cause of that contraction remains still a secret: as it has given rise to many curious speculations on the force and dispositions of the muscles, and opened a wide prospect into the wisdom and goodness of the Creator discoverable in the animal economy. And the discovery, that the nerves have a considerable share in this action, leads us a step nearer to the original cause, gives us a more enlarged view of the subject, and affords considerable assistance in restoring this motion, when it is impaired, by pointing out the seat of the injuries to which it is exposed. Now to neglect all these discoveries, because we cannot advance farther, is plainly ridiculous.

Est aliquid prodire tenus, si non datur ultra. Hor.

It is acknowledged by all, that Galileo greatly improved philosophy by showing that the power which we call *gravity*, causes bodies to descend to the earth with an accelerated velocity, and to describe the curve, called a parabola, when projected forwards; yet we are ignorant of the cause of gravity, or what makes a body heavy. But although we are unacquainted with the secret spring of this power, we can estimate its effects, and derive many signal advantages, in the art of gunnery and the motion of spouting fluids, from the knowledge we have already obtained. Sir Isaac Newton has greatly increased our knowledge of the solar system, by demonstrating, upon indisputable geometrical principles, that this power of gravitation extends to the moon and to the most distant body in the solar system, regulating and determining all their complicated motions. As Galileo laid open the effects of this unknown power in the motion of bodies near the surface of the earth, more fully than any philosopher before his time; the towering genius of the incomparable Newton pursued it through all the planetary regions, nay to the most distant retreats of the flaming comets, and thereby exhibited the various intricacies of their motions to open view, which no astronomer from observation alone could ever have explained: as they never dreamt of the cause, and consequently never examined the principle, by which all their motions were governed.

Yet still we remain ignorant of the cause of gravitation, and cannot tell whether it be an essential property of all matter, or is only produced by the operation of some cause, which is hitherto undiscovered. But because the proper method of philosophizing, in the investigation of natural causes, requires that we should not indulge groundless conjectures, but rather keep within the extent of our faculties, and be more ready to acknowledge their weakness than to conceal it under the vain ostentation of hard names, occult qualities, and groundless hypotheses,—shall we therefore reject the evident advantages we have derived in geography, navigation and astronomy, merely because we are not able to account for the operation of this wonderful principle? No: rather let us thankfully receive the improvements that have been made, and proceed with the same caution and circumspection, in the investigation of the hidden powers of nature, which has enabled others, and particularly the prince of philosophers, the great sir Isaac Newton, to make such amazing progress in the knowledge of nature.

Although from his accurate observations, sir Isaac Newton found reason to conclude that gravitation was a universal property belonging to all the perceptible bodies in the universe, and to every particle of matter; yet he nowhere asserts this property to be essential, but on the contrary has given some hints as to the cause of it; which he has left for the investigation of others. It is extremely difficult to determine what properties are essential to matter, and what depend upon the frame and composition of bodies alone: what properties belong to every particle of matter however altered and changed by composition or dissolution, and what only arise from the union and composition of these particles with other portions of matter. And since we have no other method of making discoveries in philosophy, but by gradual inquiries into nature, we must first admit all the properties we observe, and then endeavour carefully to distinguish between those, wherewith the very substances themselves are induced, and those, which only exist in appearance; while this appearance at the same time depends upon the structure of compound bodies. And we must never forget that as some of the properties of matter have been gradually discovered from time to time, more of them may yet be found out, by the researches of future philosophers; and that possibly we are not provided with sufficient methods of perception, to discover them all. But still, whatever properties occur, whose existence we have fully and clearly proved, are to be considered as the proper objects of natural philosophy; the design of which is to explain the various phenomena of the natural world. And it is the business of the philosopher, by the most accurate and careful methods in his power, to distinguish between those, that are original and essential to every particle of matter, and those, that may be derived from the operation of some more remote cause, perhaps yet unknown to him and the world.

In this method, we shall endeavour to conduct your inquiries into the laws of nature; and give you such solutions of the various phenomena, that we observe in the material world, as have been generally received by philosophers of the most approved characters; requesting you, in the mean time, to admit of no solution, but what appears to be founded on the established principles and properties of matter, and sufficiently confirmed, by an adequate number of observations and experiments. And although we shall meet with many phenomena in this extensive science, which we are yet unable to explain; yet we hope to lay before you the first principles of natural philosophy, and to give you so many solutions, in its various branches, as will be sufficient to open to your view the conduct of nature in her leading operations; to give you a relish for philosophical inquiries; and to excite that thirst for farther investigations; by which a foundation may be laid, for your future additions to the stock of general knowledge, and your acquiring the singular honour of becoming the patrons of science, the ornaments of the republic of letters, and the benefactors of mankind.

And there is ample room for the exercise of the most penetrating genius, and unwearied application. There is more in a single vegetable, than has yet been adequately accounted for by the ablest philosopher that ever lived. And when you survey the extensive fields of useful knowledge, which are opening to your investigation, through the unexplored deserts of this new world, and the improvements that are daily making by the sons of science, you must naturally feel your hearts swell with an honest and laudable ambition, to distinguish yourselves amongst the most celebrated inventors of those useful arts, which have a tendency to embellish and exalt the understanding, alleviate the miseries of human nature, and promote the happiness of mankind. Animated with this expectation, I cannot but promise myself your patient and candid attention; and I shall account myself happy, if I shall be able, in the course of these lectures, to contribute any thing to the republic of letters, in your future advancement and distinction in the world.

LECTURES.

NATURAL experimental philosophy is a science, which investigates the reason and causes of the various phenomena which occur in the material world, and makes the truth of the solutions given, evident to the senses, by plain and adequate experiments.

By the phenomena of nature, we mean all the situations, motions, and appearances of natural bodies, which are evident to the senses, and not immediately dependent upon the voluntary agency of an intelligent being; by which we do not exclude those appearances, which are found in animal bodies, so far as they do not depend upon the volition of the animal, but arise by the instrumentality of second causes. All these situations and motions are produced by certain fixed and determinate rules, which are denominated the *laws* of nature; as they are the invariable appointment of the First Cause of all things, whereby he determined that certain natural causes should always, in the same circumstances, invariably produce the same effects. And in the production of the effect, they constantly observe this fundamental rule: that the effect shall be always proportionable to the whole power and efficiency of the cause. Thus the same quantity of fire always burns, by a natural necessity, with the same degree of intensity; and gravitation always causes a heavy body to descend from a given height with the same invariable force. In this consists the difference between a *natural* and an *intelligent* cause: as the latter may produce its effects, very much disproportioned to the whole of its power. Thus a man may raise a greater or less weight, just as he pleases. The one, acting with evident design, may alter, increase, or lessen the effect at pleasure; but the other, acting without design, has no such power to alter the effects produced. It is from this invariable proportion between the cause and the effect in natural agents that we estimate the force of the cause, from the known quantity of the effect, and vice versa; and therefore it should be carefully observed, as we shall have frequent occasion for it, in our subsequent lectures, while we endeavour to investigate and ascertain the cause of any particular effect from the established proportion that must subsist between them.

In the investigation of the laws of nature, many philosophers proceeded upon false and sophistical principles, until the incomparable Newton appeared, and laid down the few following rules of philosophizing; which are so plain and solid, as to gain the approbation of every intelligent person, who desires to make any safe progress in the knowledge of nature.

1. That no more causes are to be admitted, than what are true and sufficient to explain the phenomena.

A variety of causes, to produce the same effect, is needless, and therefore cannot be supposed to take place, in the workmanship of Him, who does nothing in vain. As we have no safe method of investigating any cause, but by its known effects; when one cause is sufficient to explain the phenomena, we have no possible means of knowing that there are two or more;

and therefore it is unphilosophical, to assume the liberty of imagining that there really does exist more than one cause of the effect.

2. That natural effects of the same kind are to be ascribed to the same cause.

This is a direct consequence of the first rule, and needs only to be explained. Thus the descent of heavy bodies in different parts of the world is to be ascribed to the same cause. Respiration in men and other animals is to be accounted for on the same principles. The reflection of light from different bodies is owing to the same cause.

3. That those qualities of natural bodies which cannot be lessened or augmented in the bodies, on which we can make experiments, are to be deemed the universal properties of all bodies whatsoever. Without this rule we could make no progress in the study of nature. We have no means of knowing the properties of bodies, but by experiments, and from thence drawing conclusions and reasoning by analogy, concerning bodies that are beyond the reach of our examination. The only caution necessary in this case is, that the experiments and observations, from whence we would draw conclusions, be numerous enough; and that proper regard be paid to the objections that occur. This is the method of *induction*, which is farther explained by the following precept.

4. That whatsoever is thus collected by induction be received, notwithstanding any conjectural hypothesis to the contrary, until it be contradicted or limited by other more adequate observations on nature.

Mere hypotheses, the figments of a warm imagination, unsupported by observation and experiment, have no place in the newtonian philosophy. Although the arguing from induction is no demonstration of general conclusions, yet it is the best that the nature of the thing admits of in many cases, where mathematics cannot be applied. It is not therefore to be rejected, but used with caution, remembering that the degree of evidence it affords is always proportioned to the completeness of the induction.

If we survey the material world by these rules, we shall find that we have no sufficient experiments, to prove that there are different kinds of matter in the universe, but rather that all the diversity, which we observe in natural bodies, arises from the various modifications and different connexions of the same primogenial particles; as all bodies, on which experiments have been made, may be reduced, by a chemical analysis, partly into water, or air, or oil, or earth, or salt: which have therefore been denominated the five elements or principles of all bodies; yet all these original particles of matter have the same common properties.

Matter thus variously modified is found, as far as our experiments reach, to be possessed of these common and universal properties; viz. EXTENSION, SOLIDITY, FIGURABILITY, DIVISIBILITY, a CAPACITY OF MOTION, a VIS INERTIÆ, or power of *resisting* any effort to change its state of motion or rest, ATTRACTION, and REPULSION. These must be explained.

EXTENSION is that property of body, whereby it occupies and fills a certain portion of space, which portion is denominated the place of the body. If this extension lie in one direction it is called a line; if in two, a superficies; and if in three, it is termed a solid. It is this, that constitutes the magnitude of a body: and as all bodies, on which we can make experiments, are evidently possessed of this property, we conclude, by the third and fourth rules of philosophizing, that the stars have this property also; although no experiments confirm it; as our best telescopes leave them without any sensible magnitude, and represent them as shining points. The sensible magnitude which they seem to have to the naked eye is only an optical illusion; and the smallest particles of matter, that float in our atmosphere, are sufficient to hide them for a moment from our view, which by the by occasion their twinkling appearance.

Solidity is that property of body, whereby it excludes every other body out of the same place at the same time. According to this definition, the softest bodies are equally solid with the hardest; as they both equally exclude every other body from the place they occupy. We acquire the idea of solidity only by the touch, and transfer it to such bodies as we cannot feel. It does not arise from the idea of extension: the image of an object, which has no solidity, appears equally extended with the object itself, and may appear also in the same place. Air and water are equally solid with the hardest metals.

Figurability is that property of body, whereby it is rendered of some particular shape. This quality arises from its limited extension. Since matter cannot be infinite, it must be circumscribed within certain bounds, which determine its figure. Upon this property many important phenomena of nature depend. Different areas and magnitudes may be comprehended under the same lines or superficies. The hardness and softness of bodies depend upon the different figures of the small particles, whereby they may touch in more or fewer points. From this property of body also, we derive a probable argument for a *vacuum.* Because it is demonstrable from the principles of geometry, that no assemblage of figures can fill any given space, without any remaining vacuity, but some one or more of the five platonic bodies, viz. the *tetrahedron* of 4 equal triangular sides, the *hexahedron* of 6 equal square sides, the *octahedron* of 8 equal triangular sides, the *dodecahedron* of 12 equal pentagonal sides, and the *icosahedron* of 20 equal triangular sides. Now the assertors of a *plenum* are under the necessity of proving, that, although matter is liable to an infinite variety of forms, yet every particle must be of one or other of the five above mentioned; for otherwise there must of necessity be a vacuum.

DIVISIBILITY is that property of body, whereby its parts are separable from each other. That matter is divisible *ad infinitum* is capable of being demonstrated various ways. Let a given particle be placed between two parallel lines; and from a given point in one of them let diagonal lines be drawn to the other crossing and dividing the given particle of matter. Now it is evident that the longer these diagonal lines are, the more of the particle will be divided off, and yet that they never can coincide with the parallel line from whence they are drawn, and therefore can never pass through the extremity of the particle and divide it off.

This is true in theory: and in many of the operations of nature and art we observe surprising divisions of matter into its minute parts.* What an inconceivable number of particles flow from an odoriferous body, for many days together, without any sensible diminution of its weight or bulk? How minutely are the particles of tallow divided, that are emitted from a lighted candle in a single vibration? As a candle can be seen at the distance of four miles, the flaming particles of tallow are disseminated through a sphere of eight miles in diameter in an instant. A single grain of copper dissolved in the spirit of sal ammoniac and diluted with water, is thereby divided into many millions of times its own bulk, and tinges the whole. A grain of gold may be beaten into fifty inches square and then divided into two millions of visible parts: and by the help of a microscope which magnifies the diameters of objects but 140 times, this grain may be divided into 40 thousand millions of parts. Nay a grain of gold may be divided into twenty times the number above mentioned, by first gilding the eighth part of an ounce of silver with it, and then drawing it out into a wire of more than seventeen hundred feet long. By the microscope we discover animalcules which lie beyond the reach of the most penetrating eye; the particles of whose circulating juices must be inconceivably small; and yet the particles of light are as much smaller than these, as *they* are smaller than the whole earth. For otherwise, coming from the sun with

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* One grain of gold may be divided into two millions of parts; and a cubic inch into nearly nine billions and five hundred and twenty four millions of parts, each distinctly visible to the naked eye. If a cubic inch were divided into a million of parts, each of

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these parts would contain more than two millions of such particles as a single grain of copper might be divided into, and yet all be visible to the naked eye. Nay, if a cubic inch were divided into forty trillions of parts, the particles of some odoriferous bodies would still be less than these. Take the smallest grain of sand that is visible to the naked eye, and it would contain more of the globules that float in the fluids of animalculæ discovered by the microscope, than two hundred and sixty six of the highest mountains in the world would contain of such grains of sand.

the amazing velocity of 12 millions of miles in a minute, they would tear in pieces the tender coats of the strongest eye.

From this infinite divisibility of matter,* we deduce the surprising corollary: viz. That it is possible, that any given particle of matter, however small, may be formed into a sphere, that shall fill any given space, however large, and yet not leave a pore in the whole, that shall exceed a given magnitude.

That matter is CAPABLE OF BEING MOVED or translated from one place to another, none have ever seriously denied; not even the cartesians, who notwithstanding assert, that there is no such thing in nature as a vacuum. This property of matter, which is so evident to all our senses, necessarily proves that there must be some space void of all matter: for when a body moves, it must be either through a space absolutely full of matter, or a vacuum; or at least the motion must be in a circle. If the first be asserted, then two bodies may be in the same place at the same time; which is absurd, and contrary to all our ideas of matter: and the last is contrary to all our experience; and therefore there must be such a thing as a vacuum.

The VIS INERTIÆ of a body, which may be termed its *resistance*, is that property, whereby it endeavours to continue in its state of motion or rest, and resists the actions and impressions of all other bodies, which tend to generate or destroy motion in it. This property does not depend upon its gravity or tendency to its proper center. For if there were but one body in the universe, it would require a certain degree of force to put it in motion, and the same to destroy that motion again: and whether it be acted upon by gravity or not, it will, when put in motion, pass over the same space in a given time in the direction of the impressed force; for otherwise the effect will not be proportional to the moving force; which must always be the case.

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* The infinite divisibility of matter is to be considered as only existing in the mind, where it may indeed be divided into a number of parts greater than any that can be assigned. And the difficulty of conceiving of this proposition is no solid objection to its truth.

ATTRACTION and REPULSION are properties of bodies, whereby they accede to, or recede from, each other. This property was originally given to matter by the omnipotent power of God, and does not seem necessary to its existence. We do not presume to assign the cause of this property, but only to prove its existence and reality, in all the bodies that fall under our notice. Various hypotheses have been formed by ingenious philosophers, to account for this strange phenomenon; but as they are all unsupported by experiment and observation, they should have no place in a system of philosophy, which appeals to demonstration and experiment. We cannot indeed conceive of a body acting where it is not, any more than of its acting after it has ceased to exist; and therefore it is supposed that there may be some subtile and elastic medium diffused through our whole system, and pervading the pores of all bodies, by the tremors and undulations of which the attractive and repulsive virtue of bodies is excited and propagated from one to another. But however this may be, it is our business at present only to explain the effects and laws of its operation. In order to which we observe,—That any power or virtue proceeding from a body, in right lines and in all directions, must decrease in its strength and in-whose diameter is but one half of that quantity. This is a necessary consequence of that law,—That the power of attraction is proportional to the quantity of attracting surface in contact with the fluid. Now according to this law, the smallest tube can raise but half the quantity of fluid suspended by the other, as the number of the particles in its internal circumference is but half the number of particles in the other. But the altitude of the fluid in the lesser tube must be just double to what it is in the larger, in order to have the quantity in the lesser tube just one half of what it is in the larger; as the quantity increases with the square of the diameter. The whole fluid in the tube is suspended by the attractive force of a ring of particles in the tube near the surface of the fluid, as the lower parts of the tube draw the fluid as much downwards as upwards; so that it is of no consequence, what the form of the immersed part of the tube may be, whether straight or crooked, wide or narrow. This ring of particles having a determinate force of attraction, can suspend only a certain quantity of the fluid, whose gravity is an exact counterpoise to the attraction of the tube. Hence the decreasing velocity of the ascending fluid, as the quantity raised gradually approaches nearer to an equality with the attractive force of the tube. The same principle accounts for the form of the hyperbolic curve in the water rising between two glass planes set together in a small angle; as every column may be considered as a capillary tube, differing in diameter from the contiguous column, according to its distance from the angular point of the planes.

The capillary syphon works by the same laws, that are observed in a capillary tube, raising the fluid to the bend of the syphon, by the force of corpuscular attraction; and the same power continuing to exert its energy in conjunction with the gravity of the fluid, causes the fluid to descend through the longer leg of the syphon till the whole be decanted.

The *philter* is only a compound capillary syphon, the threads of which it consists lying near together form long slender vacuities, resembling so many capillary tubes, which attract and raise the fluid. The wick of a candle or lamp is no other than a philter, which attracts the particles of oil or melted tallow. Such also are the tubular fibers in the roots of trees and vegetables, which act as so many philters, attract the moisture of the earth, and imbibe the nutritive juices destined for the growth of vegetables.

Thus also the fine ramifications of the lacteal, lymphatic and sanguiferous vessels, disposed through all the substance of the glands and viscera, are so many fine capillary tubes, which assist the circulation of the blood and juices in an animal body, in order to their various secretions; which is an effect supposed vastly too great for the pulsive force of the heart.

Hence too, the more subtile parts of medicines, used for the cure of diseases, are incorporated with the juices of the body, being attracted and carried to the remotest members by the force of these capillary tubes. Thousands of these vessels terminating in the pores of the skin, conduct the juices to the surface of the body, where they must be expressed by the action of the solids, and appear on the skin in large drops of sweat, or being greatly rarified by the natural heat of the body, are attracted by the air, and carried off in an imperceptible vapour, called the insensible *perspiration.*

The motion of a drop of oil towards the angular point of two planes, or in a tapering tube towards the smaller end, shows the construction and operation of the inhaling vessels of the skin. The larger orifice of these vessels terminating in the surface of the body, attract and inhale the humid particles that either float in the air, or otherwise come into contact with them, and convey them to the internal parts of the body. Hence we see the rationale of all external applications in the cure of diseases, and the way of receiving infectious disorders.

The air is frequently impregnated with particles of a poisonous quality collected from different vegetables, stagnant waters, putrefying animals, infirmaries, hospitals, ships, gaols, and the private chambers of the sick, which are pestilential to the human frame, and contain all the seeds of diseases, to which the animal economy is subject; and which being mixed with the blood and juices of the body, by first adhering to the lungs in inspiration, and to the mouths of the fine capillary tubes that terminate in the skin, and by them communicated to the vital parts, would soon bring on the dissolution of the whole frame, were it not for the speedy assistance of the skilful physician counteracting their fatal effects.

This accounts for the manner of our receiving such diseases as are commonly called contagious and pestilential, and possibly for many others, which are not generally accounted of that kind. On the same principles we may account for the sudden death of persons, from breathing in adust air, or in that which is impregnated with pestilential damps and suffocating exhalations, so fatally experienced in wells, mines, and subterraneous places.

Some may imagine that some of the phenomena mentioned above may be accounted for from the pressure of the air, such as the spherical figure of the drops of water, the adhesion of the leaden balls and of glass planes, &c. But this cannot be the true cause, as the appearances are the same in vacuo, as in open air.

This sphere of attraction between the particles of different bodies has certain limits, and does not extend far beyond contact: and where the sphere of attraction ends, the sphere of repulsion begins, as appears from the disunion of water and oil, of mercury and glass, the walking of some insects on water without wetting their feet, and the rolling of water off the feathers of fowls, and leaving them dry. The particles of these bodies are placed beyond the sphere of each other's attraction; and cannot be brought nearer to each other but by the addition of another body, which attracts the particles of both. Thus oil is mixed and incorporated with water by the addition of soap or gall or flour. Mercury and glass are brought into contact by lead or tin. Hence the praxis of foliating glass; the mercury will incorporate with lead or a thin sheet of tin, and fill up the inequalities of its surface, so that the air being excluded from its pores, by the mercury, the tinfoil is brought into close contact with the glass and adheres to it. Mercury is used, because it is of a smooth surface, solid and opake, and therefore fit for causing the regular reflection of light. Mercury is poured upon a thin sheet of tin, laid even upon a table, and because of the strong attraction between the particles of both, the mercury will enter into the pores of the tin and dissolve its texture. The particles of tin, being lighter than mercury, will rise on the surface like a scum, which must be carefully removed with a little cotton; and a sheet of clean paper being laid on the mercury, the glass plate, being previously cleaned, dried and moderately heated, is laid on the paper, and the paper is gradually drawn from between the mercury and glass, to carry off with it the scum that is continually rising through the mercury. The mercury now incorporated with the particles of the lead or tin will adhere to the glass, which it repelled before, and cause a beautiful and regular reflection of the rays of light. On the same principles, but by a different praxis, we foliate the inside of a glass globe. For this purpose, melt one ounce of tin or lead with an ounce of marcasite of silver, and add two ounces of mercury. When the mercury begins to evaporate with the heat, take it off from the fire and pour the mixture into cold water. When the mixture is cold, let it be strained through a linen cloth doubled, and poured into the globe by means of a funnel reaching down to the bottom of the globe, to prevent its sparkling, and it will closely adhere to the globe, wheresoever it touches it, provided the globe be first heated, dry and clean. This is necessary only for the beauty and regularity of the designed reflection.

From the *repulsion*, which takes place between some bodies, such as oil and water, mercury and iron, we account for the depression of the water about a ball of tallow, or an oiled glass bubble floating in it, and the depression of the mercury round the iron. From the attraction between the glass and water, we see the reason, why the water rises round the side of the glass vessel, when only partially filled with it, and forms a concave surface; and also why the water will assume a very convex surface, and at last stand above the edge of the glass, without running over, when more is poured into it, than it naturally contains. This also accounts for the motion of a clean glass bubble towards, or from the side of the vessel, according as the water has a concave or convex surface; the bubble being more attracted on that side, on which the water rises highest. The contrary happens in each case, where there is a repulsion between the bubble and the fluid, in which it floats. Thus if the bubbles be oiled and float in water, or clean and float in oil, they will be more forcibly repelled on that side, on which the fluid rises highest. For the same reason, two oiled, or two clean, glass bubbles, when floating near each other on the level surface of water, will accede towards each other, but they will separate and fly from each other, if one of them be oiled and the other clean.

It is by the force of this attraction that the pen retains the ink, and the paper draws it forth in writing; and from the natural repulsion between oil and water, the paper, when oiled and greasy, refuses to receive the ink. It is likewise by the same power of attraction and repulsion in bodies, that the rays of light are variously refracted and reflected; as we shall see in optics.

The *hardness* or *softness* of bodies depends partly on the number of particles in contact, and partly on their fitting each other so exactly as not easily to admit of any motion among themselves, from any external pressure or cause whatsoever.

Fluidity is that state of a body, in which the force, that unites the particles together into one mass or community, is just balanced by the repulsive power which causes them to fly from and repel each other; which repulsive force very probably depends upon the action of heat: as we shall more fully see, when we come to consider the nature and property of fluids. Fluidity seems to be the greatest degree of softness, where the particles must be generally of a spherical form, that they may touch only in a few points and easily yield to any partial and unequal pressure. Doubtless many of them are placed beyond the sphere of each other's attraction and so repel each other; and when this balance between the attractive and repulsive powers is destroyed, either by the too violent action of heat, or the want of it, the fluid, in the first case, is converted into an elastic vapour, and in the other case, into a fixed and hard body, as is evidently seen in water and mercury. This produces the melting or fusion of metals; for when their particles are so far separated by the action of fire, as to touch one another but very slightly or not at all, and yet not be out of the sphere of each other's attraction, they assume the appearance of a fluid, and have all its wonderful properties.

The *elasticity* of bodies depends upon the same principle of corpuscular attraction. In the bending of a spring, the particles on the convex side are drawn apart, while those on the concave side are crowded together, so that the attraction of the particles on the convex side restores the spring to its former situation; and it is very probably assisted in this operation by the repulsion of the particles on the other side of the spring. When the particles of fixed and hard bodies are separated beyond the sphere of their attraction, the body breaks; and we cannot again reduce the particles within their sphere of attraction, but either by melting the body, if it be capable of fusion, or by the intervention of glue or solder, which entering the pores of the wood or metal and uniting closely with these substances, hold them fast together, by their attractive force.

The *solution* of one body in another is also an effect of this principle of attraction. Silver, copper, tin, &c. are dissolved by aqua fortis, gold by aqua regia, sugar and all kinds of salts by water, and other bodies by other menstruums. When the particles of the fluid and solid attract each other more strongly, than the particles of either attract each other separately; and at the same time the pores of the solid are large enough to admit the particles of the fluid to enter into them; the solid will then be dissolved in the fluid. Suppose a particle of the solid adhering to the body with a certain force; yet a particle of the fluid, being attracted by a superior force, will rush upon it with considerable impetus, while other particles of the fluid, in like manner entering the pores of the solid, will surround the forementioned particle of the solid, and adhering to it with superior force will separate it from the solid, to which it was united by a fainter attraction. Thus we must conceive of every particle of the solid, until the whole be dissolved, or the fluid be *saturated;* which is the case when every particle of the fluid has filled its sphere of attraction with those of the solid; for then it will dissolve no more. That this solution is effected by the mutual attraction of the particles is evident from the visible motion, effervescence and ebullition produced in the fluid. Nay sometimes the internal motion is so violent, as to produce a great degree of heat and flame, and even a sudden explosion. The oil of carroway-seed poured on the compound spirit of nitre kindles instantly into flame, with prodigious force, even in vacuo. Equal quantities of steel filings and crude brimstone mixed together with a little water into the consistence of a paste, will, in five or six hours, become too warm to be handled, burn and emit a flame, and if buried under ground it will explode with a great noise, and produce an earthquake in miniature. In a mixture of brimstone, salt of tartar and saltpetre, in the proportion of one two and three to each other, the particles of the different ingredients, when melted and heated over the fire in a shovel, will attract each other so violently as to cause a great effervescence, ebullition, and at last a very loud explosion. This composition is called the *pulvis fulminans.*

Precipitation is another chemical process, which is also to be accounted for, from the doctrine of corpuscular attraction. If to the solution of any solid body, in its proper menstruum, there be added another fluid or solid body, whose particles attract those of the former fluid with a greater force than that with which they attract those of the dissolved solid; then will the particles of the dissolved solid be disengaged from the former fluid, and be suffered to precipitate or fall to the bottom, by their own gravity, in the form of a powder. Thus is the solution of gold in aqua regia precipitated by the volatile spirit of sal ammoniac, or by the oil of tartar per deliquium. This powder of gold is the *aurum fulminans*, which, when heated over the fire, explodes with a prodigious noise. But this fulminating quality is totally destroyed by the addition of half its weight of crude sulphur, powdered and mixed with it. The solution of silver is precipitated by copper, and that of copper by iron; and in general of all metals dissolved in acids, by an absorbent earth, or a volatile or fixed alkali.

All the compositions, decompositions and mixtures of the chemists, whether effected by a single or double elective attraction, may be accounted for on the same principles. When two solutions of solids in their proper menstruums are mixed together, the solids may quit the former fluids in which they were dissolved, and be suspended by the other, to which they had a stronger attraction. This is called a double elective attraction. The same corpuscular attraction takes place in the mixture of different fluids: in some, without any sensible effervescence and heat, where the attraction is but slight and weak; and in others, with both heat and flame, from the violent attrition of the particles on each other. Thus, common spirit and water mix without any sensible heat; but the compound spirit of nitre poured upon half its weight of the oil of cloves, or any ponderous oil distilled from animal or vegetable substances, or upon the oil of turpentine mixed with a little sulphur to thicken it, will suddenly take flame. The same effect is produced by pouring the same spirit on the spirit of wine highly rectified. Thus also the oil of vitriol mixed with water, or the essential oil of anniseed, produces a great heat and effervescence.

In the same manner we account for the various phenomena of burning mountains, mineral corruscations, earthquakes, hot springs, &c.; which are produced by the intestine commotions, arising from the mixture of various nitro-acid and sulphureous particles, in the bowels of the earth; which separately would remain at rest, but being brought together in considerable quantities, in large cavities, by the subterraneous streams of water, rush upon each other with such rapidity and violence, as is sufficient to produce their amazing effects.

The action of fire is sometimes necessary to produce the mixture of two bodies, whose particles naturally attract each other. Thus spirit of nitre poured upon snow, will not mix with it, but only dissolve it, and thereby produce a high degree of cold; but if the snow be first melted and then the spirit of nitre be added, it will intimately mix with the snow water, and produce a high degree of heat. Thus too, the spirit of vitriol and water will not mix together when cold; but when heated, intimately incorporate with each other. The reason is, either that the pores of the one may be too small for the admission of the particles of the other, until they are enlarged by the action of heat; or rather, that the particles of each fluid attract each other more forcibly, than they do those of the other fluid, until this attraction is diminished by the action of fire, separating them to a greater distance, and thereby allowing the particles of the other fluid to attract them with a greater force, than what subsists between the particles of either fluid, separately taken.

Fermentation is another effect of the same cause; as it consists in the intestine commotion of the particles of the fermenting liquor, arising from the attraction so often mentioned. By this, bodies are either wholly dissolved, or have their parts so altered and compounded with other matter, as to change their nature, consistence, appearance and properties, together with their principles and colours. Malt by distillation yields a spirit after fermentation, though nothing like it before could be discovered in the malt. A third part of the spirits of hartshorn mixed with the spirits of wine, will immediately unite into a white coagulated consistence, called the *offa alba*. We shall endeavour to account for the change of colour produced by fermentation and chemical mixture, and the colours of vegetables, plants and natural bodies, when we come to optics.

Evaporation, together with the various operations of nature in the production of clouds, rain, snow and dews, must be resolved into the same corpuscular attraction. A fluid evaporates in air after the same manner that a solid is dissolved in its proper menstruum. Many philosophers indeed, of no inconsiderable note, have supposed that water is evaporated in the following manner: when a particle of water has its diameter enlarged by the action of heat, so as to be ten times as great as it was before, then it must become specifically lighter than air, whose specific gravity is to that of water as 1 to 860. In this situation it will rise and float in air until it ascends to that place, where the air is of the same specific gravity with itself; where it will float indifferently in any direction, until meeting and coalescing with others, and having their diameters contracted with cold, the air is no longer able to sustain them, but suffers them to descend in the form of rain, dew, hail or snow. This theory is however liable to many weighty objections. No experiments sufficiently confirm the position that the specific gravity of a body is altered by being divided and having its surface increased; because its magnitude or quantity of matter on which its specific gravity depends, continues the same. A pound of gold, whether round, or square, or divided, is still a pound. A particle of water, however minutely divided by the action of heat, remains still heavier than air; and cannot rise in it. So that evaporation cannot be effected by these means. The doctrine of attraction and repulsion alone is sufficient to explain this phenomenon. Although the particles of air constantly repel each other, so that no power is sufficient to bring them into contact, so far as any experiments we have made can reach; yet they attract many other substances, such as water, oil, salts, earth, &c. These substances therefore will be dissolved in air, as a solid is dissolved in its proper menstruum: each particle of air assuming one or more particles, and communicating some of them, that adhere with a slighter force, as being at a greater distance, to such particles of air as yet are loaded with none; in the same manner as a fluid already saturated with a dissolved solid, will dissolve more of the solid, when the quantity of the fluid is increased; the additional particles of the fluid receiving part of the burden from those that were before saturated, or new particles from the dissolving solid. While the saturated particles of air are carrying off their burden of the evaporating fluid, they will deposit part of their burden on any other body that attracts the fluid with a superior force: and hence is the collection of watry particles on the stone walls of a building, or the sides of metallic vessels in a hot day, when evaporation is going briskly on, or the air is much loaded with moisture. But still it must be remembered that the particles of most fluids attract one another more forcibly than they attract the particles of air, until their cohesion is lessened by the action of heat: so that heat assists evaporation in the same manner that it assists the mixture of two fluids, and the solution of solids. Hence the slow evaporation of water and oils when cold, and their quick evaporation when hot. Another effect of heat is to rarify and expand the air, setting its particles at a greater distance from each other, and increasing their repulsion; by which means the air when moderately heated will support a greater quantity of vapour *invisibly*, than when it is cold: as it can more easily keep the particles of the suspended fluid at a distance from each other, and prevent them from running together into larger concretions to refract the rays of light and obstruct vision. Hence the moisture carried off from the lungs in respiration is not so visible in warm as in cold air. Hence the dissipation of fogs by the morning sun, the visible wasting of clouds in a hot and serene sky, the sudden overcasting and troubled state of the atmosphere taking place of a pleasant and clear state of the the heavens, only by a stream of cold air condensing the vapours and rendering them visible. Wind also, which is another effect of heat, greatly assists evaporation, by carrying off the saturated particles of air, and bringing fresh particles in contact with the evaporating fluid, to dissolve and carry off more. Hence a wet room is quickly dried by being exposed to the open air. Hence the sudden drying of the roads after rain, by a strong wind. Because of the limited sphere of attraction belonging to each particle of air, it can suspend but a certain quantity of moisture; and therefore when it is saturated, it can receive no more, until it has discharged part of its burden to the neighbouring air. Hence air frequently breathed, at last refuses to carry off the necessary discharge from the lungs, and becomes unfit for respiration. Earth when pulverised and reduced to dust is also attracted and suspended by the air, until it be returned again by coalescing with the drops of descending rain, or dew.

Although air attracts both water and oil separately, yet, because of the natural repulsion between these two fluids, it cannot attract and dissolve them both at the same time. Hence the suffocating nature of air impregnated with oily particles, in a chandler's shop, or from lamps and the wicks of extinguished candles: the air in this situation not being able to attract and carry off the moisture of the lungs. But when air is saturated with poisonous and pestilential particles collected from putrefying substances, hospitals or gaols, it may in respiration exchange them for the moisture of the lungs, and thereby produce various diseases, by what is called by the chemists, a single elective attraction: the lungs attracting these poisonous particles more forcibly than their own moisture, while the air attracts the moisture of the lungs with greater force than that by which it attracts these noxious particles.

An easy experiment confirms this theory of evaporation on the principles of chemical solution. Place a vessel of water under the receiver of an airpump, and when the air is drawn from the pores of the water, in which it is copiously found, it will bring with it such a quantity of vapour, as is sufficient to obscure the contents of the receiver, until being drawn off with the air, or adhering to the sides of the receiver, it will leave it quite clear and transparent, as before the operation began.

ATTRACTION OF MAGNETISM.

The next species of attraction to be explained is that of magnetism, which is found to take place only between the loadstone and iron. Its principal properties, so far as they have yet been discovered, are such as these. Every loadstone, or artificial magnet, has two poles opposite to each other, called its north and south poles; because the magnet, when at liberty to move, will naturally place itself nearly in the direction of the meridian, and always with the same end towards the north point of the heavens. If a magnet be laid on a sheet of clean paper in the midst of a parcel of steel filings, they will soon be found disposed in regular curves terminating in the poles; from whence some have been led to conjecture, that the magnetic virtue, by which it attracts iron, is emitted from one pole and received by the other. Either pole of a magnet will attract iron or steel that has not previously received any magnetic virtue; but when two magnets are brought near together, they will attract or repel each other accordingly as the poles that are presented to each other are of different or of the same denomination. Though if their magnetic virtues be very different, the same poles will attract each other, but with little force, and at a small distance. A magnet exerts its virtue at the distance of several feet, constantly decreasing in strength as the distances increase, but in very different ratios in different magnets. In some the force of attraction seems to be inversely as the squares of the distances; in others as the cubes of the distances; and in others in other proportions. Its virtue is exerted in vacuo, as well as in open air, and without any sensible diminution of its force by the interposition of the hardest bodies, whose pores it easily pervades. This virtue is copiously communicated to iron* by the touch, and to no other substance yet known. This iron when properly balanced on its center will, like the loadstone, settle itself nearly in the position of the meridian, turning that end to the north, which was touched with the south pole of the magnet. This touched iron becomes an artificial magnet and will communicate the same polarity to any other small pieces of iron, without any diminution of its own virtue, and therefore answers all the purposes of the loadstone. The polarity of the needle may be reversed, by reversing the manner of touching it; that is, if the north end of the needle be touched with the north pole of the magnet it will then turn to the south: and this may be done as often as we please. A smart stroke of a hammer will also reverse the poles of the needle, when the end of the needle is only struck. And the whole magnetic virtue may be destroyed by fire. This polarity is also given to iron various ways without the touch of the loadstone: such as by a long and hard attrition in the same direction; (hence files and drills are always found to be magnetical;)—by suffering iron bars to stand long in one position;—by conducting a violent shock of lightning or charge of electricity;—by heating the iron until it become redhot, and suddenly quenching it in water, holding it at the same time in an angle of about 75 degrees below the northern horizon, and nearly in the direction of the meridian;—and lastly, by preparing and polishing two pieces of steel, and arming one of them with a cap of the same: then, if the unarmed piece be held in the magnetic meridian, and the other be drawn along it several times, one way, it will acquire a strong degree of magnetism, and be able to communicate the same to the other piece, when the armour is changed from one to the other.

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* Iron, or soft steel will more readily receive the magnetic virtue than hard or tempered steel, but will also more readily part with it, and therefore all artificial magnets, or magnetic needles should be made of the latter.

If a needle be properly balanced on its center, so that it hangs horizontally before it is touched with the magnet, it will now dip below the horizon in an angle of some degrees, towards the north; which seems to be owing to its losing some of its gravity from the south end of the needle; as the whole needle has been found by many experiments to weigh less than before the touch. This is called the dipping needle. The south end of the needle dips below the horizon in south latitudes.

This singular property of the magnet giving polarity to the balanced needle, has been of prodigious advantage to mankind, in the arts of surveying and navigation.

It was said above, that the needle settles nearly in the direction of the meridian of any place. It will however deviate a little from this direction, and this deviation is called the *variation* of the compass or needle. There is a certain irregular curve line which surrounds the earth towards the north and south poles of the world; on any point of which line the needle points exactly to the north and settles in the meridian. This is called the line of *no variation;* on the east side of which the needle points to the westward of the meridian, and on the west side of it, it points to the eastward of the true meridian. This variation increases gradually as you recede either eastward or westward from the line of no variation until it comes to its maximum; which is found to be about 40 degrees in some places, after which it gradually decreases again until you arrive at the line of no variation. Did this variation increase or decrease in any regular manner from any known meridian, it would enable the mariner to correct his longitude at sea. Dr. Halley has proposed this use of it; and has, from a number of observations made in different parts of the ocean, constructed a chart for this purpose, which has been greatly improved by later observations. And indeed any chart of this kind will constantly stand in need of alterations from time to time; as the line of no variation is not stationary, but shifts with a slow motion towards the westward or eastward; which will occasion the lines, that represent any particular variation to shift with it. Hence the variation at any place will be different in different years, but without any known proportion between the time and the quantity of variation. Surveyors generally allow about one degree for every twenty years, in running the lines of an old survey, in this city and neighbourhood, and more in some other places. There is also a diurnal variation of the needle observed in any particular place.

From about eight or nine o'clock in the morning, every day, a good variation needle moves westward from six to fourteen minutes, till about two or three o'clock P. M., when it becomes stationary for some time, and then returns to the position it had in the morning. This daily variation is greater in summer than in winter, and is always most remarkable upon sudden changes of the temperature of the weather. There is also a very small daily variation in a fine dipping needle, but not so large as that of the horizontal needle. The reason of this seems to be the lessening of the magnetic and directive virtue by the heat of the sun, and thereby leaving the needle to verge towards the colder parts of the atmosphere.

The power of a magnet may be diminished, by not keeping its poles in the direction of the meridian of the place, or in the magnetic meridian, having its north pole directed towards the north point of the heavens;

and also by not keeping a piece of iron at or near its poles. Hence the armour of a natural magnet makes it fitter for use, as it collects the magnetic power into smaller bounds, so that it will give a stronger touch and sustain a greater weight, and have its virtue thereby longer preserved. Although the power of a magnet is not diminished by communicating its virtue to a small piece of iron; yet it may be greatly diminished by communicating it to a bar of iron of much greater size than the magnet. A smart stroke of the hammer may sometimes destroy the power of the magnet intirely, as well as reverse its poles; and the hammering of a piece of iron when cold, or rubbing two pieces of iron together, or even the bending of a piece of iron backwards and forwards until it breaks, will communicate the magnetic virtue, where it was not found before.

The magnetic power of loadstones is found to decay in time, by neglect; but when impaired may be restored again, by hanging to them as much iron as they are able to sustain. And as they will be found, after some time, able to sustain an additional quantity, by appending this additional quantity their magnetic virtue will be found at length to be surprisingly increased.

It is no easy matter to explain the phenomena of magnetism; nor indeed are we yet furnished with a sufficient number of facts and observations on this mysterious power to found any certain theory for accounting for its operations. We have little but conjecture and hypothesis on this subject. What appears to me the most probable, as best agreeing with the facts and phenomena, I will briefly deliver; leaving the establishment or refutation of the theory, to future experiments and observations.

Magnetism seems to be the peculiar property of iron and its ore alone, and to be so essential to it, that it cannot be deprived of it by any means. It seems to consist of two distinct and different powers, which in their natural state remain conjoined; and strongly attracting each other remain quiescent without any signs of magnetism; but still may be separated by friction, heat or electricity, or even by standing for a long time in a perpendicular or reclining position. When these powers are thus separated in any piece of iron, one retires towards one end of the bar, and the other towards the other end, making one end of the bar point to the north and the other to the south, when balanced on its center. In this separate state they naturally attract each other as well as before; and therefore the north end of one needle will attract the south end of another, when both have their powers separated: but each of these powers repels the same power in two different pieces of iron, for the two north ends or the two south ends powerfully repel each other.

The attraction between the loadstone and iron is mutual, as appears from their approaching each other when set to swim on any quiescent fluid; but the force of attraction does not appear to be in proportion to the strength of the loadstone but to the quantity of iron to be attracted; as an ounce of lead at the end of a small piece of iron will separate it from a loadstone that is capable of sustaining a pound of lead when fixed to the end of a large piece of iron, even though the points of contact between the iron and loadstone were the same in both experiments. It acts with the greatest force at corners, points and angles, and therein has something surprisingly similar to electricity, as it also has, in the separation of its powers, by friction, or by a shock of electricity, and in the long continuance of this separation; as well as in the conjunction of these powers again in the magnetic needle by fire, &c. by drawing the north end of the magnet over the north end of the needle. This may be done by a rub or two, until the magnetic powers in the needle are conjoined and remain quiescent without any signs of polarity; but by continuing the operation a little longer the poles will be reversed.

There is also a great similarity between the magnetic and electric virtue, in the manner of their passage along a conducting substance. If a piece of iron be suspended to one pole of the magnet, and a crooked piece of iron form a communication between the poles of the magnet, the suspended piece of iron will drop off; as if the magnetic virtue were accumulated round one pole, while it was subtracted from the other; and having a constant tendency to go from one pole to the other by the shortest communication, it will pass through the connecting wire, and not through the suspended iron; which will therefore be left at liberty to fall to the ground by its own gravity. It seems then to resemble the charged electrical vial; which having one side charged positively and the other negatively, will be discharged by forming a communication between them.

I said before that the magnetic virtue may be given to a heated piece of iron, by quenching it suddenly in a position parallel to the dipping needle. In this position it acquires the highest degree of polarity; and as you recline the bar from being parallel to the dipping needle, you will find the polarity lessen by degrees, until it become perpendicular to that position when all signs of polarity vanish. And this rule will hold good, provided the whole bar be equally heated throughout, and equally cooled, whether in water or air. Hence it is that the tongs, so often heated in the fire and then suffered to cool in a reclining or perpendicular position, will in process of time have their magnetic powers separated, acquire a small degree of verticity, and then communicate it in a much higher degree to smaller pieces of iron or hardened steel, than that in which the tongs possessed it before. And it is always to be remembered that large surfaces easily communicate this virtue to small ones, but not vice versa. Take a large bar of steel and hold it in the direction of the dipping needle; then take a number of small steel bars about 6 inches long, a quarter of an inch thick and one sixteenth broad, and draw them from their middle over the end of the large bar, ten or a dozen of times, as you would touch them by an artificial magnet, and you will find them all changed into artificial magnets, of greater strength, than what the large bar exerts in any position.

How this power is raised to degrees exceedingly above the power that gave it, and then withdrawn by a contrary friction, has been long a matter of wonder and surprise; and few have attempted to account for it. I shall give you my thoughts on this phenomenon, which may be accounted for on the foregoing hypothesis of two distinct and separate powers in magnetism. While the large bar remains in the position of the dipping needle, its magnetic powers are separated in some degree by the influence of that foreign power, whatever it is, which governs and directs magnetism in general; one of them being attracted by it, while the other is repelled, as appears by the bar's exerting some degree of verticity, while in that position; which verticity is greatly increased and finally fixed by the bar's continuing long in that state. While these powers are separated even in the lowest degree in the large bar; if a small bar be drawn over the end of the large bar, the power, which predominates in the end of the large bar, attracts the other or contrary power of the small bar to the end in contact, and thereby forces the other power to recede to the other end of the small bar, until by frequent repetition a fixed and permanent polarity is produced. When a considerable number of these small bars have their magnetical powers separated and are become magnetical, they may be employed conjunctly to separate the powers of the larger bar, and produce a fixed polarity in it also. This whole theory may be in some measure confirmed by an observation open to every body: that while any piece of iron is in contact with a magnet either natural or artificial, the magnetic powers of the iron are separated, and continue so, while the contact continues.

ATTRACTION OF ELECTRICITY.

Electricity is the next species of attraction to be explained. It derives its name from Ελεκτϱον, the greek word for amber; which was the first substance, observed by the ancients, to possess the power of producing, by friction, the appearance that was called electricity. The earth, being the largest body on which we can make any experiments, seems to be the grand reservoir of electricity; and every body seems to be possessed of some portion of it, according to its quantity of matter, or possibly its surface: for the proportion is not yet accurately ascertained. Some bodies part with it more easily than others; and some shew signs of electricity on being rubbed with the hand or any dry substance; while others exhibit no appearance of any such thing. Of the first kind of bodies are glass, resin, sulphur, wax, silk, hair, quills, &c. These are therefore denominated *electrics*, or *electrics per se;* while other substances, which exhibit no signs of electricity by friction are denominated *conductors*, or *non-electrics*. When a clean dry tube of glass or stick of sealing wax is rubbed with the dry hand, or with a piece of leather, flannel, &c. it will attract light bodies towards it, such as feathers, down, chaff, paper, &c. If these light bodies be suspended by silk or hair lines, they will, after contact with the tube, be repelled by it, and fly from it. The tube or stick of wax, &c. seems therefore to be surrounded with some effluvia, in form of an atmosphere, which extends to a certain distance, and is called the *electric virtue*, or the *electric fire*, from its luminous appearance, when in motion. When a body is once attracted by the rubbed or excited tube or wax, and comes into contact with it, it flies off and is no more attracted again, until it touch some other body; after which it is again attracted as at first. In its recess from the tube, it eagerly flies to any conductor of electricity, and is powerfully attracted by it. From this appearance taking place invariably without exception, philosophers have concluded that the light body has either received some electrical fire from the tube, or has communicated some to it, or possibly both in different circumstances, occasioning this alternate attraction and repulsion; which is observed to continue a long time.

But observing, that only a small quantity of the electric matter could be collected round the excited glass tube, and that a slender and sharp metallic point drew off the atmosphere from the tube in a quick and imperceptible manner, they have contrived various machines for collecting it in greater quantities. A glass globe is made to revolve on an axis, pressing against a leathern cushion; while the electrical matter, thereby collected, is conveyed by these slender points, to a large conducting body, covered with metal, which is found to be the best conductor, and suspended by silk lines or supported on glass pillars; which being electric per se or nonconductors, prevent the electric matter from escaping, and confine it to the conducting body; round which it is collected in an atmosphere, extending to several feet, according to the surfaces of the said prime conductor. The reason of suspending the prime conductor on silk or hair lines, or of supporting it on glass pillars, is to cut off all communication with the earth; for should a communication be formed between the earth and the prime conductor, the electrical effluvia would escape by the conducting substance that had formed the communication, to the earth, the grand reservoir, as fast as it could be collected by the revolution of the globe. In this manner the electric matter is collected in large quantities at pleasure, according to the surface of the prime conductor and of bodies connected with it. So that upon presenting your finger to it, while the globe is in motion, you may draw it off instantly in a large, visible, and luminous spark of fire; from whence it derived the name of electrical fire.

The particles of this electric fire seem to repel each other, as they collect in the form of an atmosphere round the body on which they are cast, and may be sensibly felt by the hand; and the more that is thrown on the prime conductor, to the greater distance will it extend. When a nonelectric substance is brought near to this atmosphere it is attracted by it; but an electric per se is repelled or seems to be repelled, as it recedes from it. The first draws it off instantaneously with a crackling noise and visible spark of fire, when the nonelectric substance communicates with the earth to which the electricity wholly passes, without leaving any considerable remainder either in the prime conductor or in the nonelectric substance which conveys it off; but an electric per se may be repeatedly immerged into this atmosphere without attracting and carrying off a single particle. But if the nonelectric substance which conveys this atmosphere to the earth have a sharp point, it will then be drawn off silently without any noise, in a gradual and imperceptible manner; the point appearing luminous in the dark. Should such a point be fixed to the prime conductor, it will disperse the electric matter through the air, as fast as it is collected by the globe, electrifying the air where it is disseminated. This power of a point has given rise to the ludicrous experiment of the electrical horse race; which is performed by placing a parcel of horses, with sharp points in their heels, on the circumference of a circle, which revolves on its center. As soon as this machine is electrised, and the electric matter is prevented from escaping to the earth by its being placed on a glass stand, the electricity escaping by the points will by its reaction on the horses give them motion in a contrary direction; and the circle will revolve on its center. This principle of motion may also be employed for playing musical tunes, and for many other purposes, by the invention of proper machines.

This attractive power of the metallic point suggested to Dr. Franklin (whose improvements in this branch of natural philosophy far exceeded any of his predecessors) the happy means of preserving houses, ships and persons from the melancholy effects of lightning; which he has demonstrated to be the same with electricity, by actually drawing it down from the clouds, by means of a kite raised on the commons of this city, in the time of a thunder storm. The metallic point fixed in the head of his kite attracted the electricity from the cloud, which was then conveyed by the string that raised the kite, to a bottle in which it was collected. The experiment was easy, but dangerous; as the repetition of a similar experiment proved fatal to the celebrated professor Richmond of Petersburg in Russia.

Hence the doctor advised the fixing of iron rods to chimneys of houses and the masts of ships inserted to a considerable depth in the ground or water, and terminating in a sharp point eight or ten feet above the building: that by this conductor the lightning might be safely conveyed to the earth without an explosion or damage. The success of this invention has hitherto answered his expectations; for although the lightning has not always been thus conducted without an explosion, yet we have no instances of damage done to the buildings that were thus secured.

If instead of the smooth glass globe, one of sulphur or resin or wax, or even a rough glass globe, whose polish is destroyed by emery, be used, it will excite the electric effluvia, and form an atmosphere round the prime conductor, to appearance the same with what is formed by the friction of the polished glass, but possessing different and contrary properties. For although two bodies electrified by this resinous or rough globe repel each other and attract any nonelectric substance, in the same manner as if they had been electrified by the polished glass; yet they will strongly attract another body equally electrified by the polished glass globe; and upon contact all signs of electricity will vanish, and both will be left in their natural state. When this discovery was first made by Mr. Kinnersly, he communicated it to Dr. Franklin, observing that although one of these electrified *positively* and the other *negatively*, as the doctor has termed the different operations, yet he could not determine from any thing he had seen, which of the globes had the power of electrifying positively. By a body's being electrified *positively* was meant its being possessed of more than its natural quantity of electricity; but if this natural quantity was lessened, the body was said to be electrified *negatively*. Hence it was found by experiment, if both these kinds of globes were turned at the same time and worked equally well, no sign of electricity would be found on the prime conductor; and therefore one was supposed to draw off the electric fire, as fast as it was collected by the other. These and similar appearances led Dr. Franklin to frame his famous hypothesis of the *positive* and *negative* electricity, by which he endeavours to account for all the phenomena hitherto observed in this wonderful fluid, according to the following general laws, which he deduced from his experiments.

1. Two bodies in their natural state, without having their natural quantity of electricity increased or diminished, neither attract nor repel each other.

2. Two bodies equally charged with an additional quantity of electricity mutually recede from, and repel each other; whether they both be electrified positively or negatively.

3. Two bodies unequally electrified, whether positively or negatively; or one of them remaining in its natural state, while the other is electrified either positively or negatively, mutually attract each other, with a force proportioned to the difference of quantity in each.

4. Two bodies unequally electrified whether positively or negatively, or one of them remaining in its natural state and the other being electrified either negatively or positively will, upon near approach or contact, communicate to each other so much of the surplusage above, or deficiency below, their natural quantities of electricity, that this surplusage or deficiency shall be divided between them in proportion to their surfaces: and they will remain equally electrified, or in their natural state.

Upon these principles the doctor accounts for the charging of the Leyden vial, so called from an accidental discovery of the electrical shock first felt by Cuneus, and then by professor Mushenbroek of Leyden. It had been known that the electricity might be collected in a vial partly filled with water, with a wire passing through the cork of the vial into the water when it was held in the hand, and the wire presented to the prime conductor. The professor's curiosity led him to try, whether he could receive a spark from the conductor, with his other hand, while the vial was in charging; and to his surprise he felt a violent shock through his arms, and all signs of electricity in the vial and conductor vanished; the vial being thereby discharged. Being ignorant of the cause, he attributed the shock to the gun barrel, which he used as a conductor. This however passed for the solution; and a gun barrel was everywhere used as a conductor; until it was afterwards found, that a solid rod of metal, or even a piece of wood covered over with metal, could shoot in this manner as well as the gun barrel.

By a more careful analysis of the vial and of the operation of nature in charging it, Dr. Franklin found that the electric fire was not accumulated in the water or in the metallic coating on the inside of the vial, but in the pores of the glass; and that no electricity could be collected in it, unless an equal quantity was taken from the outside, either by a communication from the outside to the earth or to the cushion under the globe; and that the vial might be charged with the electricity that was drawn from the outside alone. He found also that the form of the vial made no difference; for that a pane of glass might be charged in the same manner, having more than the natural quantity on one side, and less than the natural quantity on the other; and that as soon as a communication was formed between the inside and outside surfaces, by any conducting body, such as metal, water, &c. the vial was immediately discharged with a violent explosion and flash of lightning in miniature. The vial was equally charged, whether the outside or inside communicated with the prime conductor, and the other side with the earth; for the side that communicated with the prime conductor, when the smooth glass globe was used, was found to be electrised positively, and the other side negatively; and that every thing was reversed when a sulphur or waxen globe was used.

Although every phenomenon leads us to conclude that the electric fire is lodged in the pores of the glass; as the shock may be had, when both the external and internal coatings are removed and others are applied; yet it does not appear, that it can pass through the substance of the glass; as the charge will remain in the vial for many years, when all communication between the outside and inside surfaces, by any conducting substance, is cut off: unless the attractive power of one atmosphere should reach to the other, from the outside to the inside and thereby cause a discharge of the vial. Hence it is, that a fractured vial cannot be charged; and that a vial, while hanging to the prime conductor sometimes discharges itself; when the outside coating of the vial is near enough to the prime conductor, to receive a spark from it.

This necessary *striking distance* shows that the density of the atmosphere round the prime conductor decreases, in some certain proportion to the distance from it, which is not yet sufficiently ascertained; and at the same time the stronger attraction of a sharp point of metal is fully proved by drawing off the atmosphere at a much greater distance than that at which a spark may be received from the conductor. The only advantage of filling the vial with water, or coating both surfaces with some conducting substance, is, that by touching in many points, this coating may allow the electric matter to pass more copiously into, or from, the pores of the glass.

According to Dr. Franklin's hypothesis, the process of nature, in charging the Leyden vial seems to be as follows. When the smooth glass globe is rubbed with a piece of soft leather, it attracts a small quantity of electricity from the earth, which is accumulated round the globe in form of an atmosphere, and may be sensibly felt by the hand. This surplusage of electricity is readily received by the prime conductor, which was before in a natural state, by the intervention of metallic points, which attract it at the greatest distance. The prime conductor now being electrified will part with its electricity to the inside of the vial, which was before in a natural state: while the earth or cushion having parted with some of its natural quantity to the globe, is ready to receive as much from the outside of the vial. Thus we proceed until no more can be forced into the pores of the vial, which is then said to be fully charged. Should the operation be farther continued, the electric matter thrown upon the prime conductor would swell into an atmosphere of such magnitude and density as to strike to the outer coating of the vial; and then the whole collected quantity is discharged with a violent explosion; and both sides of the vial are left in their natural state, or nearly so. There is indeed a small quantity left, which after a minute or two standing, will discover itself by a second discharge upon forming the communication between the surfaces. And this residuum has been repeatedly found after charging the vial once.

The velocity of the motion of this electrical effluvia is exceedingly quick; as it takes up no time in passing over or through a wire of some miles in length, from the outside to the inside of the vial. If the communication be formed between the two surfaces of the vial, by persons joining their hands together, while the first touches the outside and the last touches the inside or wire of the cork, they will all be shocked at the same instant: the whole charge of electricity passing through the arms and bodies of the whole company, from one side to the other of the vial.

The human body is a good conductor of electricity: and hence a person receives the shock, which professor Mushenbroek experienced, when his arms formed the communication; but metal being a much better conductor, a person may hold the wire which forms the communication, in his hand, without receiving the shock.

All the phenomena of this surprising fluid, that are hitherto discovered may be easily accounted for, from the principles and laws already delivered, upon the hypothesis of a negative and positive electricity: and by these may the various experiments that are usually made be explained; such as the dancing of paper images between two metallic plates; the shower of electrised sand rising and falling alternately; the ringing of musical bells, &c.

The air is an electric per se and will not readily conduct the electric fire, so that when it is partially removed, the electric atmosphere soon expands to a greater degree, and the charged vial under the exhausted receiver is soon discharged; the attenuated air serving as a conductor. However if the air be loaded with moisture, the watery particles conduct and disperse the electric matter; so that little of it can be collected and retained in a moist state of the air. But an absolute vacuum will not conduct it at all.

Heat possesses the power of changing the nature of electric substances and of converting them into non-electric conductors of this fluid. A piece of glass heated, melted wax, resin, or hot air become conductors, and the vial when heated cannot be charged; the electric matter passing through it as readily as through a metallic wire. A high degree of cold on the other hand changes water, when converted to ice, into a nonconductor or electric. And possibly the same may be true of other substances.

Although the electric fluid discovers no signs of heat while it is at rest, yet it produces a great degree of heat when in motion, so as to kindle spirits, set houses on fire, and melt metals. And although it thus produces common fire, yet it is essentially different from it, in their most distinguishing properties. Common fire moves slowly through metals, and seems first to pass along their surfaces; but the electrical fire moves instantaneously through the substance of the longest wire we can find, and is not felt by the hand that holds it. Common fire has no such attraction to other bodies, as this is found to possess. And sparks of it, that will kindle spirits, may be extracted from ice, which does not contain such a quantity of common fire as is necessary for this purpose.

That the electrical fire is the same with lightning is not only demonstrated by its being actually drawn down from the clouds, in the manner before mentioned, but also by their agreement in their principal effects and properties: rending the hardest bodies, melting metals instantaneously, killing animals, giving polarity to iron, emitting a strong sulphureous smell, passing through air in a crooked direction, being conducted by metal or water, rather than by any other substance, and exhibiting bright flashes and loud explosions. Possibly the air, being an electric per se, by its friction on the trees or mountains, may produce the electric fire and communicate it to the watery clouds which float in it, until they become highly electrified either negatively or positively, and then being attracted by the earth, approach towards it and discharge themselves on such eminences as are nearest to them, producing thunder and lightning. Hence the reason why thunder clouds generally float lowest in the air, and why eminences, such as mountains, the masts of vessels, and the chimneys of houses, are most frequently struck with lightning.

So far as our present knowledge on this subject extends, the following may be considered as the general laws of electricity.

1. The electric fluid exists in all bodies in a certain quantity which is natural to them, and commonly called their natural quantity.

2. The particles of the electric fluid repel each other, and hence recede from each other as far as they can.

3. When a body has either more or less of the electrical fluid than its natural quantity, it will in the first case impart some of it to the surrounding bodies, that are capable of receiving it; and in the latter case, the electrical fluid of the surrounding bodies finding less resistance towards a body negatively electrified, or having lost part of its natural share of electricity, rushes towards that body, and tends to diffuse itself through it, and thus to compose itself into an equilibrium.

4. Hence, the reason why the electric fluid existing everywhere, seems to be inactive in the common or natural state of bodies, is this; all bodies having their ordinary share of this fluid, an equal pressure exists on all sides. And if, at the same time, all should receive an additional quantity, no electrical phenomena would ensue; because the pressure being everywhere

equal, the repulsive force of the electrical particles would be everywhere equally balanced. Thus two bodies, being equally electrised, whether positively or negatively, give no spark to each other, but only recede from each other, and the farther, as their state differs more from that of the surrounding bodies. And for the same reason, two bodies suspended in a metallic vessel, by silken threads, show no signs of electricity, when the electric fluid presses equally on them from the electrified vessel on all sides.

5. All nonconducting bodies may acquire on each part of their surface a certain portion of the electric fluid, as well as conductors; but they do not allow it to pass freely through their substance, nor over their surfaces.

6. All bodies are susceptible of either positive or negative electricity indifferently, either by exciting them by friction, or by bringing them within the sphere of action of other bodies already electrised; so that insulated metallic bodies, the best conductors, may be excited by friction. The only material difference, between conducting and nonconducting substances, seems to be this: that the electricity does not spread itself so easily and completely over or through the nonconductors as through the conductors. An electrical spark, thrown on the surface of an insulated piece of metal, rapidly diffuses itself through its whole substance; and a single touch of an uninsulated conductor as instantly takes it off again, to whatsoever part of the metal it may be applied. But electricity seems to stick to that part of a nonconductor to which it is applied, spreading but slowly and unequally over its surface, and may again be taken off by degrees, by touching those parts to which it had been applied.

There seem to be also some bodies of an intermediate quality, through which the electric fluid propagates itself completely, but slowly: such as common wood, moist air, and many other bodies.

7. Nonconductors seem to acquire electricity with some reluctance, and to part with it with the same difficulty, as they hold it more tenaciously than others. A single touch takes away all the electricity of a metallic body, but does not absolutely destroy the electricity of a nonconductor, such as glass, wax, resin, &c. The metal plate of an electrophorus takes almost no electricity from the resinous cake, if it be lifted up, without having been touched when it was upon it.

8. All resinous bodies, silk, and many other bodies, retain their state of electricity more tenaciously than glass, however dry. For an excited glass is robbed of its electricity by the application of a conducting substance; but a resinous body, although touched, retains much of its electricity.

9. An insulated conducting body, being within the sphere of action of an excited nonconducting substance, or in contact with it, as the prime conductor of an electrical machine, acquires at the same time two contrary electricities, at its different extremities, viz. the same with the excited electric, at its farthest extremity; and a contrary electricity, at the end in contact, or near to the excited electric. The same thing happens to an insulated conductor brought within the sphere of action of an excited or electrised conducting body; but if the insulated conductor be brought into contact with the electrised conducting substance, it will acquire the same electricity with it, through the whole of its extension, or divide its electricity equally

with it. It seems therefore to be a law of this fluid, that when it is accumulated upon a body, and finds an obstruction in its passage to a neighbouring body, by the interposition of a nonconductor, such as dry air, glass, &c. it forces, by its repulsion, the natural quantity of electricity of any neighbouring body to its farthest extremity, so as to excite a deficiency of electricity at the nearest extremity of it; until at last the accumulation of the fluid on the electrised body becomes so great as to overpower the obstruction of the interposing nonconducting substance, through which it forces its passage and rushes upon the neighbouring body in form of a luminous spark of fire. Glass makes a greater resistance to its passage than air; yet it frequently overcomes even this and forces its passage through it. When thrown upon the surface of a pane of glass, coated with tinfoil on both sides, its accumulation on one side forces the electricity of the glass from the other side upon any conducting substance near enough to receive it, and convey it off until the accumulated quantity on the first surface counterbalance the resistance of the glass, and force its passage through it, rending the substance of the glass, in order to diffuse itself over the other surface, upon which a deficiency of electricity or kind of vacuum was produced. This is called the charging and discharging of the glass. But when the glass is thus fractured, it cannot again be charged, unless it be repaired; as the hole does not close of itself to give a new resistance, as is the case with the air, being an elastic fluid.

From what has been said of the difficulty with which electric substances acquire a state of electricity either negative or positive, and their reluctance to part with it, when acquired, we may easily explain the operation of the *electrophorus;* which is a cake of resin or pane of glass, placed between two metal plates; one or both of which may be removed from the cake at pleasure by an insulating handle. When the cake, lying on one of the metallic plates is excited negatively by friction, and the other plate is laid upon it, the electricity of the plate, finding less resistance on the side next to the cake, rushes towards it, leaving a deficiency of electricity in the upper side of the metal, which will therefore receive a spark from any other conducting substance brought near to it. If this plate be now removed from the cake, by its insulating handle, it will carry with it the additional quantity it received and appear to be positively electrised, and therefore give a spark to any body fitted to receive it. The resinous cake, being more tenacious of its state of electricity, does not receive from the metal the electricity it had received, but remains nearly in the state, in which it was when the plate was laid upon it, and therefore suffers the plate to carry off with it, the additional quantity it had received from the touch of the conducting substance.

If the resinous cake be excited positively, by passing the wire of a charged vial over it, its acquired electricity will force the electricity of the metallic plate to its farther surface, which being there accumulated will readily pass off to any body that will conduct it to the earth, and thereby leave the plate negatively electrised; for the cake will not readily part with its superabundant electricity to supply this defect in the plate. The plate therefore, when removed from the cake, will be in a negative state, and thus receive a spark from a conducting substance brought near to it.

Thus may a vial be charged either negatively or positively by the *electrophorus*.

What happens to the metallic plate placed upon the cake, happens also to the other plate upon which it is placed; save only that when one plate is electrised positively the other is electrised negatively, if the electrophorus be placed on an electric stand.

The same also happens to the two coatings of a vial or pane of glass, when charged and discharged in the common way. They are then said to be restored to their natural state: and to appearance they are so. But the upper coating connected with the prime conductor has not only lost, in the discharge, what was thrown upon it by the conductor, but also some of its own electricity driven into the glass by the force of charging; and therefore if separated from the glass in this situation it will appear in a negative state. And the under coating, connected with the earth, has recovered by the discharge, not only what had been forced through it into the common stock, but also that additional quantity which that surface of the glass, being brought into a negative state in charging, had absorbed from the coating; so that when separated from the glass, it will appear to be positively electrised. Consequently the two coatings, thus separated from the glass and brought together, by insulating handles, will give a spark to each other.

From the principles already laid down, we are enabled to account for the *aurora borealis* and *australis*. As heat and cold are sufficient to change electric substances into conductors and vice versa; cold must increase the electric power of such substances as are already electric; that is, very cold air, glass, resin, &c. when dry, will be more electric than when warm.

With respect to air, this is evident, from considering, that clear, frosty weather is, of all others, the most favourable for electrical experiments. Now the air all round the globe, at a certain height above its surface, is found to be exceedingly cold, and as far as experiments reach, very electrical also. And as the inferior parts of the atmosphere, between the tropics especially, are much heated in the day time, by the reflection of the sun's rays from the earth, this air will become a conductor in some measure, and more readily part with its electricity to the clouds and vapours that float in it, than the colder air above, or towards the poles. Hence the violent thunders and lightnings in hot countries. Immense quantities of electricity are thus communicated to the earth, and the inferior warm atmosphere, being exhausted of its electricity, must again receive a supply from the colder air above. Hence the French mathematicians, on the top of the Andes in South America, often observed the clouds sinking below them into the warmer regions of the atmosphere, where they appeared to be highly electrified, and discharged themselves in violent thunder and lightning, while *they* enjoyed a clear, cold and serene sky on the summits of the mountains. In the frigid and temperate zones, the atmosphere never being so heated as in the torrid zone, its inferior parts will not so readily part with its electricity to the earth, nor require such a supply from above. Now, as the warm air, near the surface of the earth in the torrid zone, is perpetually bringing down great quantities of electricity from the colder air above, and no such thing happens in the colder regions towards the poles, it follows, that the upper parts of the atmosphere, about the equator, will continually need a supply of electricity from the regions towards the poles. This easily shows the necessity of an electrical current from the poles towards the equator, as we always observe in the aurora borealis; at the same time that it accounts for its more frequent appearance in winter than in summer; and also in the colder regions than in the warmer. The inferior parts of the air not being so much heated in winter as in summer, it will retain its electricity better, and needing no supply from above, or very little, the abounding electricity of the upper region must almost wholly pass off towards the equator, to supply the deficiency there, as it cannot pass to the earth through air, that is nearly as cold as the air, which it deserts. In Siberia, the coldest country in the world, travellers agree that the aurora appears in greatest perfection, and its corruscations the most terrifying, being seen constantly from October to January.

In confirmation of this, the experiments made with the electrical kite prove, that the air is much more electrical in winter than in summer, though the clouds are known to be more highly charged with electricity in summer than in winter: a proof, that when the air is most heated it most easily parts with its electricity to the clouds.

The slow motion of the corruscations of the aurora may be supposed an objection to this theory, when we consider the rapidity with which the electric equilibrium is restored. But in answer to this it must be observed, that although the passage of electricity by a good conductor is instantaneous, yet through a bad conductor it is slow and gradual. And the air, when moderately heated, still continues but a bad conductor; and therefore the motion of the electrical fluid through it must be slow, when there is no good conductor to accelerate its motion.

Some animals, such as the torpedo and Guiana eel, are evidently possessed of the power of giving the electrical shock at pleasure, and stroke after stroke in quick succession, as often as they please; and while they emit the electrical matter from one part of the body they receive it again in another, being conducted by the water in which they float. Providence has armed them with this amazing power for their defence against the attacks of larger fish, and for killing or stunning the smaller fish for their food. This power has been long observed in these fishes, without any suspicion of its being the same with electricity; and philosophers have attempted to account for it from the structure of their muscles, of which you may see an account in Chambers's dictionary.

One of these eels having been brought to this city, by a poor man, and showed for money, I went to see it among others; and upon receiving the first shock, immediately proclaimed it to be the electrical shock, from the sameness of the sensation it excited in my arm. This led us to make experiments for the confirmation of the conjecture; and the success answered my expectations. When a single person put his hand in the water and irritated the eel, he received the shock, which he felt as high up as the wrist or the guard of the arm, according to the violence of it. The same happened when he irritated the eel by a rod of metal of any kind. I then proposed to try whether the shock could be communicated through a number of persons connected together by joining their hands; and, to our satisfaction, we found the whole circle shocked in the same manner as by the Leyden bottle.

This was a convincing proof, that the power, which the eel possessed was the same with electricity. Little fishes being thrown into the water with the eel, it first shocked and then swallowed them: those that were too large to be swallowed, it stunned and then neglected. We tried to render the spark visible, in its passing between two wires set nearly together, and forming part of the communication; but we could not succeed in this experiment, though all the persons, whose hands were joined felt the shock. We made many other experiments and often repeated those above mentioned; and the result of all was a full confirmation of the truth of our hypothesis. The owner of the eel would not suffer us to take it out of the water, that we might ascertain the particular part of its body, in which this unusual virtue resided, or whether it was diffused over the whole. This has been since done with respect to the *torpedo*, by Mr. Walsh, who went over to the coast of France to make similar experiments on that fish; as many of that species of fish are found there. He found that the shock received from it, was the electrical shock; and that the fish possessed the power of shocking only in two parts of its body directly opposite to each other, and near to the head. A spot on the back and another on the belly opposite to the former, being of a different colour, led him to make the experiment; and he found that the electrical virtue was confined to these, and that any other part of the fish might be handled, without receiving a shock while it was out of the water. Either of these places separately might be handled, without the shock being received, until a communication between them was formed. This makes it appear probable, that the same may also be the case, with the

Guiana eel. One of these spots must therefore be always in the positive, and the other in the negative state; or possibly each of them may be alternately in the positive state, while the other is in the negative state: or rather they are both generally in the natural state, until by an effort of the fish's will, they are suddenly put into different states; as we frequently found, that the hand might be in the water, which formed the communication, without receiving any shock; which cannot be the case with the Leyden bottle when charged, which suddenly discharges itself, upon forming the communication.

Whether there be any electric atmosphere round these spots in the torpedo, we cannot tell; as we had no opportunity of examining this matter in the eel, nor have we heard whether Mr. Walsh made any experiments for ascertaining this matter.

There is another hypothesis of Mr. Eels of Ireland, which solves the various phenomena of electricity, and which I will here explain, and leave you to your own judgment and future experiments, to adopt or reject it as you may find sufficient reason.

Observing that bodies electrised negatively, as Dr. Franklin calls it, really repel each other with every appearance that takes place, when they are electrised positively, he accounted it absurd to suppose with Dr. Franklin, that they repelled each other the more forcibly, the more they were deprived of their repelling power, especially as every phenomenon proved that they were surrounded with an atmosphere as well as those that are electrified positively.

This led him to conclude, that there are two species of electricity, the *vitreous* and the *resinous*, so called from the substances which excite them by friction:

and that they exist in equal quantities in every body, strongly attracting and condensing each other in the natural state, so as to show no signs of electricity in that state, while they are in equilibrio; but while they attract each other they as powerfully repel, each the same power in any other body; that is, the vitreous repels the vitreous, and the resinous repels the resinous, when they are excited in equal quantities. When the natural equilibrium is destroyed by the increase of the one and the diminution of the other, the increased power will expand itself into an extensive atmosphere, decreasing in density and attractive force with the increasing distance, round the excited body; which atmosphere attracts any other atmosphere excited by the other power, and repels an atmosphere of the same kind. These powers are inherent in every body, and when a body is said to be electrified, no more is meant, than that these powers are separated by friction. No part of either of these powers can be taken away from any body, but an equal quantity of the other must at the same time be given in exchange for it, which it will retain until the first subtracted quantity be restored and an equilibrium formed as at first. Friction is found to separate these powers in glass, resin, wax and other bodies which are electrics per se; but the powers of a nonelectric body are separated without friction, by being brought within the atmosphere of the excited electric: as is evident from the conductor, whose end next to the glass globe is electrified with the resinous power, while the other end is electrified with the vitreous power; and the contrary happens, when a rough glass or sulphur globe is used. The same separation of powers takes place in any body brought near to the prime conductor, if the body be nonelectric. If it be brought with its end towards the electrified conductor, it will be found to be electrified with the different powers at its opposite ends; that is, with the resinous power at the end next to the prime conductor when it is electrified with the vitreous power. Now this separation of powers is absolutely necessary, previous to any exchange of them in any body. Silk, wax, glass, and all other electric substances, have their powers also separated, while in an electric atmosphere, in the same manner with nonelectric substances, but not so suddenly; and they are found to be electrified with the contrary power, to that, which they excite by friction. Their powers do not change so soon; which constitutes their principal difference from nonelectrics.

According to these principles, Mr. Eels accounts for the charging of the Leyden bottle in the following manner. The glass globe has its powers separated by friction, and parts with its resinous power to the cushion in exchange for its vitreous power. And this deficiency of resinous power in the glass is supplied by the prime conductor in exchange for the vitreous power thrown upon it by the globe; and at the same time the deficiency of the resinous power in the conductor is supplied from the inside of the bottle in exchange for the vitreous power, which it receives from the conductor. This accumulation of the vitreous power in the inside would go on to produce a separation of the powers in the bottle, but it being an electric per se, the exchange is not made so easily; so that the vitreous power is chiefly retained; while through the pores of the glass, or round by the wire to the outside coating, some part passes off to the earth, together with the vitreous power of the coating, in exchange for the resinous power from the earth or cushion, according as the connexion is made between the outside and the earth or cushion; until at length, the resinous power on the outside becomes a balance, in attraction, to the vitreous power on the inside; which therefore strongly attract each other, and confine them to the glass until the bottle be discharged in the common way.

ATTRACTION OF GRAVITATION.

THE fourth species of attraction is that of gravitation, which is observable chiefly between the larger bodies of our system, such as the sun and planets that surround him, and regulates all their motions. Sir Isaac Newton, observing that when a body had been projected upwards, its velocity continually decreased until it was destroyed, and then the body began to return with an accelerated velocity to the earth, supposed that the same power, which caused the heavy body to descend to the earth, might also extend to the moon; and upon examining the effects of this power, which he called gravitation, he found that the moon might be retained in her orbit by the very same force, which attracts a heavy body, provided that it decreased in its energy and strength in the duplicate proportion to the distance inversely; and farther, that all the bodies which compose the solar system were governed by the same attraction, acting according to the following laws, which he found to take place in every experiment he could make.

1. Gravitation is common to all bodies, and mutual between them.

2. It is exerted every way from the center of the attracting body in right-lined directions.

3. It is directly proportional to the quantity of matter in the attracting bodies.

4. It decreases in strength, as the squares of the distances from the center increase.

From the first of these laws we learn, that there is no such thing as absolute levity, or any body indued with the property of lightness, or want of gravity. All bodies, on which we can make any experiments, have the property of gravity; and it would be absurd to suppose that there were others destitute of it, especially if their motions and other affections were the same that they must necessarily be, on the supposition of their being possessed of this property. Whether the particles of light, which appear to be matter, are indued with this property, we cannot tell; as we cannot collect them together in sufficient quantities for the discovery.

From the second law we see the reason, why all heavy bodies fall to the earth in right lines perpendicular to its surface.

From the third law we learn, that there is not an absolute *plenum* in nature, as the Cartesians affirmed; but that there must be some vacuities, or spaces void of all matter, in the composition of natural bodies. For, if the gravity or weight of bodies be proportional to their quantities of matter, there must be much less matter in a cubic inch of cork than one of lead, as it is much lighter, and therefore in its composition there must be vacuities, or spaces void of all matter.

From the same third law we see the reason, why all bodies fall equally fast in vacuo; the attraction of the earth being always proportional to the quantity of matter in the falling body. If one body contain an hundred times the quantity of matter that is in another, the earth will exert an hundred times more force upon it, than upon the other, and therefore will bring it down in the same time and with the same velocity. This may be easily apprehended by conceiving the larger body to be divided into an hundred parts, each equal to the smaller body: the attraction of the earth on each of these parts is equal to its influence on the smaller body; and their union or separation can neither increase nor diminish the earth's action upon them.

From the fourth law we learn, that if the earth were a hollow sphere, a heavy body would be perfectly at rest, when placed any where within it, being equally attracted in contrary directions. The particle C, fig. 1. placed any where within the hollow sphere ABDE, is attracted with a force proportional to the number of particles in the bases of the cones AB and DE, or the areas of these bases; which areas are as the squares of the distances from C respectively, as is evident from the third law, above mentioned. And by the fourth law, the particle C is attracted in a contrary direction, with a force, which is inversely as the squares of the distances. But the direct and inverse ratios of the distances or squares of the distances, when compounded together, make a ratio of equality. Therefore the body C, being equally attracted in contrary directions, must rest indifferently in any part of the hollow sphere; for what is demonstrated of the point C is equally true of any other.

A body placed on the surface of two spheres of equal density, but unequal diameters, would be attracted by them with forces proportional to the said diameters. Let the two spheres be represented by A and B, fig. 2.

their diameters by D and d, and the attractive forces of the spheres by F and f: then, by the fourth law, the attractive forces are as the squares of the distances, or double distances from the center, inversely; that is $F : f :: \frac{1}{D^2} : \frac{1}{d^2}$. But, by the third law, these forces are directly as the quantities of matter in the spheres, which are as the cubes of their diameters; that is, $F : f :: D^3 : d^3$. and by compounding these ratios, we have $F : f :: \frac{D^3}{D^2} : \frac{d^3}{d^2} :: D : d$; that is, the attractive forces of the spheres are directly as their diameters. Q.E.D.

Hence if the earth were of the same density throughout, and perforated through the center, the gravity of a body descending through it would continually diminish as it approached the center; because in every point of its descent, it might be considered as on the surface of a sphere, whose diameter is double the distance from the center; and the parts behind being only a concave shell, which attracts equally in all directions, can no way affect the descent of the body. Yet it would acquire such a velocity at the center as would be sufficient to carry it up to the surface on the other side, if all resistance of the air be removed. Hence it would continue to vibrate in this perforation, without ceasing.

N. B. The weight of a body depends both on its bulk or magnitude, and its density, or the compactness of its parts; therefore when neither are given, $W=MD$. Hence $M=\frac{W}{D}$, and $D=\frac{W}{M}$.

MOTION IN GENERAL.

As all motion in bodies depends on the principle of attraction, whose general laws we have explained, we now proceed to consider the various affections and species of motion.

MOTION is the translation of a body from one part of space to another, and may be considered under different views, either as absolute or relative, equable or accelerated. Absolute motion is the change of absolute place, but relative motion is the change of relative place, when compared with some other place or body. A person on board of a vessel at sea is absolutely in motion, although he may be relatively at rest, with respect to other objects in the same vessel. When a body in motion passes over equal spaces in equal times, the motion is said to be equable; but if the spaces passed over in equal portions of time, be unequal the motion is said to be accelerated or retarded, according as the spaces increase or decrease. If the increase or decrease be equal in equal portions of time, the motion is said to be equably accelerated or retarded. Equable motion is generated by a single impulse, as the motion given to a ball by the action of gunpowder; but accelerated or retarded motion depends upon the continued action, or constant exertion of the moving force every instant. Thus the motion of a falling body is continually accelerated by the attractive force of the earth; which acting upon it every moment, and communicating equal increments of velocity in equal times, the motion is thereby equably accelerated. For the same reason, the motion of a body, projected perpendicularly upwards, is equably retarded; and if the resistance of the air be neglected, the velocity of the ascent and descent of the body in any given point, must be constantly the same, as also the whole times of the ascent and descent; since the attractive force of the earth generates or destroys equal quantities of the motion in equal times.

The general laws of all motion are reduced by Sir Isaac Newton to these three.

1. As a body at rest continues at rest until it be put in motion by some external force, this motion will continue until it be altered by some external force.

2. The motion produced is always in the direction of the moving force, and invariably proportional to it.

3. When one body acts upon another, its action is equalled by the contrary reaction of the other. Or, action and reaction are equal and contrary.

The first of these laws depends upon the vis inertiæ of bodies, or the resistance they give to all change of situation or condition, whether of motion or rest. Daily observation convinces us, that any body, which we once see at rest, never puts itself in motion, but continues always in the same place, till removed by some power applied: and when in motion, having no power within itself either to generate or destroy its motion, the motion continues after the moving force ceases to act: and if it continue in motion but a single moment after the moving power leaves it, there can be no reason assigned, why it should stop without some external force.

This shows us the reason why the planetary motions are so long continued, by the influence of the projectile force, which first set these worlds in motion, through spaces void of all resistance.

The meaning of the second law of motion is this: If a certain force be sufficient to move a body with such a velocity as to carry it over any given space in a given time, double that force applied to it would carry it over double the space in the same time; and treble the force, over three times the space. This is evident enough when only one force is applied to move the body; but the application of the law, when two or more forces are applied at the same time in different directions, will need some explication. Suppose on a body at A,* a force were exerted that would carry it over the space AB in a given time, in the direction of that force, and at the same time, another force exerted its influence on the same body in the direction AC, with sufficient efficacy to carry it over the space AC in the same time. Now these two forces in the given time must produce their separate effects: that is, the body must be carried the distance of AB from the point A, and also the distance of AC from the same point, and in the directions of the forces. But this can be no otherwise effected, than by their joint energy causing the body to move in the diagonal of a parallelogram, whose sides are in the direction of the two forces, and proportional to their strength. So that at the end of the time the body will be found at D, having passed over the space AD, in the same time, and by their combined influence, that would have been required to pass over either of the spaces AB or AC by the respective forces alone, which acted in these directions.

This second law of motion, therefore, teaches us how to compound and resolve various forces, and to estimate their joint or separate effects, namely, by describing a parallelogram, whose sides are in the directions of two of the forces and proportional to their separate influence, and taking the diagonal as the measure of their joint effect and the direction of the motion produced. This diagonal, being considered as the measure and direction of two forces, may be compounded with a third, by making it the side of a new parallelogram, of which the third force constitutes the measure and direction of the other side; so that the diagonal of this second parallelogram will exhibit the joint effect of the three forces, acting on the body at the same time. And thus may any number of forces be compounded into one.

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* See Plate I. Fig. 3.

MOMENTUM.

As we are now on the subject of the forces requisite to give motion to bodies, it is necessary that we be very precise in our ideas of motion; by which we are not to understand the velocity, with which a body is moved, but the force, with which it moves. As if two bodies A and B, being in motion, when twice the force is required to stop A as to stop B, then the motion of A is said to be double the motion of B.

In moving bodies, we must carefully distinguish between the velocity with which they move over a given space in a given time, and the quantity of motion, with which they press upon any obstacle in their way. The quantity of their motion is sometimes called their momentum; and depends upon the velocity of the body and the quantity of matter it contains. If M represent the quantity of motion, Q the quantity of matter, and V the velocity of a body in motion; then $M=QV$; or, the quantity of motion is proportional to the rectangle of the quantity of matter, and the velocity of the motion.

The third law of motion is evident both from sense and experiment. A stretched cord, being cut asunder in the middle, flies towards both ends. If a boat and ship be connected by a rope, they approach each other with an equal quantity of motion, let either end of the rope be drawn. When a horse draws a load, the harness that is stretched between them, presses as much upon the breast of the horse as upon the load; and the progressive motion of the horse is as much retarded or hindered, as the motion of the load is promoted.

OF NONELASTIC BODIES.

If a body that is perfectly hard or soft strike against any other body, it will give some degree of motion to it, and lose just the same quantity. Suppose the two bodies equal: from the time they meet, both are to be moved by the single motion of the first; but now, as both are to be moved by that force, which moved the first only, the ensuing velocity will be the same, as if the power which moved the first had been applied to both at the same time; hence they will proceed with half the velocity which the first moving body had; that is, the first body will have lost half its motion, and the other will have gained just as much. Therefore, if the bodies be both in motion, and moving the same way, the sum of their motions towards the same part, or in the same direction, will be $M+m=QV+qv$. But if they move in contrary directions, the sum of the motion in the direction of the first will be $M-m=QV-qv$. M and m representing the momenta; Q and q, the quantities of matter, and V and v, the velocities of the bodies. Hence supposing the bodies to remain contiguous after their impact, we have the following rule for their common velocity, x. Divide these momenta by the quantity of matter in both, and you have the velocity after collision. $\frac{QV+qv}{Q+q}=x$.

OF ELASTIC BODIES.

BODIES upon striking each other suffer an alteration in their figure, having their parts pressed inwards by the stroke, which recoil again as the bodies endeavour to recover their former shape. This power, whereby bodies recover their former figure, is usually called their *elasticity;* and when it acts it forces the bodies from each other and causes them to separate. Now the effect of this elasticity, when it is perfect, is such, that they recover their figure with as much force as that with which they were compressed, and in the same time; hence this power will separate the bodies, as swiftly as they before approached; and acting on both equally—on the one in a direction contrary to its motion, and on the other in the direction of its motion, it will take from the former, and add to the latter, equal degrees of velocity, when the two bodies are moving in the same direction.

The change of velocity which is made in any case of congress, when the bodies are perfectly elastic, is double the change which would be made, in like circumstances, where the bodies are void of elasticity. For if two bodies A and B strike one another, whatever motion A loses, by the stroke, B gains, if they be void of elasticity. But by the stroke the sides of both are pressed inwards, and, as they recover their former shape with the same velocity and in the same time that was required for their compression; while the side of A next to B is returning to its former situation it must impress B with a force equal to which relative velocities with which they separate, will still be the same as that by which they approached each other; the body that moved fastest and overtook the other, will, after collision, move slowest; and the other having received an increment of velocity from the stroke, will now move forward and leave the other behind as fast as it was overtaken by it before: so that their velocities will be interchanged. Hence if one body strike an equal body at rest, the percutient body will communicate all its motion to the other, and remain at rest.

From what has been said we may determine the velocity of the bodies after impact, in the following manner. Let $Q =$ the quantity of matter in the body A, and $q =$ that in B; V and $v =$ their respective velocities, and let $x =$ the quantity of motion lost by A and gained by B. Then $QV-x$, and $qv+x$ will be their respective motions after the stroke. Consequently $\frac{QV-x}{Q}$ and $\frac{qv+x}{q}$ will be their velocities. Now the difference of these velocities, making that the least, which was greatest before collision, will still be equal to the difference of their velocities before they met; supposing the bodies moved the same way.

$$V-v=\frac{qv+x}{q}-\frac{\overline{QV-x}}{Q}=v+\frac{x}{q}-V+\frac{x}{Q}$$

$$QqV-Qqv=Qqv+Qx-QqV+qx$$

$$2Qq\times\overline{V-v}=\overline{Q+q}\times x$$

$x=\frac{2Qq}{Q+q}\times\overline{V-v}=$ the quantity of motion lost by A and gained by B.

Now substituting this value of x in the expressions for the velocities of the bodies A and B, we have

$V-\overline{\frac{2q}{Q+q}\times\overline{V-v}}=$ the velocity of A after collision.

$v+\overline{\frac{2Q}{Q+q}\times\overline{V-v}}=$ the velocity of B after collision.

N. B. If the bodies move in contrary directions, v must have a contrary sign, because B is supposed in these theorems to move in the same direction with A. If B be supposed at rest, then $v=0$. And if either of these expressions should be negative, then the motion of that body is contrary to what it was supposed to be before their congress, in these theorems.

EXAMPLES.

Let $Q=3$, $V=8$, $q=9$ and $v=2$, when both bodies move in the direction of A forwards.

$8-\frac{108}{12}=8-9=-1=$ the velocity of A backwards.

$2+\frac{6\times6}{12}=5=$ the velocity of B forwards.

Let $Q=1$, $V=5$, $q=3$, $v=11$, when B moves contrary to A.

$5-\frac{6\times6}{4}=-19=$ the velocity of A backwards.

$-11+\frac{2\times16}{4}=-3=$ the velocity of B forwards, or contrary to the direction of A.

These theorems may be easily remembered by comparing the quantity of motion lost and gained, by bodies that have no elasticity, with what is lost or gained by elastic bodies, and remembering that the action of the latter is always double the action of the former.

These theorems therefore may be expressed in the following

RULE.

Compute the motion lost and gained by the collision on the supposition that the bodies were nonelastic; double this quantity and add it to the first motion of the body that has gained motion by the stroke, and subtract the double loss of motion from the first motion of the body that has lost motion by the stroke, and you will have the motion of both after collision.

N. B. When a body loses its whole motion and acquires a contrary motion the other way, these two motions must be collected into one sum, in order to have the motion that is lost; and when the quantity to be subtracted exceeds the quantity of motion before the stroke, the lesser is still to be subtracted from the greater, and the difference is the motion, in the contrary direction.

From the foregoing theorems and computations it will always be found that the quantity of motion towards the same point is not altered by the stroke, either in elastic or nonelastic bodies: the motion in the contrary direction being always subtracted. In elastic bodies, if the body A of one pound, and 15 degrees of velocity strike against B at rest of four pounds weight, B will move forward with 6 degrees of velocity; so that its quantity of motion will be 24; which is nine more than the motion of A at first; but this 9 must be subtracted from 24; because the body A after the collision is reflected back with that quantity of motion. It will also be found that if the bodies be equal, they will either move forward, or be reflected backwards, with interchanged velocities. If one, two, three, or four bodies, all equal, strike against any number of equal bodies, the motion will be communicated through them, and as many will fly off.

while the others will remain at rest. If one strike against a number at rest, it will move the first with its own velocity and remain at rest; the second moves the third in like manner and remains at rest, and so on to the last; which, having none to strike, flies off with the velocity acquired. In like manner you may conceive of all the rest.

If a small body strike a larger body at rest, the greater body acquires more momentum than the smaller body had at first, viz. as much more as that with which it is reflected back: and the motion of the larger body is increased by the interposition of a body of an intermediate bigness; and this increase of motion is greatest when the intermediate body is a geometrical mean between the other two. Let the quantities of matter in the three bodies be as 1,2,4, and the first strike upon the second with the velocity 18, it will communicate to the second a velocity of 12, and a motion or momentum of 24; which motion will communicate to the third a velocity of 8, and a momentum of 32. But for the want of perfect elasticity, the error in experiments, where there is a double percussion, will be more sensible, and the momentum in the above case will in ivory balls, scarcely exceed 30.

MECHANICS.

On the doctrine of the momenta of bodies, depends the operation of the mechanic powers: but before we can explain these, it will be necessary to premise a few things, concerning the centers of magnitude, of motion, and of gravity. The center of magnitude is that point of a body which is equally distant from all the external parts of it: and in uniform bodies, it is the same with the center of motion, which is that point which remains at rest, while all the parts of the body revolve round it, in any direction. The center of gravity is that point, about which all the parts of the body exactly balance each other; those in opposite directions having equal momenta. So that if a body be suspended by this point, it will rest indifferently in any position; for then the momentum of any particles to carry the body round is counteracted by an equal momentum of other particles in the opposite direction. As long as this point is supported, the whole body is prevented from falling or descending; but if it be at liberty to descend, the body falls or tumbles downward, however to appearance it may seem to ascend, as in the case of the rolling cones, whose center of gravity really descends while the ends of the cones seem to roll up an inclined plane. Another property of this point is this: if a body be suspended by any other point, it cannot be at rest, in any position, excepting two, namely, when the center of gravity is directly above, or directly below the point of suspension. Hence the solution of many curious phenomena: as for example,—why some bodies stand more firmly on their bases than others,—why some stand better in an inclined position,—why a wagon loaded with iron or heavy materials is carried safely along a shelving road, where one loaded with hay would be overset,—why we stand firm, while the center of gravity of the human body (which is in the pelvis) is perpendicularly over the base line of the feet, but tumble down, when it is removed from that position, —why equilibrists and wiredancers perform such wonders, by a nice adjustment of this point over the wire—why our feet must be brought under this point before we can rise from our seats,—why fowls stretch their heads forwards, when they walk up an hill,—and how a bucket may be formed, that shall turn its mouth downwards when empty, and upwards when full. Hence also you see the reason of suspending a bucket of water upon a short stick laid on the edge of a table, &c.

You may easily find the center of gravity of any irregular body, that has a plane surface, by suspending it by any point, and hanging a plummet from the point of suspension; for the center of gravity is in this line: then suspend it by any other point, and the plumbline from this point must intersect the former line in the center of gravity.

In homogeneous bodies, such as circles, squares &c. it is the middle point of the line that connects any two opposite points: in a triangle, it is in a line drawn from the vertical angle bisecting the base, and one third of the length of this line from the base: in the axis of a cone, one third of the length from the base, if the cone be hollow, but one fourth, if the cone be solid. In two bodies connected together, their common center of gravity is between them, and at such a distance from each, that their respective distances multiplied into their weights shall produce equal momenta or products. If this point of an inflexible line, which connects the bodies be supported, it is evident, the bodies must be in equilibrio, and cannot descend, while it remains at rest. The force of each of the bodies, to cause the system to descend, depends upon its weight and velocity, which constitute their respective momenta; and their velocities, being measured by the spaces they pass over in equal times, while they librate up and down on their common center, are proportional to the arches of the circle they describe;

or to their radii, which are their respective distances from it. Now their weights being multiplied into their velocities or distances from the common center producing equal momenta or products, the force of one body to cause the whole system to descend is exactly balanced by the other; so that the system is at rest, and the said point is the center of gravity.

This is the fundamental proposition of mechanics; and enables us to compute the force of any machine, whether simple or compound. You may find this point by the following

RULE.

By the sum of the weights, divide the product of the distance and one of the weights, and the quotient will be the distance of the other weight from the center of gravity.

OF THE MECHANIC POWERS.

THERE are but five simple mechanic powers; and of these all the machines in the world are compounded. The *lever*, the *pulley*, the *wheel* and *axle*, the *inclined plane* or *wedge*, and the *screw*.

They are called mechanic powers, because by the use of them our power over matter is increased; and by a small exertion of our natural strength we are thereby able to move large quantities of matter. But since the momentum of matter arises from its vis inertiæ or quantity, and its velocity of motion; and the first of these being unalterable, and the other variable, any contrivance, to diminish the velocity of its motion, will give us an advantage over its weight; but from what we have already said, we see that there is no other means to accomplish this, than by increasing the velocity of our own exertions exactly in the same proportion that we diminish that of matter to be moved. For the products of the weight and velocity of matter, and of our strength and velocity of exertion must still be equal, when there is an equilibrium between them. Now the human force, being generally less than the quantity of matter we wish to move, may be represented by a small weight, which we are able to raise without the intervention of any machine, and is therefore by the writers on this subject, usually called the *power;* while the larger quantity of matter to be moved is denominated the *weight.* The power and the weight, thus understood, are always applied to the same machine, at different places; and the point on which the machine turns, which we have hitherto described under the name of the center of gravity, is commonly called the *fulcrum;* because by the distances from this point the velocities of the power and weight are respectively measured and determined.

THE LEVER.

The lever, the most simple of all the mechanic powers, and that to which all the rest may be referred, is an inflexible rod, supposed, in theory, to be without weight, and supported on an immovable point, called the fulcrum; while the power is applied at one end, and the weight to be moved at the other. Now, in order to increase the velocity of the power above that of the weight, it must be applied to that end which is most distant from the fulcrum, that it may pass over a larger space than the weight in the same time. As there will be an equilibrium between the power and weight, when their momenta are equal; it follows, by turning this equation into a proportion, that the power will be to the weight, inversely as their velocities or distances from the fulcrum, or perpendicular ascent and descent in the same time: and this proposition holds true in all the mechanic powers.

The hammer, for drawing nails out of timber, is no more than a bended lever, where the hand is the power, the resistance of the wood is the weight, and the point on which the hammer rests in drawing the nail, is the fulcrum. Levers are of four different kinds: 1. Where the power and weight are applied to the ends; 2. Where the fulcrum is at one end, the power at the other, and the weight between them; 3. Where the fulcrum is at one end, the weight at the other and the power between them; 4. When the fulcrum is placed at equal distances between the power and weight, and then it is called the *balance*, whose use is not to raise a weight, but to compare one weight with another, called a *standard.* If the arms of the balance be unequal in length, and this inequality be compensated by the greater weight of the shorter arm, or by the difference of weight in the scales, so that the beam may hang in equilibrio with the scales, it will be deceitful; and this deceit may be detected by changing the scales, or the articles put into them, from one scale to the other.

When the arms are nicely equated in length and weight, and divided into equal parts, it becomes the proportional balance, to be used without scales: and when the products of the different weights into their respective velocities, added together on one side, are equal to the sum of the like products on the other, there will be an equilibrium formed. The steelyard is a lever, which shows the comparative weight of bodies by a common weight to be moved to different distances from the fulcrum.

The effect of the compound lever is computed, in the same way, as that of the simple lever.

THE PULLEY.

The pulley is a small wheel, movable round a fixed center, while a rope passes round a groove on its circumference, to one end of which the weight is fixed, and the power to the other. If the pulley be single, and fixed to a beam above, it contributes nothing to the raising of the weight, for the power and weight must be equal, to hang in equilibrio; since they have equal velocities as they pass over equal spaces in equal times. This pulley only changes the direction of the power.

But if one end of the rope were fixed to a beam above, and from thence passed round the pulley below, and the weight fastened to the pulley, the beam would sustain half of the weight, and the power would sustain the other half. For while the weight is raised one foot, each rope is shortened one foot, and the power of consequence passed over two feet. Therefore to constitute an equilibrium, the power being to the weight as the velocity of the weight to the velocity of the power, one will be double to the other. If the rope therefore instead of being held in the hand above, pass over a pulley fixed to the beam at top, and from thence pass round a second pulley fixed in the lower block, to which the weight is fastened; this second pulley would give the same advantage, with the first, in raising the weight. Now, while the weight rises one foot, each of the four ropes that go round the two pulleys of the lower block will be shortened one foot, and the hand or power will pass over four feet, and have four times the velocity of the weight; and therefore need be no more than one fourth of the weight to stand in equilibrio with it. So that, let there be ever so many pulleys in the lower block, each of them affording an advantage equal to the first, in raising the weight, the velocity of the power will be increased beyond that of the weight, in the proportion of unity to the number of ropes that pass round the lower block. There will then be an equilibrium, when the power is to the weight, inversely as their velocities, or, as unity is to the number of ropes going to and from the lower block.

There are various other combinations of pulleys, which are easily understood from what has been said. For in all of them, the power is to the weight inversely as their velocities.

THE WHEEL AND AXLE.

The axle and wheel is another simple mechanic power, for raising a weight fixed by a cord to the axis, while the power is applied to the circumference of the wheel by another cord wound about the wheel in the contrary way. Now that the power and weight may be in equilibrio, the power must be to the weight, as the circumference of the axis to the circumference of the wheel, or as their diameters, because these are as their velocities. For while the power is passing over a space equal to the circumference of the wheel, the weight has passed over a space equal to the circumference of the axis, by the cord winding once round it. Now, these spaces being as the velocities of the power and of the weight, it is evident, that the same proportion holds good in this machine, as in the lever and pulley; viz. The power is to the weight inversely as their velocities. The wheel and axis is indeed no other than the lever, or at least may be reduced to it, by considering the center of the wheel as the fulcrum, while two opposite radii, one of the wheel and the other of the axis, constitute the longer and shorter arms of the lever; and as these radii are the respective distances of the power and weight from the common center of motion, or fulcrum, the power and weight are to each other inversely as these radii or distances from the fulcrum; as was shown to be the case in the lever.

It is usual to combine a multiplicity of wheels together in various machines, such as clocks, watches, &c. and the power of the whole combination may be computed from the proportion given above. If, for example, the teeth of a wheel, which are in number sixty round the circumference, drive the wings or pinions of an axis, which are only six in number; it is evident that the wheel must move with one tenth of the velocity of the axis; or that the second axis must move ten times as fast as the first wheel, and therefore will raise ten times the weight of the power, supposing the weight raised to be applied to an axle of the same diameter; but if the cord that raised the weight were wound round an axle of only one fourth of the diameter supposed above; one pound applied to the circumference of the first wheel would raise forty pounds applied to the axis of the second: and so for any number of teeth in any combination of wheels, and any diameters of the wheels and axles.

THE INCLINED PLANE AND WEDGE.

The inclined plane is a machine for raising weights by being rolled up the plane. By this machine a considerable part of the weight is supported, while the remainder is drawn or pushed up by the power. To know when there will be an equilibrium between the power and the weight, we must consider the velocities of each, or the spaces, that are passed over by each in the same time. Now, it is plain, that when the weight has been rolled up the whole length of the plane, and the power, connected with it by a cord, has descended the same distance, the weight is only advanced, in perpendicular altitude, the height of the plane, while the power has descended through a space equal to the length of the plane. These spaces therefore passed over in the same time are the measures of their respective velocities. Hence, as there will be an equilibrium between the power and the weight, when they are to one another inversely as their respective velocities; the power sufficient to sustain a weight on any inclined plane, will be to that weight, as the height or perpendicular altitude of the plane is to the length of the plane: e. g. If the length of the plane be ten times its height, then a power of ten pounds is sufficient to sustain one of a hundred on such a plane.

THE WEDGE.

The wedge is no other than two inclined planes joined together; and is used for splitting timber, or raising weights, by being driven under them by a stroke, whose momentum is here called the power, while the resistance of the timber, or the gravity of the body to be raised is called the weight. When it is therefore put under any weight, the force with which the wedge will lift that weight, when driven under it, by a blow on the end, will bear the same proportion to the force, wherewith the blow would act on the weight, if directly applied to it, as the velocity which the wedge receives from the blow, bears to the veloty, wherewith the weight is lifted by the wedge. herefore the power is to the weight, as half the hickness of the wedge is to its length.

THE SCREW.

The screw is also applied for raising weights, by pressing upon them, when fixed in a frame; and while the power which is applied to the handle of the screw passes over the circumference of the circle in which it moves, the weight is raised only the distance between two contiguous threads of the screw, and these distances are proportional to their respective velocities. Therefore the power is to the weight, as the distance between two contiguous threads of the screw is to the circumference of the circle described by the power in one revolution. For they are to one another inversely as their velocities.

These are all the simple mechanic powers, and of these are all the machines in the world composed, by such various combinations, as may best answer the intention of the artist.

Instead of raising weights, the design of the artist is sometimes only to cause some light body, whose gravity is not regarded, to move with a certain determinate velocity, such as in clocks and watches, orreries, &c. which are usually constructed with wheels, whose circumferences are divided into a certain number of teeth playing in the pinions on the axle of another wheel; so that by the proper adjustment of the number of the teeth and pinions the velocity of the wheels, with respect to one another, may be increased or diminished in any proportion.

These may be easily computed by the proportions above laid down.

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What we have said, in our explanation of the effect of these simple mechanic powers, proceeds upon the supposition, that the parts of the machine suffer no retardation in their motions, by friction, or the rubbing of one part upon another. But great allowances must be made on this account, as it generally destroys one third part of the effect of those machines, that are designed for raising weights. This friction is always in proportion to the weight of the parts of the machine that move on others, and to the velocity of their motion. Now it is of great consequence in the construction of machines, that the parts be as well polished and smooth as the nature of the materials will admit of; and that the velocity of the parts that rub on others, be as much diminished as possible. The friction of the end of an axle is much lessened, by inserting a gudgeon in the end, that it may revolve on this; as the velocity of the rubbing parts is diminished in the proportion of the diameter of the axle, to the diameter of the gudgeon. And the velocity may be still farther lessened, by laying the gudgeon on the circumferences of two wheels, which are carried round by the gudgeon, without any friction, but what arises from the slow motion of the axes of these friction wheels.

Hence we see the reason of laying a heavy body on wheels or rollers, that it may be transported from place to place, with little friction, while the wheel or roller sustains the weight. Wheel carriages meet with less resistance than any other kind of carriages, from the inequalities of the road. The larger that the wheels are, so much easier is the draught, until the radius becomes equal to the height of the horse's breast. The larger that a wheel is, at so much the greater

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distance from the point of contact between the wheel and the ground will it reach an obstacle, over which the load is to be drawn, and thereby affords the advantage of an inclined plane of a greater length, for raising the weight over the obstacle. Besides this, the larger wheels have less friction on their axes, because their velocity is less, than that of small wheels, in passing over any given distance, in a given time. And small wheels sink deeper in a soft road, than larger wheels, and thereby increase the difficulty of transportation. Hence a carriage of four large wheels is drawn with less force, than one of two large and two small wheels. And a carriage with unequal wheels is drawn with less force, when the load is laid on the axis of the larger wheels, than when laid on the axis of the lesser. But these advantages of larger wheels are in some measure counterbalanced by the difficulty of turning in a small and narrow compass. Hence carriages are generally constructed with unequal wheels. And this construction has this farther advantage, that the line of traction through the axis inclines to the horizon before the horses and below their breasts, so that in drawing they in some measure lift the load over any obstacle, while a contrary inclination would press the load into the earth and increase the difficulty of drawing it along.

OF COMPOUND MACHINES.

In a combination of levers, the power will sustain the weight, when the power is to the weight, as the product of all the distances of the weight from the fulcra is to the product of all the distances of the power from the fulcra; or as the product of all the shorter ends of the levers, is to the product of all the longer ends. And in a combination of wheels and axles, the power will sustain the weight, when the power is to the weight as the product of all the diameters of the axles is to the product of all the diameters of the wheels. And in general, in every machine, the power will sustain the weight, when they are to each other inversely as their velocities. So that what is gained in power is lost in time; and it is impossible to do otherwise.

OF FRICTION.

In estimating the power, the weight of the matter of which the machine is made, and the friction of its parts, must be added or subtracted, according as it is for or against the power. The softer or rougher that bodies are which rub upon each other, the greater, in general, is the friction; if their pressure be increased the friction will be increased in the same ratio: and if their velocity be increased, the friction will also be increased, but not in the same ratio. In general, the increase or diminution of smooth surfaces makes no alteration in the friction while the weight and velocity continue the same. Wood slides more easily on the ground in wet weather than in dry, and more easily than iron in dry weather, but iron more easily than wood in wet weather. A cubic piece of soft wood, eight pounds in weight, moving at the rate of three feet per second on a smooth plane of soft wood, has a friction equal to two thirds of its weight nearly; but moving on a smooth plane of hard wood, its friction is equal to about one sixth of its weight: and hard wood upon hard wood has a friction equal to about one eighth part of its weight. Two thirds of the friction of wood upon wood may be destroyed by oil or grease properly applied. The naves of wheels have four times less friction, when greased than when wet. When polished steel moves upon polished steel or pewter properly oiled, the friction is about one fourth of its weight; on copper or lead, about one fifth; and on brass, one sixth of its weight.

The friction of ropes moving on machines depends upon their stiffness, tension, grease and state of the weather. All other circumstances being equal, the difficulty of bending a rope, is as the square of its diameter and its tension directly, and inversely as the diameter of the cylinder or pulley, about which it goes. A rope of one inch diameter tended by five pounds and passing over a pulley of three inches diameter requires a weight of one pound to bend it. In the pulley, the friction is directly as the weight, the velocity, and diameter of the axis; and inversely as the diameter of the pulley. A single pulley loaded with 14 pounds at each end of the rope, requires one pound to be added to one end to raise the weight at the other. And a power of one hundred pounds that would sustain a weight of 500 pounds with a combination of pullies of 5 ropes requires an addition of 50 pounds to raise the weight. In the wheel and axle, the friction on the axle is as the weight, the velocity, and diameter of the gudgeon. On the inclined plane, the friction is according to the matter and surface of the rubbing bodies, and the bending of the rope and rubbing of the pulley, as explained above.

EQUABLE MOTION.

Equable motion is that which is generated in a body by a single impetus; by which the body will pass over equal spaces in equal [illegible] The spaces therefore must be proportional to the times; that is, if it pass over a mile in an hour, it will pass over two miles in two hours, if the velocity continue the same without variation. The spaces also, that are passed over in a given time, must be proportional to the velocity of the motion; for if with a certain velocity it pass over a mile in an hour, with double that velocity it would pass over two miles in the same time. Hence as the spaces depend on both the time and velocity of the motion, they will be proportional to the rectangle of both. Let $S=$ the space, $T=$ the time, and $V=$ the velocity; then $S=TV$. $V=\frac{S}{T}$ and $T=\frac{S}{V}$. Hence the velocity will be directly as the space and inversely as the time; and the time will be directly as the space and inversely as the velocity. And if the spaces be equal or given, the times and velocities will be in an inverse ratio to each other; that is the less the time is, the greater must be the velocity in order to pass over the given space.

ACCELERATED MOTION

IN BODIES DESCENDING BY GRAVITY, IN VACUO.

Accelerated motion is produced by the continual operation of the moving force every instant; generating an additional velocity every moment. And as gravity is such a force as continually exerts its influence on descending bodies, to bring them in right lines towards the center of the earth, their motion must be continually accelerated: and as it acts with equal force on the same body at all times, the increments of velocity produced by its exertion must be equal in equal times; that is the motion of heavy bodies descending freely near the earth's surface, by the force of gravity, in spaces void of all resistance, must be equably accelerated. Secondly, the increments of velocity generated in every indefinitely small portion of time, being equal to one another, the aggregate or sum of them, at the end of the fall must bear a constant proportión to the time in which they were produced: that is, the last acquired velocity is always proportional to the time of the fall. Thirdly, the spaces passed over by descending bodies are as the squares of the times, or as the squares of the last acquired velocities. For the spaces depend both on the times and velocities of the body passing over them; but these times and velocities are proportional to each other; so that one may be substituted for the other. Therefore the spaces will be as the squares of the times, or squares of the last acquired velocities. Hence the times or velocities will be as the square roots of the spaces. Fourthly as the velocity begins from nothing and gradually increases to the end of the fall, it is evident that if the body had moved with the last acquired velocity during the whole time of the fall, it would have passed over a space double to what it passed over in its descent, or it would have passed over the same space with the mean velocity it had acquired in the middle point of the descent. Fifthly, it is found by accurate experiments that a heavy body falls by the force of gravity, in spaces void of resistance, 16.15 feet in the first second of time. Hence by such a fall it would acquire a uniform velocity of 32.3 feet per second. Sixthly, as gravity would generate in the descending body an uniform velocity in the first second, which would in the same time carry it over twice 16.15 feet in another second, it is evident, that if gravity ceased to act, the body would have passed over thrice 16.15 feet at the end of two seconds of time; but as it continues to act with the same force which it exerted in the first second, the space passed over must be four times 16.15 feet at the end of this second. It has then acquired an uniform velocity, that would carry it over eight times the space of 16.15 feet in the third second, if gravity ceased to act; but during the third second gravity continues to act as formerly and will carry the body over another 16.15 feet; so that at the end of the third second, it has descended through nine times 16.15 feet. But in the two first of these seconds, it had passed over four of these spaces, and in the third five of them. Hence the spaces descended through in the separate second of time, will be as the odd numbers, 1.3.5.7.9, &c. And hence as the square of 1 is 1. and the sum of 1 and 3, viz. the sum of the spaces passed over at the end of two seconds is 4, the square of two seconds; and the sum of 4 and 5, the spaces passed over at the end of three seconds, is 9, the square of three seconds; and the sum of 9 and 7, the spaces passed over at the end of four seconds, is 16, the square of four seconds, &c. It is evident that the spaces descended through at the end of any time must be as the square of the time, as we have seen before.

Now as these odd numbers grow large, they approach nearer to an equality, and consequently as they represent the spaces passed over in each separate portion of time, the motion approaches nearer to an equable motion, the longer it is continued.

Bodies projected perpendicularly upwards are retarded by gravity in the same manner as they are accelerated by gravity in their descent. Hence they spend the same time in rising to a given height, as in falling from it. The heights to which they will rise, when projected with any given velocities will be as the squares of the velocities, or the squares of the times of ascent. The times of ascent and descent will be equal; and so will the velocities at any particular point of the line of ascent and descent; and consequently at the place of projection and the end of the fall.*

If the descent be in a resisting medium, the motion will sooner become uniform, the denser the medium is; and as the quantity of matter in the falling body is lessened. But to understand this matter more clearly, we must next consider

THE MOTION OF BODIES IN A RESISTING MEDIUM.

The whole doctrine of resistance in any medium may be comprised in the following articles, which may be easily understood, when considered separately.

1. If a globe move in two mediums of different den-

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* As the spaces descended are as the squares of the times, if a heavy body fall through 16.15 feet in a second of time; in $60''$ or one minute it would fall through the square of 60 such spaces, or 3600 times 16.15 feet = 58140 feet, almost ten miles, at the surface of the earth.

But if the falling body were at the distance of the moon, or 60 semidiameters of the earth from its center, then the force of gravity there would be diminished as the square of the distance was increased, and therefore it would be 3600 times less at the distance of the moon. Consequently it would bring down a heavy body towards the earth but a 3600th part of 58140 feet in a minute, which is 16.15 feet. So that a heavy body at the distance of the moon would require a minute of time to fall as far, as it would fall in one second, at the surface of the earth. Now if the projectile force of the moon was destroyed and it was at liberty to fall towards the earth, it would fall through 16.15 feet in the first minute; and this being exactly equal to the force by which the moon is retained in her orbit, it proves that the moon is retained in her orbit by the force of gravity: as we shall see hereafter.

sity, with the same velocity; it is plain, that the number of particles which it strikes in a given time mus be proportional to the density of the medium, and consequently the resistance will be in the same proportion Let R= the resistance and D= the density of the medium; then R=D.

2. Let the globe move in any resisting medium witl different velocities; the quicker it moves, it strikes th greater number of particles in a given time, and it strike every one of them with a greater force; and as actio and reaction are equal and contrary, the resistance wil be proportional to the velocity, on a double account viz. on account of the number of particles and the forc with which each particle is struck; that is the resistance will be proportional to the square of the velocity R=VV.

3. Suppose two globes of different diameters, bu containing the same quantity of matter, move in th resisting medium with the same velocity, they wil strike every particle with the same momentum, but th number of particles struck in a given time will be proportional to the surfaces or half surfaces of the globes and these surfaces are as the squares of the diameters therefore the resistance in this case will be proportiona to the squares of the diameters. R=dd. (d=diam.)

4. But if we consider the globes as containing different quantities of matter under equal surfaces, on being, for instance, of lead, and the other of cork, bu of the same diameter; the resistance they meet witl will be inversely as their quantities of matter; becaus the greater that the quantity of matter is in a bod moving with a given velocity, the more easily will overcome the resistance by its greater momentum, an the less will it be retarded in the resisti

Now the quantity of matter in globes of different diameters will be as the cubes of their diameters. Therefore the resistance in this case will be inversely as the cubes of their diameters. 1 $R=\frac{1}{ddd}$.

5. Now if we combine these four cases together, and suppose that two globes of different diameters and densities or quantities of matter, move through resisting mediums of different densities, as air and water, and with different velocities; the whole resistance will be in the direct ratio of the density of the medium, the square of the velocity, and the square of the diameters; and in the inverse ratio of the cubes of the diameters. But, because the direct ratio of the square of the diameters combined with the inverse ratio of the cubes of the same makes only the inverse ratio of the diameters simply, the resistance will be proportional to the density of the medium and square of the velocity directly, and to the diameters of the globes inversely. $R=\frac{Dvvdd}{ddd}=\frac{Dvv}{d}$.

In all that I have said on this subject, I have neglected the small resistance arising from the asperity of the moving body, or the tenacity of the fluid medium, which is subject to no certain rules of computation, and is very small in comparison with what arises from the vis inertiæ of the particles of the medium, which we have only considered in the foregoing articles.

From these propositions we are able to account for many phenomena of bodies in motion in a resisting medium; as why the accelerated motion of a body falling by the force of gravity through air or water becomes uniform, when the resistance of the medium, which increases with the squares of the velocity, becomes equal to the constant [illegible] of velocity generated by gravity,—why a bullet will go farther and faster through air, than the same weight of small shot, with a given charge of powder, as the resistance increases with the surfaces of the moving bodies, and as the surface of the small shot is much greater in proportion to its weight than that of the bullet, the shot is most resisted,—why a ball of lead will go farther, than a ball of cork, of the same diameter, with a given force of powder, through the air or in water: the resistance being universally as the quantity of matter in each, or the cubes of their diameters; whereas, if they moved in an unresisting medium, the cork ball would fly quickest and farthest,—why the motion of a body projected downwards through a resisting medium continues uniform throughout; when the velocity with which it is projected is equal to the velocity of a uniform descent in that medium; but if the projectile velocity were greater than that of a uniform descent, the motion would be retarded, and if less, it would be accelerated. Hence an arrow spends less time in rising than in falling, the latter part of its descent being either uniform or nearly so, according to the weight of the arrow. Hence also the slow descent of dust, feathers, and light bodies through the air. And lastly we see the reason why a ball shot with considerably velocity, passes through a board slightly supported, without throwing it down; because of the resistance of the air behind it: and why it might be flattened on the surface of water, without entering far into it, when projected perpendicularly to the surface; or reflected from it, when the direction is oblique to the surface.

If a body be projected upwards in a resisting medium, such as air or water, with a given velocity, it will not rise to the same height it would do in vacuo, because of the resistance of the medium; and consequently falling from a less height, than what is necessary to acquire the first velocity, its last acquired velocity, at the end of the fall, will be something less than that with which it was projected upwards: and it will be still less, on account of the resistance it meets with in its descent.

MOTION ON INCLINED PLANES.

As the motion of bodies on inclined planes is occasioned by gravity, it must be of the same nature with the motion of bodies descending perpendicularly towards the surface of the earth. Part of the gravity of the body that rolls* down the plane is supported by the plane, and the remainder of it, which is called the *relative gravity* accelerates its motion down the plane in an equable manner.

In order to determine what proportion there is between the absolute weight of the body, and that part of its weight which carries it down the plane. Let DFE† be an inclined plane, CFE a body resting on it in the point ECE being perpendicular to the plane, while CFB is perpendicular to the horizon. Now since the body gravitates in the direction CFB, let CF represent its absolute force of gravity. This force may be resolved into two, by the second law of motion, viz. CE and EF, of which CE being perpendicular to the plane, and being destroyed by the reaction of the plane, is that part of its gravity that is supported by the plane, and EF represents its *relative* gravity by which it is carried down the plane. Now by the similarity of triangles EFC, and BFD, CF : FE : : DF : FB. That is, the absolute force of gravity is to the relative force on the plane, as the length of the plane DF, to its perpendicular altitude FB. If the plane therefore be elevated, so that its altitude is half of its length, one pound, hanging perpendicularly over the end of the plane, will be in equilibrio with two pounds resting on the plane.

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* In the theory of bodies moving down inclined planes, as the effect of friction is not taken into the account, they must be supposed to *slide* and not to *roll*, since this kind of motion can only be produced by friction.

† See Plate I. Fig. 4.

Now since the absolute is to the relative force of gravity as the length of the plane is to its height, the spaces over which they would carry the body in the same time must be in the same proportion; for the effect must be proportional to the cause: that is the space passed over by the absolute force of gravity in a perpendicular descent is to the space rolled down the inclined plane by the relative force of gravity, as the length of the plane to the height of it; or as BF : FI. So that if BI be perpendicular to the plane, drawn from the horizon, FI will be the space the body will roll down the plane, in the time that it would fall perpendicularly through its altitude.

Hence, since every angle in a semicircle is a right angle, if a semicircle be described on the altitude of a plane as its diameter, it will intersect all the planes that have the same altitude in the points to which a heavy body would roll down, in the time it would fall through the perpendicular altitude or diameter of the circle. That is, a heavy body will fall through the diameter AB* of the circle, in the time that it would roll down AF on the plane AFC, or AG, on the plane AGD, or AI, on the plane AIE.

Hence a heavy body will roll down any of the chords of a circle, that are connected with the lowest or highest point of the diameter, in the same time that it falls through the diameter. This is the fundamental proposition of the doctrine of pendulums, as we shall see, when we have said something farther of the inclined plane.

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* See Plate I. Fig. 5, 6.

As the motion on an inclined plane is uniformly accelerated by gravity, it must observe the same laws with bodies falling freely by the force of gravity. That is to say,

The spaces passed over, or the lengths of the planes, must be as the squares of the times, and as the squares of the last acquired velocities at the end of the descent: for the velocities are always as the times, when the planes are similarly situated.

As the velocity acquired at the lowest point, in perpendicular descents, is always the same from any given altitude, so the velocity acquired by rolling down any planes, that have the same perpendicular altitude, must also be the same. And hence it follows, that if a body roll down any number of planes through the same perpendicular altitude, the velocity acquired at last, will still be the same as if it had fallen perpendicularly; it may therefore roll down a curve, which may be considered as an infinite number of planes, having one given altitude, though not in the same time, it would fall through the perpendicular, yet acquiring the same velocity at last.

The times of rolling down planes of different lengths, but of the same altitude, must depend on the lengths of the planes: for the longer that the plane is, the less it is inclined to the horizon, and the more of the weight of the body will it sustain, leaving the relative velocity less, by which alone it is carried down the plane; and therefore as the force is less in proportion to the length of the plane, in the same proportion must the time necessary for rolling down such a plane be increased: that is, the times of descent along planes of the same altitude, but different lengths, must be proportional to their lengths. It is the same relative gravity, that acts on the body when rolling down planes similarly situated, and then the times of descent will be proportional to the square roots of the lengths of the planes. But when the body descends along planes of the same altitude and differently inclined to the horizon, the relative gravity lessens with the inclination of the plane, and the body must therefore move slower, in the same proportion. As the length of the plane or its inclination to the horizon does nothing but lessen the force of gravity, it follows, that the effect, viz. the space passed over, must be lessened in the same proportion. That is, if a certain force of gravity would carry a body down an inclined plane 10 feet in a second, half that force would carry it down but 5 feet in the same time; and therefore this half force must require two seconds to carry the body 10 feet down the plane; and this half force is occasioned by the increased length of the plane. Therefore the times of descent through planes of different lengths, are as the lengths of the planes, when the altitudes are the same.

The velocity acquired in falling down the different chords of a circle, connected together at the lowest point of the diameters, is as the lengths of these chords. Because these velocities are as the square roots of the perpendicular altitudes, or versed sines of these chords; and the square roots of these versed sines are as the lengths of the chords. For $BF^2 = AB \times BH$, and since AB is a given quantity, BF is proportional to $\sqrt{BH}$, which is as the velocity.

We have now prepared the way for considering the important doctrine of the

MOTION OF PENDULUMS.

A PENDULUM is any body suspended by a thread and movable about any fixed point. The most material properties of the pendulum are such as these that follow, viz.

1. The vibrations in very small arches are performed nearly in equal times. Could the pendulum vibrate in the chord of a circle, then all its vibrations would be performed exactly in equal times; as the time of falling through all the chords of a circle is the same. And when the arches are very small, they nearly coincide with the chords. But when it vibrates in larger arches, it will lose time, as they are considerably longer than their correspondent chords, and require a longer time for their description by the pendulum.

2. The velocity of the pendulum at the lowest point is nearly as the length of the chord of the arch it has described in its descent: and is just sufficient to carry it up to the same altitude on the other side, did it meet with no resistance from the air.

3. The time of a complete vibration, from its greatest altitude on one side to its greatest altitude on the other, is equal to the time of descent through the diameter of a circle whose radius is four times the length of the pendulum, or through a space that is eight times the length of the pendulum.* For in half the time of a

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[*] It is demonstrable, that the time of one entire vibration in a very small arch of a circle, or in any arch of a cycloid, is to the time of a heavy body's descending through half the length of the pendulum, as the circumference of a circle to its diameter. Or, which is the same thing, the constant number 4.93482 multiplied by the length of the pendulum, will give the space through which a body will descend by its own gravity in the time of one vibration.—*Ed.*

vibration, a heavy body would descend through the diameter of a circle, whose radius is the length of the pendulum, and in the other half of the time, the heavy body continuing its motion, will have passed over four times the space counted from the beginning of the descent; that is eight times the length of the pendulum.

4. The times of vibrations of different pendulums are as the square roots of their lengths. Because they are proportional to the times of descent through spaces equal to eight times the lengths of the pendulums, and these times of descent are as the square roots of the spaces.

5. Since the lengths of pendulums are as the squares of the times of vibration, if a pendulum, whose length is 39.2 inches vibrate once in a second, as it is found to do, then a pendulum of one fourth of this length, that is 9.8 inches, will vibrate in half that time.

6. The rod of the pendulum is here supposed without weight; but if a uniform body, one third longer than the pendulum, were suspended by the end, it would vibrate in the same time with the pendulum. For the whole force of such a body is the same as if the whole matter was collected in a point two thirds distant from the point of suspension. This point is called the center of oscillation, or of percussion; because it will give the greatest stroke possible.

The length of a pendulum and the time of its vibration depending upon each other, it cannot be an exact measure of time, unless we could secure the continuance of the same length in all the varieties of heat and cold. If the rod of the pendulum be made of metal, it will expand and lengthen by heat, and contract by cold; so that in the first case the pendulum will lose time, and in the last gain time. By many experiments made with the *pyrometer*, it has been found, that with the same degree of heat, the expansion of several kinds of metal will be in the following proportion; viz. Iron 80; steel 85; copper 89; brass 110; tin 153; lead 155.

Of all metals iron, being least expansible by heat, is properest for the rods of pendulums. Wood does not lengthen much by heat; and therefore, when well dried, it will answer the purpose better than metal. But glass will answer still better, if kept of the same length.*

Various methods have been suggested to remedy this inconvenience in pendulums. If a pendulum be made of a glass tube partly filled with mercury, while the tube expands by the heat and sinks the center of oscillation, the mercury will also expand, and rising in the tube will raise the center of oscillation as much as it was depressed when they are properly adjusted to each other; and thereby the length of the pendulum and consequently the times of vibration continue invariable.

Pendulums have also been compounded of different bars of iron and brass, in such a manner, that while the center of oscillation is depressed by the expansion of some, it is equally raised by the expansion of others. In this manner the pendulums for astronomical clocks are generally made.

The loss of time occasioned by the vibrations being performed in larger arches at some times than at others, by an unequal application of force to the wheels, has been in some measure prevented, by a flat board fixed on the pendulum, to give it a retardation from the resistance of the air, to moderate the velocity given by an unequal application of the weight to the wheels.*

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* Glass will expand regularly and equably by heat; whereas metals expand irregularly, when they have imbibed a certain quantity; and the increase of length in a metallic pendulum is not exactly proportional to the degree of heat.

* If a pendulum be made to vibrate in a cycloid, all its vibrations, whether through smaller or larger arches, will be performed in equal times.

If a circle perform a revolution on a right line, any point in the circumference will describe on an opposite plane, a curve called the cycloid; whose axis is equal to the diameter, and whose base is equal to the periphery of the generating circle.

If the generating circle (see plate 2, fig. 1.) DBI move along the line AEF, any point as A will describe the curve ABVGF, called the cycloid. From the generation of the curve, it is evident, that the arch of the circle $BD=AD$, and the arch $BI=DE=KL=BN$, that the cords BD,NE are parallel, as also BI,NV, that BD is perpendicular to the curve, and that BI is a tangent to the cycloid at the point B, because DBI is a right angle.

The arch of the cycloid BV, cut off by a line BNG parallel to the base, is double of the corresponding chord BI or NB of the generating circle.

Let POR be drawn infinitely near to BNG, cutting VN produced in S, and the generating circle VNE in O, and OT be drawn perpendicular to VS; draw also the tangents QV and QN of the circle at the points V and N, then the triangles QNV,ONS will be similar as QV is parallel to SO, and because QN and QV are equal, SO, and ON will be equal, and NT the increment of the chord VN, will be one half of NS or BP the simultaneous increment of the arch VB, as $VT=VO$. Therefore as the arch of the cycloid increases twice as fast as the corresponding chord of the generating circle it must, in every situation, be double of the chord Q.E.D.

If then a pendulum be suspended at the point H by a flexible thread applying to the semicycloidal cheeks AH and FH, so that it may vibrate in the cycloid AVF, all its vibrations through larger or smaller arches will be performed in the same time.

If the pendulum descend from R or G, the last acquired velocity will be as OV and NV respectively, or as $2OV=RV$ and $2NV=GV$. Therefore, when the velocities of any kind of motion are as the spaces moved through, the times of description must be the same. Hence whether the pendulum descend from R or G, it will arrive at V in the same time.

MOTION OF PROJECTILES.

A PROJECTILE is a heavy body thrown in any direction. The force with which it is thrown is called the *impetus* and as its velocity is measured by the height, from which a body must fall to acquire this velocity, this height is also called the impetus. The *random* or *amplitude* of the projection is the horizontal distance to which the body is thrown. The angle which the piece of ordnance makes with the horizon is called the *elevation:* and the perpendicular height to which the ball rises above the horizon is called its *altitude.*

A projectile is acted upon by two forces in different directions, viz. the force of gravity, which acts in right lines directed to the center of the earth, by which alone, it would pass over spaces proportional to the squares of the times; and by the projectile force, which acts by a single exertion, and therefore produces an equable

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To make a pendulum vibrate in a given cycloid DKG;* let two semicycloids ABD, and AG, of equal magnitude with the given cycloid be described, whose vertices shall be at D and G the ends of the base, meeting together in the axis of the given cycloid produced in A, making AK double of the axis FK. If a pendulum be suspended at the point A, and of such a length as to reach to K, and in its vibration apply to the semicycloidal cheeks it will still be in the cycloid, in every part of its vibration.

Let the pendulum be in the direction ABP; then, because ABD=ABP=AK.BD=2 DC=BP=2 BE; therefore BC and PI will be equally distant from the base DF, and will cut off equal portions DC and IF of the generating circles, and the chords DC and IF will be equal and parallel, the tangent BP being parallel to both: therefore EF=PI. And since BC or DE=the arch DE or the arch IF, PI or EF will be equal to the remainder of the arch IK: and the point P will be in the cycloid DPK.

* See Plate II. Fig. 2.

motion, in which the spaces are proportional to the times, and consequently the squares of the spaces are proportional to the squares of the times.

Now as the projectile is acted upon by two forces in different directions, it will, by the second law of motion, describe the diagonal of a parallelogram whose sides are proportional to the forces. But as one of the forces produces an equable motion, and the other an accelerated motion; the spaces passed over by these forces must have the different properties of these motions; that is, one must be proportional to the square of the other; and the sides of the parallelogram must have the same property. The projectile will therefore move in the diagonal of a parallelogram, of which one side is constantly proportional to the square of the other. Now this is a known property of the parabola, whose abscissæ and ordinates are in this proportion. If these parallelograms be supposed therefore to be infinitely small, their diagonals will coincide with the curve of the parabola, and therefore it will move in that curve, or at least nearly so. The difference will be hereafter accounted for.

If ADB* be a parabola, AF a diameter, and AE a tangent at the point A, then AF will be an abscissa and FD parallel to the tangent will be an ordinate, and from the known property of the parabola AF=DE will be proportional to the square of FD=AE.

Let a body be projected from the point A in the direction AE, with such a force as would carry it over the space AE in a given time, then AE=FD will be proportional to the time, and the square of it to the square of the time. But while the body is carried over the space AE by the projectile force it is carried over the space AF=ED by the force of gravity, and this space, by the laws of falling bodies, is proportional to the square of the time, or to the square of AE or FD. Therefore the body will be found at the end of the time at D, in the curve of the parabola. Q.E.D.

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* See Plate II. Fig. 3.

The principal properties and laws of this kind of motion are such as these.

1. The horizontal ranges or amplitudes are proportional to the rectangle of the sines and cosines of the angles of elevation, or to the sines of double the angles of elevation; as these are well known to be the same.

2. The elevation of 45 degrees will give the greatest possible horizontal range.

3. The elevations equally above and below 45 degrees give the same ranges.

4. The times of flight through the air are proportional to the sines of the angles of elevation.

5. The greatest altitudes of projections are proportional to the squares of the sines of the angles of elevation, or to the versed sines of double the angles of elevation.

6. The velocity of a projectile at the point of projection, and when it meets the horizon, is such as it would acquire by falling through one fourth of the parameter of the diameter that passes through the point of projection.

7. The greatest range, at an elevation of 45° is double the height fallen through to acquire the velocity of the projection.

These propositions may be demonstrated in the following manner.

Let s and c = the sine and cosine of the angle of elevation EAC to the rad. 1.

a=16.1 feet, the space a body falls in a second of time.

d=the space the projectile passes over in a second with its given velocity=V.

T= the time of flight.

H= the greatest altitude of the projection.

x=AC or AB, the horizontal random.

As $c : x :: 1 : \frac{x}{c} = AE$.

$d : 1'' :: \frac{x}{c} = AE : \frac{x}{dc} = T$ = the time of flight, and $TT = \frac{xx}{ddcc}$.

$1''^2 : a :: TT = \frac{xx}{ddcc} : \frac{axx}{ddcc} = ED = DC =$ Height of the projection=H.

$c : x :: s : \frac{sx}{c} = EC$.

Now, when AC becomes equal to AB, or x=the whole horizontal range; then ED=EC; that is, $\frac{axx}{ddcc} = \frac{sx}{c}$. Hence $x = \frac{dd}{a} \times sc$. That is, the horizontal ranges are proportional to the rectangle of the sine and cosine of the angle of elevation, which rectangle is well known to be proportional to the sine of twice the angle of elevation. This is the first proposition. As the sine of twice 45 degrees is = rad. the greatest sine possible, the angle of 45 degrees will give the greatest random possible.

As any two angles equally above and below 45 degrees being doubled are the supplements of each other, and have the same sine, these elevations must give the same range.

Again, since $T=\frac{x}{dc}$, and $x=\frac{dd}{a}\times sc$, $T=\frac{d}{a}\times s$. That is the time of flight is proportional to the sine of the angle of elevation: for $\frac{d}{a}$ is a constant quantity. When $AC=CB$, then $x=\frac{dd}{2a}\times sc$: and since $H=\frac{a}{ddcc}\times xx$, and $xx=\frac{d^4}{4aa}\times sscc$, $H=\frac{dd}{4a}\times ss$, that is, the heights of projections are proportional to the squares of the sines of the angles of elevation, or to the versed sines of double the angles of elevation.

If the projection were made perpendicular to the horizon, then the angle of elevation is 90° and its sine is equal to radius, or $ss=1$, and $H=\frac{dd}{4a}=$the height of the projection, or the distance from whence a heavy body must fall, to acquire the velocity of the projectile, when it leaves the ordnance. Now this height, commonly called the impetus of the projection or engine, is one fourth of the parameter of that diameter of the parabola that passes through the point of projection, or one fourth of a third proportional to the abscissa and ordinate of that diameter: Because $ED : AE :: AE :$ parameter; that is $\frac{axx}{ccdd}=ED : \frac{x}{c}=AE :: \frac{x}{c}=AE : \frac{dd}{a}=$ parameter, which is four times the altitude of the projection, when made perpendicularly, or the distance, from whence a heavy body must fall to acquire the velocity with which the ball is projected. But farther, since we had the horizontal range, $x=\frac{dd}{2a}\times sc$, and the height of the projection, $H=\frac{dd}{4a}\times ss$; when the angle of elevation is 45°, sc is equal to ss, and then x, the horizontal range at the elevation of 45°, is equal to twice the height from whence a heavy body must fall to acquire the velocity of the projection.

Having given the laws of projectiles with their demonstrations so far as is necessary for any computations of the ranges, altitudes, velocities, or times of flight, I shall subjoin a few remarks concerning practical gunnery, which result from the demonstrations already given.

If it were required to find the elevation of the engine necessary to strike a given point B, from the place A; join AB, and at A erect AE perpendicular to the horizon and equal to four times AL, or four times the height from whence a body must fall to acquire the velocity with which the projectile is to be thrown. Through the points A and E describe a circle that shall touch the line AB in the point A; at B erect the line BGH perpendicular to the horizon cutting the circle in G and H. Then AH or AG will be the direction to strike B.

The time of flight in the lower elevation will be less than that in the other, in proportion to the sines of the angles of elevation.

When the points H and G coincide in the point I, the direction AI is the only one by which the point K can be struck with the given velocity. AK is therefore the greatest horizontal range and is equal to twice AL or half of AE; therefore the angle IAK=45°, when K is in the horizon: According to what was demonstrated before; that the greatest horizontal range was double of the height from which a heavy body must fall to acquire the velocity with which the projectile was thrown. And when the projection is made with the elevation of 45° the height of the projection is one fourth of the greatest random.

If nothing be required of the engiueer but to hit the object, it is better to use the constant angle of 45° and let the randoms be determined by the quantity of powder; because much powder will be saved by this means, and the engineer will be more certain of hitting his mark, than at any other elevation. An error of a degree in the elevation near 45° will not produce a sensible error in the distance which is proportional to the sines of double the angles of elevation. Now the sines of the angles near 90° do not differ much from the radius. The position is then evident.

The forces necessary to give a ball any degree of velocity, being as the square roots of the distances fallen through to acquire that velocity which will carry it over the proposed distance, must be as the square roots of these distances or horizontal ranges.

If it were therefore required to find what charge at an elevation of 45° would carry a ball over 3600 feet, from a cannon whose greatest random was 8000 feet, and the requisite charge of powder 28 pounds; this proportion will answer the question. As $\sqrt{8000} : \sqrt{3600} :: 28 : 18\frac{2}{3}$ = the charge required to carry the ball 3600 feet.

Still we are to remember, that this conclusion is mathematically true only upon the supposition that all the powder exerted its force in the bore of the piece. But we find, that some of it does not take fire, especially in high charges, until it be out of the mouth of the gun; which will occasion some variation, and an allowance must be made for it in practice. It is necessary also to be remembered, that the heating of the cannon, after two or three shots, will lessen the cohesion of its particles, and make it more liable to burst; while the sudden drying of the powder will occasion more of it to take fire in the bore of the piece, and produce a greater force, either to overshoot the mark or split the cannon. On both of these accounts, the charges to be successively used, in a small space of time, should be gradually lessened.

If you wanted to find the impetus of a cannon, with a given charge of powder, let the cannon be shot off at the elevation of 45°, and half the horizontal range is equal to the impetus, or distance from which a heavy body falls to acquire the velocity of the ball when it leaves the mouth of the cannon. This velocity is thus determined.

In order to find the velocity at parting from the cannon, we must remember, that the last acquired velocities in perpendicular descents are as the square roots of the spaces passed over, and that a descent through 16.15 feet will generate a uniform velocity of 32.3 feet per second. Therefore say, as $\sqrt{16} : 32 :: $ the square root of the impetus: the velocity of the ball at parting from the cannon; or so is the square root of half the greatest random, to the velocity. Or shorter, thus; since $\frac{32}{\sqrt{16}}=8$; eight times the square root of half the greatest range is the velocity of the ball, both when it leaves the cannon, and when it meets the horizon. Now this velocity multiplied by the weight of the ball, will give the momentum, or force, with which it strikes the object.

But in throwing bombs, which are not designed to do damage by their projectile force, we do not so much regard their momentum, as the time of their flight, by which the length of their fuses is to be determined, that they may burst as soon as they fall to the horizon. Now, in order to compute the time of flight, we must remember that it is equal to the time of ascent and descent through the altitude of the projection: and that the times of descent through spaces, by the force of gravity, are as the square roots of the spaces; and also that a heavy body falls 16 feet in one second. Therefore say, as $\sqrt{16}=4 : 1$ second of time : : the square root of the height of projection : seconds of time spent in the ascent; and this doubled is the time of flight in seconds when the height is measured in feet. Therefore half the square root of the height is the time of flight in seconds. But when the projection is made at the elevation of 45° the height of the projection is one fourth of the greatest random. Therefore at this elevation, one fourth of the square root of the greatest random is equal to the time of flight in seconds. And for other elevations, say, as sine 45 is to one fourth of the square root of the greatest range, so is the sine of the elevation, to the time of flight in seconds; the ranges being measured in feet.

The line of direction is always a tangent to the parabola in the place of the engine; because gravity, acting constantly upon the body as soon as it leaves the cannon, must bring it down towards the earth: in half a second causes it to descend 4 feet, in a whole second, 16 feet, and in two seconds, it will be found no less than 64 feet below the line of direction, when that is horizontal. Hence there is no such thing as a point-blank shot, as it is commonly called. Let the projectile motion of the ball be ever so rapid, gravity notwithstanding acts upon it as much as if there were no horizontal motion at all.

The line of elevation being a tangent to the curve, in the place of the engine, will bisect the angle between the zenith and the focus of the parabola, as is demonstrated by the writers on conic sections. It is also a known property of the parabola that the distance of the focus from any point in the curve is one fourth of the parameter of that point, or equal to the impetus of the engine; from whence it follows, that if a circle be described round the place of the engine with a radius equal to the impetus, it will pass through the focus of the parabola described with that impetus, and consequently through the foci of all the parabolas that can be described with that impetus, with any elevation. Hence also it follows, that when the elevation is 45°, the focus of that parabola will be in the horizon, and the foci of any two parabolas, that have the same horizontal random will be equally above and below the horizon, and the one perpendicularly above the other. It follows also, that if two circles be described round any two points of a parabola, with radii respectively equal to one fourth of their parameters; if they intersect each other it will be in two points, which will be the foci of two parabolas that pass through the centers of the circles; and these foci will be equally above and below the line that passes through the intersections of the parabolas or centers of the circles: but if the circles only touch one another, it will be in the focus of the only parabola that can pass through both the points, being described by the same impetus. Now should one of these points of the parabola be considered as the place of the engine, and the other an object to be struck, either above or below the horizon; the focus of the parabola will be in the line that joins the engine and object, when the object is at the greatest distance that can be reached, on any plane either ascending or descending, with a given impetus. And the direction to hit the object will be the line that bisects the angle between the zenith and object. But if the object be at a less distance, the foci of the two parabolas that pass through the object will be equally above and below the line that connects the object and engine, and the directions to strike it will be equally above and below the line which bisects the angle between the zenith and the object.

MOTION OF BODIES ROUND A CENTER.

In our lecture on the motion of projectiles, we supposed that the moving body was acted upon by an impressed force in a certain direction, called the *projectile force*, at the same time that gravity acted upon it in right lines parallel to each other; and consequently that the central force was placed at an infinite distance, and that it acted continually with the same degree of strength without any change arising from the projectile's approach to the center of motion, or recess from it. Both of these suppositions are necessary to cause the projectile to move in a parabola; though neither of them is strictly true: for gravity does not act in lines parallel to each other, but in lines directed to a center, and it increases in strength as the squares of the distances from the center decrease. But because of the great distance of the center of the earth from its surface, and the small distance to which we can throw any body, there is no sensible error committed in computing the projectile's motion in a parabola.

But when the distance of the central force bears a considerable proportion to the distance which the projectile passes over, and the central force acts in right lines directed to a center, the case will be different, and the path of the projectile will be no longer a parabola, but some other curve, such as a circle, or ellipsis. The effect of the central force is to cause the moving body on which it acts, to deviate gradually from the straight line in which it would proceed, if undisturbed by any other force, and to describe an incurvated line, which will always be concave towards the central force; whether it approach towards, recede from, or keep at the same distance from the center of force.

Suppose the line AB* to be a tangent to the circle described about the center C, the place of the central force, and the point of contact to be A. If a body were projected by any force in the line AB, it would continue to move in it, if undisturbed by the central force: but as soon as this begins to act, it will gradually cause the projectile to leave the direction AB, and to move in some other line, between the line AB and the line AC; viz. in the diagonal of a parallelogram, whose sides are proportional to the two forces, that acted upon it. If the central force be supposed to act incessantly, while the projectile force acts by a single impulse, there must be a continual deviation from the direction AB, and even from the diagonal of a parallelogram; so that the diagonal will degenerate into a curve which is concave to the central force, and convex towards AB.

The distance of the projectile from the central force may continue invariable, through the whole of its progress, when the central and projectile forces are so balanced, that the one draws it as much towards the center of force, as the other draws it from it. What this balance is, we shall hereafter see, and demonstrate to you, that when the velocity of the projectile is such as it would acquire by falling freely one half of the way to the center, it will, in that case, keep continually at the same distance from the center, and revolve in a circle; of which this is a known property, that in every point a ray drawn from the center is perpendicular to the direction of the projectile, which is a tangent to the circle.

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* See Plate III. Fig. 2.

If this angle, instead of being a right angle, be obtuse on the side to which the body moves, then it will recede from the center of force, and continue to fly from it, as long as it remains obtuse; but if it be acute, the moving body will approach the center, as long as the said angle continues to be acute; the effect of the said central force being in the first case to retard the recess of the moving body from the center, and in the other case to accelerate its approach.

If, while the projectile increases its distance from the center of force, the central force retain sufficient strength to make the obtuse angle above mentioned to become less and less obtuse, till it terminate in a right angle at last, that is, when the line from the center of force to the moving body becomes perpendicular to the curve; from that moment the moving body will no longer recede from the center, but in its subsequent motion it will again descend, as the angle now becomes acute; and in its approach to the central force it will describe a curve, in all respects similar to the curve described in its ascent; provided the central force continues to act every moment with the same strength at equal distances from it. Because if we suppose that the motion of the projectile was stopped, when the aforesaid angle became a right angle, and the body thrown back in a contrary direction to that in which it was moving, and with the same velocity, it must, by the joint operation of these forces, describe the same curve over again, which it described in its ascent, and acquire the same velocity with which it was projected at first. The curve therefore which it describes in its progressive motion, while it is approaching the center of force, being described by the operation of the same forces, must be altogether the same.

Now if the curve, described from the time that the said angle is a right angle, until it become a right angle again, first increasing in its obtuseness to its maximum, and then decreasing, be a complete semi-ellipsis, described in absolute space, then the other part of the curve, which the projectile will describe, in its progress, while the said angle increases, and then decreases, in the degree of its acuteness, must also be a complete semi-ellipsis; and the body will have arrived at the precise point, from whence its motion began. And this being the case in every subsequent revolution, the points where the said angles become right angles, which are called the *apsides* of the orbit, will remain stationary.

If the portion of the orbit described within the aforesaid limits be less than a semi-ellipsis, the projectile will have arrived at its greatest distance from the center of force, before a complete revolution round the central force was finished in absolute space. From this point however, where its distance is greatest, called its *aphelium*, it will begin to descend again towards the center; and in its next revolution, it will come to its greatest distance before it arrives at the same point again in absolute space; and this being the case in every subsequent revolution, the aphelia or apsides will have a retrograde motion, and will be found in all directions, om the center, after a certain number of revolutions of he projectile.

But if, on the other hand, the portion of the orbit decribed between the times, when the revolving body is t its greatest and least distances from the center of orce, be more than a semi-ellipsis, it will have performd more than a revolution, when it has arrived at its reatest distance from the center, where we suppose its notion began; and this being the case in every subseuent revolution, its apsides will have a progressive notion, in the direction of the moving body, and by his progressive motion, they will perform in due time complete revolution round the center of force.

Thus we see how a revolving body, which is contantly attracted by a central force in any given point, nay by its projectile velocity be kept from falling into he center of force, and describe about it an endless ircuit; sometimes approaching towards the center, and gain receding from it, within certain limits; while the entral force preserves the same force at the same disance. When we come to astronomy, we shall see that his central force resides in the sun, and that it regulates ll the motions of the planets and comets that belong to he solar system.

Before Sir Isaac Newton, no man ever advanced any olerable and consistent hypothesis to account for the notions of the heavenly bodies. He found by considerng the nature and effect of that power, which caused a eavy body to descend in a right line directed to the enter of the earth, that it was sufficient to retain the noon in her orbit*, and indeed not only the earth and moon, but also all the planets and comets in their orbits round the sun as their center; and therefore concluded that matter was endowed with such a force. Hence he called this central force, by which the planets and comets are retained in their orbits, and all their motions governed, *gravitation;* without pretending to explain its nature or the mode of its operation. He only called it by the name of gravitation, because he found it to be precisely the same with that principle of gravity, which causes a heavy body to descend towards the center of the earth.

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* If $AE=$the space which the moon describes in a minute of ime, it will be found to be $33''$ nearly. And AF will be the versed

But whether this power be mechanical or not, we cannot determine, with sufficient certainty. From accurate experiments and mathematical inductions, he has accurately explained the laws of its operation, and every day's experience confirms the conclusion he has drawn, from what he had observed, concerning this extensive principle of motion; which reaches the whole solar system, and possibly all the systems of the universe. Observation proves that this principle increases in force, as the squares of the distances decrease, from the central

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sine of 33″, equal to 16 feet in the moon's orbit. She performs her revolution in $27^d\ 7^h\ 43'=39343'$. If $39343' : 360^\circ :: 1' : 33''$ of a degree. As the radius of the earth is 3985 miles, the radius of the moon's orbit is 60 times as great, viz. 239,100 miles = 1,262,448,000 feet. Now AC : AF :: rad. vers. s. 1,33″ :: 1,262,448,000 : 16 feet nearly. Therefore the force which keeps the moon in her orbit, would cause her to descend 16 feet in a minute of time, if her projectile force were destroyed. We have proved before, in considering the descent of heavy bodies by the force of gravity, that a heavy body would fall 16 feet in a minute of time, if it were removed 60 semi-diameters from the center of the earth. Therefore the force of gravity is sufficient to retain the moon in her orbit. See Plate IV. Fig. 1.

body, in which it resides; as light from a luminous body, which we know to be material, is found to do.

Hence some have been led to conclude, that gravity was produced by some subtile medium pressing towards the sun and planets, and acting by contact on them, as all mechanical causes act. But on the other hand, when it is considered, that it is proportional to the quantity of matter in bodies, without any regard to their surfaces, and that it acts as freely upon the internal, as on the external parts of bodies; it seems to surpass the power of mechanism, and to be the immediate effect of the divine agency; or at least the result of an original law, impressed upon all matter, by the hand of Omnipotence. Every particle of matter about us, however minutely divided, is possessed of this principle, in exact proportion to its quantity of matter; as we find, that, in vacuo, all bodies, whether great or small, fall in the same time and with equal velocities; which is a necessary consequence of the supposition, that the earth acted upon every body near it, with a force, proportioned to the quantity of matter it contained. But as all matter is found to be inert, some are of opinion, that it is incapable of receiving any law impressed upon it, even by Omnipotence. It seems to me however, as reasonable to suppose, the Deity capable of impressing any law he pleases upon matter, as to believe that he gave it extension, solidity, or existence.

The manner of both is equally inconceivable by us, but neither of them implies a contradiction in our ideas; and therefore both may be within the acknowledged power of Omnipotence.

But however this may be, and however difficult it is to explain the cause of gravity, or the mode of its operation; yet Sir Isaac Newton has gone a great way towards it, by showing, that every particle of matter is indued with this force; and that a globe attracts any body, in the same manner, and with the same degree of force, as if the whole quantity of matter in the globe, were collected into its center. The united observations of astronomers conspire to prove, that from the continued operation of the central force, which is called gravitation, and the original projectile impetus, given by the hand of the Almighty, to the planets and comets, their curvilineal motions round the sun are continued. What is the particular degree of strength, in the central force, and its particular adjustment to the projectile force, to produce this effect, is the business of mathematics to determine: and we must deliver you the fundamental theorems and propositions, which are necessary for understanding the sublime science of astronomy.

PROPOSITION I.

The central force sufficient to retain a revolving body in a circle is equal to the square of any arch of that circle described in any given portion of time divided by the diameter of the circle.

Let a body at A,* revolving in the circle AED, about the central force at C, move from A to E in a very small portion of time. Let AH represent the space it would pass over in the direction of the tangent, in the same time, by virtue of the projectile force alone, and let HE or AF represent the space it would pass over by the central force alone in the same time, which force is sufficient to retain the revolving body in the circumference of the circle. These spaces therefore will represent the respective forces. We have seen from the second law of motion, that the revolving body cannot move either in the direction AH, or AF, but in the diagonal of a parallelogram, whose sides are proportional to these forces; that is, in the arch of the circle AE, supposing it indefinitely small, and coincident with the diagonal. But by 8.E.6. AD : AE : : AE : AF $=\frac{AE^2}{AD}=$ the central force. Q.E.D.

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* See Plate IV. Fig. 1.

Now if we put C= the central force, and 2R= the diameter, we have $C=\frac{AE^2}{2R}$.

1. Cor. Since the arch AE is proportional to the velocity with which it is described; if we put AE=V, we shall have the central force proportional to the square of the velocity directly, and inversely as the radius of the circle; that is $C=\frac{VV}{2R}$.

2. Hence if two bodies revolve in equal circles about different central forces, these central forces must be proportional to the squares of their velocities. C=VV. That is, if the projectile velocity of one body be twice as great as that of the other, the central force of the first must be four times as great as that of the second, to retain them in the same circular orbit.

3. If two equal bodies revolve with the same velocity in different circles, their central forces must be inversely as the radii of the circles; $C=\frac{1}{2R}$, when the velocity is given, or the same in each.

PROPOSITION II.

The space a body must fall through, by the action of the central force, to acquire a velocity sufficient to balance the central force, and make it revolve in a circle, is equal to half the radius.

Let this space be represented by x. Now the velocity acquired at the end of any fall is such as would carry the body over double the space in the same time, therefore the velocity of the projectile may be represented by twice the space it must fall through, to acquire it, that is, $2x=V$, and consequently $4xx=VV$. But farther, the space that a body passes over by the central force alone, is proportional to that force, in a given time: therefore, by the first proposition, $C=\frac{VV}{2R}=x$. Now substituting for VV its equal 4xx, we have $\frac{4xx}{2R}=x$, or $x=\frac{R}{2}$; that is, the spaces passed over by the action of the central force alone, to give the revolving body sufficient velocity to keep it in the circle, is equal to half the radius of the circle. Q.E.D.

Now this is the exact adjustment between the central and projectile forces, necessary to cause a body to revolve in a circle. Hence it is evident, that if this projectile velocity be increased in any degree, the revolving body will leave the arch of the circle, and revolve in a curve between it and the tangent; and if it be diminished, it will fall within the circle, and move in a curve more distant from the tangent; the central force being supposed to continue the same. From hence we derive an indisputable argument to prove, that the orbits of the planets and comets are not circular at present; for had they ever been such, their mutual action upon one another, sometimes increasing and other times diminishing their projectile velocities, must have gradually destroyed the circular form of their orbits.

PROPOSITION III.

The periodical time of a planet is directly proportional to the radius of its orbit, and inversely proportional to the velocity.

If the planet move with any given velocity, it is evident that the greater the radius of its orbit is, the greater must be the circumference of the circle it describes, and the greater must be its periodical time. Therefore in this case $P=R$.

And the greater that the planet's velocity is, the sooner will it complete its revolution, and therefore $P=\frac{1}{V}$. But when neither the distance nor velocity is given, the periodical time must depend upon both, and be in a ratio compounded of them; that is, $P=\frac{R}{V}$.

Cor. 1. Hence the velocity is as the distance directly, and periodical time inversely. $V=\frac{R}{P}$.

Cor. 2. The central forces of two bodies, by which they are retained in their circular orbits, at different distances from the center of force, are as their distances directly, and as the squares of their periodical times inversely. For, by Prop. 1. $C=\frac{VV}{2R}$, and by this Prop. $V=\frac{R}{P}$, or $VV=\frac{R^2}{PP}$. Therefore $C=\frac{R}{2PP}$.

PROPOSITION IV.

If we suppose that the central force decreases as the squares of the distances increase, then the squares of the periodical times will be as the cubes of the distances.

For, by Cor. 2. Prop. 3. $C=\frac{R}{2PP}=\frac{1}{RR}$, by the supposition. Therefore $P^2=R^3$.

But as none of the orbits of the solar system are exactly circular, it is unnecessary to pursue this theory any farther, we shall therefore proceed to other propositions, which will be found in fact to agree with what we find in the heavens.

PROPOSITION V.

If one body revolve round another by a force directed to any fixed center, it will describe, by a ray drawn from the center of force, areas proportional to the times.

Let the body be supposed to move in the direction AI* by the projectile force alone; instead of going in this direction it will, by the influence of the central force placed in S, move in the direction AB, and if permitted to move on in that direction, without any farther disturbance from the central force, it would move from B to C, in the same time that it moved from A to B, making BC=AB. But by the influence of the central force, which alone is sufficient to carry it to N, it will neither proceed in the direction BN nor BC, but in the direction BD: and at D it will have such velocity as would carry it over the space DE=BD, in the same time: but by the action of the central force, it is prevented from moving in the direction DE, and bent into the line DF, the diagonal of the parallelogram DKFE; for by the approach of the planet to the center of force, it has now acquired such strength as to carry the planet over the space DK by its influence alone. In the next instant of time, it would move over FG=DF in the direction DF, with the velocity acquired in F. But at the end of the said time, it will be found in the point H, having described FH the diagonal of the parallelogram FLHG.

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* See Plate IV. Fig. 2.

Now I say that all these spaces ASB,BSC,DSE,FSH, described in equal times, are equal: For, because AB=BC, the triangles ASB and BSC are equal, as having the same base and altitude; But BSC=BSD, having the same base BS, and being between the same parallels BS and CD, therefore ABS=BSD. For the same reason, BDS=DSE, having the equal bases BD and DE and the same altitude; and DSE=DSF, having the same base DS, and being between the same parallels DS and EF. Therefore, &c. Q.E.D.

Now if these bases AB,BD,DF,FH, be supposed indefinitely small, or in their nascent state, they will coincide with the curve described by the revolving body.

Cor. 1. The velocity of the revolving body is everywhere inversely as a perpendicular to the tangent of the orbit in the place of the body, from the center of force; because the velocities are as the bases of the equal triangles ASB,BSD,DSE, &c. described in equal times; but the bases of equal triangles are inversely as their altitudes; that is, inversely as the perpendiculars to the tangents, for the tangents and bases coincide in their nascent or evanescent state.

Cor. 2. The times of describing the equal areas are directly as the said perpendiculars, because they are inversely as the velocities; for the greater the velocity is, by which any arch is described, the sooner the revolving body passes over it.

Cor. 3. As any area of the orbit is to the time of its description, so is the area of the whole orbit to the periodical time; and therefore the periodical time of a planet's revolution is proportional to the area of the orbit directly, and inversely to any sector described in a given time.

In our lecture on projectiles we found, that a body would move in the curve of a parabola, if the central force acted with an invariable degree of strength, and the projectile velocity were such as would be acquired by falling through a space equal to one fourth of the parameter of that diameter, which passes through the point of projection; now, one fourth of the parameter is equal to the focal distance. The projectile will therefore move in a parabola, when thrown with a velocity acquired by falling through the focal distance; and this is the necessary adjustment between the central and projectile forces, in the description of a parabola. But we have also seen, that the projected body will move in a circle, whose center is the focus of the parabola, and whose radius is the focal distance, if projected with the velocity acquired in falling through half that distance. Therefore, since the velocities acquired in falling through any given distances are proportional to the square roots of these distances; it follows, that the velocity necessary for the description of a circle is to the velocity necessary for the description of a parabola, as $\sqrt{1}$ to $\sqrt{2}$, the radius of the circle; and the focal distance of the parabola being the same.

These two curves then require this exact adjustment between the central and projectile forces for their description; and the reason is, because the central force is supposed in both these cases to act with the same invariable degree of strength, being always at the same distance from the body revolving in a circle, and being at an infinite distance from it when moving in a parabola, as the center of this curve is at an infinite distance from its vertex.

As the parabola is the utmost limit of an ellipsis on the one side, when the curve flies open, and the center and focus recede to an infinite distance from the vertex; and the right line is the utmost limit of the ellipsis on the other side, when the conjugate axis vanishes; it is evident, that any velocity less than what is necessary for the description of a parabola, will make the revolving body move in an ellipsis, which therefore requires no certain adjustment between the central and projectile forces. If the velocity necessary for the description of a parabola be diminished in any proportion between the ratio of $\sqrt{2}:1$. the ellipsis described will lie within the parabola, and without the circle, and the center of force will be in the focus nearest to the place of projection, as the body moving without the circle recedes from the center of force: and if it be diminished in a still greater proportion, even down to nothing, the ellipsis described will lie within the circle, until it finally degenerate into the radius of the circle, and the center of force will be in the focus most distant from the point of projection. Lastly, if the parabolic velocity be increased in any degree, the projected body will move in a hyperbola by virtue of a centrifugal force, placed in the focus of the opposite hyperbola; as the focus of the ellipsis, after having receded to an infinite distance, when it flew open into the parabola, comes to a finite distance on the other side; and the curve becomes convex to the center of force. Now, as neither the parabola nor hyperbola are curves that return into themselves, if the projectile move in either of these, it is evident, that it will continually recede from the center of force, and never more return to the place of projection.

But in order that one body may revolve in the curve of an ellipsis round another placed anywhere in the axis

PROPOSITION VI.

If a line be drawn from the place of the revolving body to the focus of the ellipsis, and this line be intersected by another, drawn from the place of the central force, parallel to the tangent of the ellipsis at the place of the revolving body, then the cube of the distance between the said point of intersection and the revolving body will be to a parallelopipedon, whose base is the square of its greatest distance from the center of force, and altitude its distance from it at the given time, as the strength of the central force at its greatest distance, to the strength of it at any other given time or place.

Cor. 1. If the place of the central force be supposed to be the center of the ellipsis, its strength must increase and diminish in exact proportion to the varying distance of the revolving body, to keep it continually in the elliptic orbit; for then $xx=zz$, for PZ becomes equal to $PE=CA=x$, and then $C:c::x:y$.

Cor. 2. If the place of the central force be the focus of the ellipsis, then the central force is everywhere as the square of the distance inversely from the revolving body. Because, when L and S coincide, $z=y$, or $PZ=PS$, and $C:c::zz:xx$.

We find, both from theory and observation, this to be the law of that central force, which regulates and governs all the motions of all the planets and comets, which belong to the solar system; viz. that they all revolve round the sun in elliptic orbits, and that the sun is placed in one common focus of all the ellipses they describe. This will be made still more evident from the agreement that is found between their motions and the following propositions.

PROPOSITION VII.

If several planets relove in separate ellipses round a central force placed in a common focus belonging to all, and which is reciprocally as the squares of the distances of each from it; their periodical times will be directly as the area of each elliptic orbit, and inversely as the square root of its latus rectum.

Let P=the periodical time of a planet's revolution, A=the area of its elliptic orbit, L=its latus rectum, and S=any sector described in any portion of time. Then

By Prop. 5. Cor. 3...... $P=\frac{A}{S}$.

Per conics, $CA^2 : CB^2 :: 2CA\times IA : HI^2=\frac{2CB^2}{CA}\times IA$ $=L\times IA=L\times C$, because the central force $C=IA$.

But $SA\times HI=2S=SA\times\sqrt{L\times C}=\frac{1}{\sqrt{C}}\times\sqrt{L\times C}=\sqrt{L}$.

Because $C=\frac{1}{SA^2}$ by hypothesis $\therefore SA=\frac{1}{\sqrt{C}}$.

Hence $2S=\sqrt{L}$, and $P=\frac{A}{S}=\frac{A}{\sqrt{L}}$. Q.E.D.

Cor. 1. The area of each ellipsis is proportional to the periodical time and sq. root of the lat. rectum.

Cor. 2. $S=\sqrt{L}$.

PROPOSITION VIII.

The periodical times of different planets about the same center are in a sesquiplicate ratio of the greater axis of the ellipsis they describe; or in other words, the squares of their periodical times are as the cubes of their mean distances.

Per Prop. 7. $PP=\frac{A^2}{L}=\frac{aabb}{L}$, a and b, being the transverse and conjugate axes.

But $L=\frac{2bb}{a}$, therefore $PP=aaa$. Half the greater axis is the mean distance. Q.E.D.

Cor. 1. Hence in every species of an ellipsis that can be described on the same transverse axis; from the circle, where the foci unite in the center, to a right line, where they recede to the ends; the periodical times must be the same, being in the sesquiplicate ratio of the same axis.

PROPOSITION IX.

Upon the supposition of the same law of the central force, the velocities of planets in their different orbits are as the square roots of the parameters of their greater axes, divided by a perpendicular let fall from the center of force to the tangent passing through the place of the planets.

The velocity V, at any point P, is as the arc QP described in any indefinitely small part of time; that is, $V=QP$. But $QP\times SY=$ the area of the sector SQP described in the same time, which by Cor. 2. Prop. 7. is proportional to the square root of the lat. rectum; that is, $QP\times SY=\sqrt{L}$. Therefore $V=QP=\frac{\sqrt{L}}{SY}$. Q.E.D.

Cor. 1. If L be given, or the orbits be the same, the velocity is everywhere reciprocally as the perpendicular from the center of force to the tangent. As per Prop. 5. Cor. 1.

Cor. 2. If L be given, or the orbits be the same, the velocities of the planet in the aphelion and perihelion will be reciprocally as the distance of these points from the center of force.

Cor. 3. If L be given, the velocity of the planet at its mean distance is to its velocity at any other point of its orbit, as the square root of the perpendicular from the center of force to the tangent, is to the square root of the perpendicular from the other focus to the same tangent at the place of the planet. Because the perpendicular at the mean distance is equal to the semi-conjugate axis, which is a mean proportional between the perpendiculars from the foci to any tangent of the ellipsis. If then V=the velocity at the mean distance, and v=the velocity at any other point, we shall have $V : v :: \frac{1}{c} = \frac{1}{\sqrt{SY \times UX}} : \frac{1}{SY} :: \frac{1}{\sqrt{UX}} : \frac{\sqrt{SY}}{SY} = \frac{1}{\sqrt{SY}} :: \sqrt{SY} : \sqrt{UX}$. Q.E.D.

Cor. 4. The velocity of a planet, at its mean distance, in the ellipsis, is equal to the uniform velocity of a body moving in a circle, whose radius is equal to the mean distance.

Because the perpendicular from the center of force to the tangent at the mean distance is now the semi-conjugate axis, and the square root of the latus rectum divided by the semi-conjugate axis, is equal to the square root of 2 divided by the square root of the semi-transverse axis; that is $V = \frac{\sqrt{L}}{c} = \frac{c}{c} \times \sqrt{\frac{2}{t}} = \sqrt{\frac{2}{t}}$. And the circle is that species of the ellipsis, whose foci coincide in the center, and whose latus rectum is equal to its diameter, or to twice the mean distance of the planet, and its radius being perpendicular to the tangent is by supposition equal to the mean distance of the planet, or to half its transverse axis. Therefore the velocity in the circle is equal to $\frac{\sqrt{2t}}{t}=\sqrt{\frac{2}{t}}$, the same with the velocity at the mean distance in the ellipsis.

Cor. 5. If a body revolve in a circle, whose radius is equal to the aphelion or perihelion distance of a planet, the velocity of the planet revolving in the ellipsis at these points, will be to the velocity of a body revolving in the circle, as the square root of the latus rectum of the ellipsis, to the square root of the diameter of the circle. Because, in these points, the tangents of both curves, and the perpendiculars to the tangents coincide, and the latus rectum of the circle is equal to its diameter. Therefore the velocities being proportional to the square roots of the latera recta, when the perpendiculars are the same, they are to one another as the square root of the latus rectum of the ellipsis, to the square root of the diameter of the circle. $V : v :: \sqrt{L} : \sqrt{2R}$.

PROPOSITION X.

If the central force, which causes an ellipsis to be described round its focus, should decrease more or less in proportion, than as the square of the distance of the revolving body increases, the body would leave the ellipsis, unless the curve itself be supposed to revolve on its focus; and on this hypothesis, the greater axis would move in the same direction with the body, when the central force becomes too small, and in a contrary direction, when it becomes too great, to be in proportion to the squares of the distances inversely.

The law of the central force residing in the focus of an ellipsis, being, to increase in strength, as the squares of the distances decrease; if any alteration should happen in this law, the distances of the revolving body will be either greater or less, than what they would be, without such an alteration, and consequently the revolving body will be either without or within a fixed and immovable ellipsis. Suppose that the body moves from the aphelion towards the perihelion, and if the central force do not increase so fast as the squares of the distances decrease, the central force will attract the body with less power, and therefore suffer it to move without the ellipsis, unless by revolving on its focus, the superior apsis of the ellipsis advance in the same direction with the body, and with a velocity proportioned to the want of due strength in the central body, so as that some point of the curve may still coincide with the place of the body.

And on the contrary, if the central force increased faster than the squares of the distances decreased, it would urge the revolving body too forcibly towards the focus, and cause it to move within the ellipsis; unless the superior apsis receded backwards, and thereby caused some point of the ellipsis nearer to the focus to coincide with the place of the body. So that in the first case, when the central force is too weak, the superior apsis moves forward in the same direction with the revolving body; and in the other case, when it is too strong, the superior apsis will have a retrograde motion in a contrary direction.

We shall have occasion for this proposition in astronomy, to explain the motion of the apsis of the moon. For although the central force of the earth, which retains the moon in her orbit, would make her describe an immovable ellipsis round the earth, placed in one of its foci, if she were not attracted by any other force; yet we shall see, that she is also attracted by the sun, and this additional force, sometimes conspiring with, and sometimes acting against, the force of the earth, greatly disturbs her motion. At the new and full moon, the force, with which the sun acts upon it, causes its gravity towards the earth to decrease, in the departure of the moon, and to increase in its approach, faster than in the inverse proportion of the squares of the distances. From whence it will follow, that as the attraction of the sun is contrary to that of the earth, at the new and full moon, the attraction of the earth will be too weak to retain her in an immovable ellipsis; her distance from the earth will be increased, and therefore the apsis of her orbit will move forward, in the same direction with her own motion, which is from west to east.

But at the quarters, the disturbing force of the sun, conspiring with the attractive force of the earth, urges the moon more forcibly towards the focus, and causes her to descend within the ellipsis which she began to describe at her higher apsis; and therefore her orbit is to be considered, at these points, as a movable ellipsis, whose apogee recedes backwards in a direction contrary to her own motion.

HYDROSTATICS.

HYDROSTATICS is that branch of natural philosophy, which explains the nature, gravity, pressure and motion of fluids, whether elastic or nonelastic.

Although nothing seems, at first sight, more easily determined than the essential distinction between a fixed and fluid body, and every person sees the difference between them, yet the more that this subject is examined, the more difficulties occur in the determination. They both evidently consist of the same kind of matter, and the original constituent particles of both are the same, as they are so easily and frequently changed from one form to the other, as in the fusion of metals and freezing of water. Hence, the ancients supposed, that the difference consisted in this; that a fixed body retained its figure, while the fluid, unless confined in some vessel, exerted a continual influence to change its form. This however cannot be the true definition of a fluid, because it will also agree with a fine powder, which none ever called a fluid.

Descartes thought, that he had given a much better definition of a fluid, when he supposed, that the essence of fluidity consisted in the intestine motion of the particles; however this motion might be imperceptible by our senses.

Yet, although we grant, that the particles of a fluid have a constant tendency to motion, it is still a hasty conclusion, that they are actually in motion.

Sir Isaac Newton and his followers, rejecting the definition of Descartes, have endeavoured to ascertain the nature of a fluid from the principles of mechanics, and have determined that fluidity consists in these three qualities following.

1. The exceeding smallness of the particles: 2. Their roundness, or figure necessary for constituting volubility: 3. A very slight cohesion, or rather none at all. But these conditions are not of themselves sufficient to constitute fluidity; for, unless we add the gravity of each particular particle, there cannot be a tendency to flow. Yet all these four conditions are found in the calx of tin and minium, when sufficiently pulverized; and still they do not flow, until they begin to vitrify with a sufficient degree of heat.

To avoid this difficulty, our modern philosophers, neglecting every other property of a fluid, excepting its cohesion and gravity, say, that a fluid is that, whose smallest particles do not cohere with a greater force, than what is equal to the weight of a single drop. But still it may be inquired, what are these smallest particles, which cannot be divided into lesser?

As matter is in its own nature infinitely divisible, it is impossible to assign its smallest particles. Besides, it is evident to our senses, that the force of cohesion in small drops is much greater than their gravity, and that they do not separate until, by the constant accession of other particles, they increase to such a magnitude, that the gravity of the whole drop is more than sufficient to be a counterbalance to their cohesion.

These things then, failing to give satisfaction to the philosophic mind, let us attend to the operation of nature, and see, if we cannot investigate the precise difference between a fixed and a fluid body, from some of the most obvious properties of a fluid.

1. At first sight, there is this remarkable difference between a fixed and fluid body, that the particles of the former are so closely connected together, that they exert only one common force by their gravity; but besides this force of gravity, which all fluids possess in common with fixed bodies, the particles of a fluid are not so strictly bound together, but that they are at liberty to exert a very remarkable force upon each other. And hence it is that the particles of a fluid constantly tend to an equilibrium, and, when unrestrained by an external force, compose themselves into a plane surface; which we never find to be the case with powders, however levigated and subtile their particles may be.

2. Although the particles of a fluid are not so strictly connected together as to hinder a remarkable exertion of force upon each other, yet they have a considerable cohesion and tendency to union, when the fluids are unelastic; and this cohesion is greater in some than in

others. This affords a remarkable difference between water and oils; and even among oils, some are more viscid than others. While the particles of some cohere very slightly, those of others have such a strong cohesion, that they possess a degree of consistency, bordering upon fixity; as in butter, wax and tallow. Yet some degree of cohesion seems so essential to fluidity, that it constitutes the principal difference between a fluid and an impalpable powder, at the same time that it differs from the cohesion of a solid only in the degree of force.

3. While an unelastic fluid is confined in a close vessel, and pressed with an external force, it still admits of the freest motion of its particles among themselves, and of any bodies immersed in it, so that they are at perfect liberty to follow the laws of their own specific gravities. This property we find in no fixed body. For powders, the most subtile and impalpable, when pressed together with any considerable force, admit of no motion among their particles, nor of any body placed in the midst of them.

4. It is also a remarkable property of fluids, that they tenderly embrace the most brittle bodies immersed in them, and defend them from all external injury, with whatever weight they may be pressed. A piece of the thinnest glass, or an eggshell, when immersed in a fluid remains unbroken even when the fluid is pressed with an assignable weight.

5. All fluids, except air, or elastic vapour, are incompressible by any force; as is proved by the famous Florentine experiment, of compressing a globe of gold, filled with water, through the pores of which globe, although exceedingly small, the water passed, rather than be reduced into a narrower space. That water was compressible in a small degree, and that it suffers a con-

stant compression, by the weight of the atmosphere, some philosophers have rashly concluded from the following experiment. Let a glass bubble, having a small neck, be filled with water and placed under the receiver of the air pump; and when the air is exhausted, the water will be seen to rise in the neck of the bubble. This rise of the water in the neck of the bubble may be owing to the expansion of the air, which lodges in the pores of the fluid, when the pressure of the external air is taken off.

6. The free motion of the particles of a compressed unelastic fluid among themselves, and of any bodies immersed in it, induce us to believe that the particles never touch one another, but are kept separate by some intervening substance or power, and that they cannot come into contact, until this be removed. We see from the lastmentioned property that no external force can bring them closer together. Yet water is extremely obedient to the action of heat, by which it immediately expands, and fills a larger space; and it as certainly contracts its dimensions, upon the diminution of heat or its total removal. This intervening substance, therefore, which keeps the particles of a fluid separate from each other, seems to be heat, or at least to depend upon the agency of heat for the degree of its force.

7. Since a high degree of cold has been found to fix mercury into a hard and malleable substance, it seems scarcely to be doubted, but that some degree of heat is absolutely necessary to fluidity: and that it is always proportionable to the degree of heat, according to some law, which is not yet sufficiently determined by experiments. We evidently observe in water, that under a low degree of heat, it assumes the appearance of a fixed body, which is called ice, that under a greater degree of

heat it is fluid, and that by a still greater, it is converted into a subtile and elastic vapour. The same is observable in other bodies, such as metals, namely, that they are fluid with an intermediate heat, between what is necessary to fixity and evaporation on the contrary extremes. So that if it were possible to exclude every degree of heat from bodies, there would be no such thing as fluidity in nature, but every body would be fixed and rigid.

Cor. From the properties of fluids already enumerated, we may venture to deduce this definition. *Fluidity* is that state of a body, in which the attractive force which unites the particles together into one mass, or community, is just balanced by the repulsive force, by which they recede from each other; which repulsive force probably depends upon the action of heat. While this balance is preserved, water remains fluid; but when the attractive prevails over the repulsive force, it is converted into ice; and on the other hand, when the repulsive power exceeds the attractive, the particles fly off in a very elastic vapour.

By this balance between the attractive and repulsive forces of a fluid, we do not mean a strict mathematical or mechanical equilibrium. The word must be taken with some latitude. In some bodies we find various degrees of fluidity, from their first beginning to flow, till they fly off in an elastic vapour. This gradual increase of fluidity does not proceed with regular advances, until the particles of the fluid begin to fly off in an elastic vapour. Water, for instance, seems to be as fluid at fifty degrees of heat as at an hundred, when it evaporates plentifully, and the repulsive power remarkably predominates.

8. Fluids yield to the least unequal and partial pressure. This we find by constant experience, and it must

arise from the particles being exceedingly small, round and smooth, or from their being kept separate from each other, by the action of heat.

9. The weight of fluids being proportional to their quantity of matter, as we have already proved of all bodies whatsoever, from their equal descent in vacuo; no particle can lose its gravity when immersed in a fluid of the same kind: or in other words, all fluids gravitate in proprio loco, or in fluids of the same kind. As the weight of a bucket of water is not felt by the person who draws it out of the well, nor the weight of the atmosphere upon the hand, some philosophers have rashly concluded, that water had no weight while it was in water. But the reason why the weight of the bucket of water is not perceived, while it is immersed in the well, is not because it has no gravity, but because it is supported by the pressure upwards, as we shall presently see. The truth of this proposition may be made evident to sense. For if a vessel which contains an ounce of water, be weighed in water when empty and corked tight, so that no water can enter it; and the cork be then pulled out, and the vessel filled; it will weigh an ounce more, than when it was empty. Hence, the water, which in air weighs an ounce, will also weigh the same in water.

10. The pressure of fluids in all directions, upwards downwards, and laterally is the same, at any given depth. The uppermost particle in any column presses upon the next to it with its whole weight, and is equally pressed upwards by the reaction of the second. This is also the case with every particle in any column, that it presses upon the next under it with all the weight of the superincumbent particles, and is reacted upon with an equal force in a contrary direction. And if it were

not pressed with an equal force in every other direction, it would move in the direction of the greatest pressure. This may be confirmed by an experiment. Let a glass tube, hermetically sealed, be immersed with the open end in water, the water will rise in it to heights proportionable to the altitudes of the fluid above these heights, let the figure of the immersed part of the tube be what it will. If a hole be made in the side of the vessel, the water spouting out of it with the same velocity in all directions, upwards, downwards, and horizontally, shows that the pressure at the given depth of the hole, in all directions, is equal.

11. The surface of a fluid, that is quiescent, must be smooth and regularly convex, being part of a sphere of the same magnitude with the earth. Hence a vessel filled with a fluid at the center of the earth, would run over, through the whole of its ascent to the surface; as the convexity of the surface of the fluid would continually decrease, as the vessel receded from the center. Hence also, the hulls of vessels at sea, disappear, before the sails and masts, when they are receding from the eye; as the surface of the sea is convex. And as the semidiameter of the earth is near 4000 miles, a vessel on the sea must recede from the eye near two miles before we sensibly lose sight of any part of her; because for that distance the convex surface of the sea nearly coincides with a right line.

12. The pressure upon the bottom of a vessel is equal to the product of the area of the bottom multiplied into the perpendicular altitude of the longest column of the fluid: and in two vessels of equal bottoms, the pressure on the bottom, will be proportionable to the perpendicular altitude alone.

The truth of this proposition is evident at first sight, when the vessel is of the same diameter from top to bottom. For if the fluid be supposed to be divided into perpendicular columns, the base of each will be pressed by the weight of water that it sustains, which weight is proportionable to the area of the bottom multiplied by the altitude; and as this is equally true of every column, the proposition is manifest upon this supposition. But when the vessel is wider or narrower at the bottom than at the top, the matter is not so plain. Let it first be considered as wider at the bottom than at the top, so that some of the perpendicular colums will be shorter than others. The difficulty, however, vanishes when we reflect, that any particle in the shorter column is pressed equally, with a particle in the longest column, at the same depth below the surface, and in all directions. For otherwise the fluid could not be quiescent. The slanting side of the vessel above the tops of the shorter columns presses them downwards as much as they would have been pressed, had each of them been extended to the same altitude with the longest. For if a hole were made in the slanting side, and a tube inserted in it, the fluid would rise in the tube to the altitude of the longest column. And when the base is narrower than the top of the vessel, the slanting side of the vessel sustains the weight of all the shorter columns, and the base is only pressed with the weight of the longest columns above it. The pressure, therefore, upon the equal bases of two vessels of very different capacities is proportional to the altitudes of the longest columns of the fluid; and when the altitudes are equal, the pressure upon the bottom is the same.

Cor. 1. From this property of fluids, it follows, that any weight, how great soever, may be counterbalanced by a small quantity of a fluid, by only increasing its perpendicular altitude. If water be poured into a perpendicular tube, inserted into the uppermost of two circular boards, connected together with pliable leather, it will counterbalance any weight laid upon them, by only proportioning the altitude of the fluid to the weight. This is called the hydrostatic bellows. If a given quantity of water in the tube would counterbalance ten pounds, the same quantity would fill the tube, if its diameter were ten times less, and its altitude ten times greater, and then it would sustain an hundred pounds; as its pressure is now increased ten times greater than it was at first. It might in this manner be increased in any proportion whatsoever.

Cor. 2. The pressure upon the bottom of an inclined cylindrical vessel is equal to the pressure on the bottom of a vessel, whose sides are perpendicular to the horizon, if the bottoms and perpendicular altitudes of the fluids be the same in both vessels. Because the pressure of the shortest column on the base is equal to the pressure of an equal column whose altitude is the perpendicular altitude of the fluid. And therefore generally the pressure on the bottoms of all vessels, whatever be their form, is proportional to their bases and perpendicular heights.

13. The pressure upon the side of a vessel is one half the pressure upon the bottom of equal area with the side. By the 10th proposition, the pressure of any particle of the fluid is equal in all directions. Now the lateral pressure of any particle against the side of the vessel, being equal to its pressure downwards, must of consequence be proportional to its depth below the surface of the fluid, by the last proposition. This being the case with every particle in contact with the side of the vessel; the pressures on the side must increase from

nothing to the greatest pressure of the lowest particle which is equal to its pressure upon the bottom. Now the sum of all these pressures must be equal to half of the amount of them, had they been all equal to the greatest.

Suppose these pressures to be represented by the terms of an arithmetical progression, the pressure of the uppermost particle will be 0, as its distance from the surface is 0. The pressure of the second particle may be represented by 1, of the third by 2, and so on to the bottom. The terms of the progression representing the pressures of the particles will be $0+1+2+3+4$, &c. Suppose the number of particles in any perpendicular column to be 5, the sum of these five terms is equal to 10. But as the bottom is by supposition equal to the side, any line drawn along it, will also consist of five points, each of which is pressed with a force equal to the greatest pressure of the perpendicular column$=4$. Therefore as $5\times4=20$, which is double to 10; any line in the side of the vessel is pressed with a force that is equal to half the force that presses an equal line on the bottom. And as the bottom and side are equal, one contains as many such lines as the other, and the pressure on the side will be equal to half the pressure on the bottom.

Now as a cubical vessel has four sides and a bottom all equal to each other, and the pressure on one side is equal to half the pressure on the bottom, it follows, that the pressure on all the four sides is double of the pressure on the bottom alone, and that the whole pressure on all the sides and bottom is triple to the pressure on the bottom; and since all the pressure is produced by the gravity of the fluid, and the pressure on the bottom lone is equal to its gravity, the gravity of the fluid pro- uces a pressure, three times as great as itself.

Cor. 1. Since the pressure on any part of the side or ottom of a vessel is equal to a column of the fluid, hose base is the part pressed, and whose altitude is he depth of the part pressed, below the surface of the uid; it follows, that if the area of the part pressed be iven, the pressure will be as its depth below the sur- ice; and when the depth of the part pressed is given, he pressure will be proportionable to the area of the art. Thus a square inch taken one foot below the sur- ice of the fluid, in the side of the vessel, is pressed rith only one quarter of the force which presses a square nch four feet below the surface: and any two equal ortions of the side of the vessel, taken at different lepths, are pressed with forces proportional to their lepths. But a foot square in the side is pressed with 44 times the force that presses upon a single inch quare, at the same depth below the surface of the luid.

Cor. 2. Hence we see the reason, why vessels de- igned to contain a fluid, should be made stronger at he bottom, than at the top; as the lower parts sustain greater pressure.

Cor. 3. To find the pressure on a milldam; multiply he area of the dam in square feet by half the depth, nd that product by 62.5 pounds, the weight of a cu- ic foot of water.

THE SPECIFIC GRAVITIES OF BODIES
IMMERSED IN FLUIDS.

THE specific gravity of a body is its weight under a iven bulk, when compared with the weight of an equal ulk of some other body, which is made the common standard. Rain water is commonly made the standard; and when we say that the specific gravity of gold is 19, the meaning is, that any given bulk of gold is nineteen times as heavy as the same bulk of rain water. In like manner, when we say that the specific gravity of silver is 11, the meaning is, that, bulk for bulk, or under equal dimensions, silver is eleven times as heavy as rain water; and that the weights of gold and silver, in equal bulks, are to one another as nineteen to eleven.

Hence, the weights of bodies of the same magnitude are as their specific gravities. And the weights of bodies of the same kind, or specific gravity, are proportional to their magnitudes. If neither the magnitudes nor specific gravities of bodies be the same, the weights will be as the rectangle of their specific gravities and magnitudes. Let W=the weight of a body, S=its specific gravity, and M=its magnitude, then $W=MS$, and $S=\frac{W}{M}$ and $M=\frac{W}{S}$.

But in our lecture on the attraction of gravitation, we had $W=MD$, or the weight of bodies was proportional to their magnitudes and densities. Hence we see that the specific gravities of different bodies are proportional to their densities. $W=MS=MD.$ $\therefore$ and $D=S$.

1. If a body be immersed in a fluid, the lateral pressure of the fluid, on its opposite sides, is equal; and therefore no motion, in a horizontal direction, can be produced by that pressure. But it is pressed upwards more than it is pressed downwards; and the difference of these two pressures is equal to the weight of as much of the fluid, as the body has displaced.

The pressure upwards is proportional to the depth of the lower side of the body from the surface of the fluid, which would be just counterbalanced by the column of fluid, reaching from the surface to the same depth, and of equal base with the body; but the pressure upon the upper side of the body is proportional to the column of the fluid above it; and the difference between these two columns, is a quantity of the fluid equal in bulk to the body. Therefore, the difference between the pressures upon the upper and under sides of a body, immersed in a fluid, is equal to the weight of as much of the fluid as the body has displaced.

2. A body immersed in a fluid, of the same specific gravity with itself, will rest, in any part of the fluid indifferently, wherever it may happen to be placed.

As the weight of the body is the same with the weight of an equal bulk of the fluid, by the supposition, it is evident, that it presses downwards on the particles of the fluid in contact with its under surface, with the same force, that they would have sustained, had the solid been removed, and its place occupied by an equal bulk of the fluid. And therefore it cannot move in any direction, there being now no unequal pressure in any direction. The superior pressure upwards, above what is downwards, is just counterbalanced by the weight of the body, which is equal to the weight of the same bulk of the fluid. The column, which contains the solid, is of the same weight, with any other adjacent column. And therefore all being of the same weight, they must mutually sustain each other, without any intestine motion. It may indeed appear a difficulty to the young philosopher, to reconcile this proposition with the last, and to conceive how the immersed body should be at rest, when it is pressed upwards by the fluid, more than it is pressed downwards by it. But the answer is easy. We must remember that the body has some weight, by which it presses downwards; and that this weight when added to the pressure of the fluid downwards, is just an exact counterbalance to the pressure upwards; because, by the supposition, this weight of the body is the same with that of an equal bulk of the fluid, and this weight was, by the last proposition, the difference between the pressures upwards and downwards. And therefore, when the weight of the body is added to the pressure of the fluid downwards, the sum of the pressures downwards is just equal to the pressure upwards. Therefore, no motion can be produced by these equal and contrary pressures, in the direction of either; nor can the body move horizontally, as it is pressed equally on the opposite sides.

3. If a body immersed in a fluid be specifically lighter than the fluid, it will rise in it, but if specifically heavier, it will sink to the bottom.

This proposition necessarily follows from the reasoning in the last; for if the weight of the body, when of the same specific gravity with the fluid, must be added to the pressure of the fluid downwards, in order to counterbalance the pressure upwards; then if this weight of the body be too small, for this purpose, or if the body be specifically lighter than the fluid, there will be a superior pressure upwards, and the body will move in that direction, or rise in the fluid. But if the body be specifically heavier than the fluid, its weight, when added to the pressure of the fluid downwards, will be more than the pressure of the fluid upwards; and the body moving in the direction of the superior force, will sink to the bottom.

4. Though a body be specifically lighter than the fluid, in which it is immersed, yet it will not rise in it, if the pressure upwards be any way prevented: nor will it sink beyond a certain depth, although specifically

heavier, if the pressure of the fluid downwards be prevented. If a piece of a board or pane of glass be laid on the bottom of a vessel, which it fits exactly, and mercury, which is specifically heavier, be poured upon it, the board or pane of glass cannot be pressed upwards, as there is no pressure of the fluid under it; and it is kept down both by its own weight, and the pressure of the fluid above it. The only thing which buoys up the solid is the superior pressure of the fluid upwards, and if this be prevented, the solid has nothing in itself to raise it in the fluid. There is therefore no such property belonging to bodies as absolute levity, by which they are made to rise from the center of the earth, in the same manner, as they are made to descend towards it, by their own gravity. For when bodies ascend in fluids, they rise because they are lighter than the fluids, and not because they have a tendency contrary to that of gravity. If the board floated in the mercury by any absolute levity of its own, it ought also to float in it by the same property, when the pressure of the mercury upwards was prevented, as the removal of the mercury could not destroy any absolute property of the solid.

The truth of the other part of the proposition, viz. that a body specifically heavier than the fluid will not sink beyond a given depth, if the pressure of the fluid downwards be prevented, will appear if we reflect, that it is acted upon by three forces, the two pressures, upwards and downwards, and the weight of the body. But if the fluid, which is above the body, and presses it downwards, were removed, by any contrivance, there would be only two of these forces left, the pressure upwards and the weight of the body itself. Now if it were sunk so far in the fluid, as to make the pressure of the fluid upwards just equal to the weight of the body, these two equal and contrary forces would destroy each other, and the body would be supported and at rest. Hence loaded vessels float in the ocean, when the water is prevented from pressing upon their upper surface, but immediately sink, when they are filled with the water.

5. If a body specifically lighter than the fluid float upon it, so much of the fluid will be displaced by it as is equal in weight to the whole weight of the body. Because then the column of the fluid, which contains the solid will press upon the bottom, and laterally, with a force equal to that of any other adjacent column; and the whole will be at rest. The upper part of this column is as much pressed downwards by the weight of the body lying upon it, as if it had been pressed by a quantity of the fluid sufficient to raise it to the same altitude with any adjacent column, and this quantity of the fluid, which was displaced by the solid, must therefore be of the same weight with the solid.

6. The whole bulk of the solid, that floats upon a fluid, is to the immersed part of it, as the specific gravity of the fluid, to that of the solid. The bulk of the fluid that is of the same weight with the solid, is equal to the immersed part of the solid. We have seen before, that the weights of two bodies were as the rectangles of their bulks and specific gravities; therefore, when the weights are equal, these rectangles are also equal, hence, their magnitudes are inversely as their specific gravities; that is, the magnitude of the solid is to the magnitude of the fluid displaced by it, or the magnitude of the immersed part, as the specific gravity of the fluid is to that of the solid. $W=MS$, and $w=ms$; but $W=w$,

therefore MS=ms. Consequently $M : m :: s : S :: \frac{1}{S} : \frac{1}{s}$. Hence the weight of the part of the solid which floats above the surface of the fluid, is what the immersed part wants, to make it of the same specific gravity with the fluid. For if it were removed, and its weight added to that of the immersed part, it is evident, that the immersed part would press upon the fluid just as much as the fluid presses upwards, and therefore they must, on that supposition, be of the same specific gravity. The difference therefore of their specific gravities is the weight of the part, that is immersed, and floats above the fluid.

7. If a body be made to float in two fluids, each specifically heavier than itself, the quantities of the fluids that it displaces, or which is the same thing, the parts of the body immersed, will be to each other as the specific gravities of the fluids inversely.

Here the weights of the two fluids displaced are the same, as being equal to the weight of the solid; but these weights are proportionable to the rectangles of the magnitude or bulk, and their specific gravities; therefore these rectangles are equal: hence, the magnitudes or bulks of the quantities of the fluids displaced, or the parts of the solid immersed, are inversely as the specific gravities of the fluids. For if W,M,S and w,m,s, represent the weight, magnitude, and specific gravities of the two fluids respectively; we shall have W=MS, and w=ms, but W=w, therefore MS=ms, therefore $M : m :: s : S :: \frac{1}{S} : \frac{1}{s}$. that is, the bulk of one fluid displaced by the solid, or the parts immersed in one fluid are to the bulk of the other fluid displaced, or the parts of the solid immersed in it, inversely as their specific gravities.

Upon this principle depends the method of trying the strength of various liquors by the *hydrometer*. Their strength being inversely as their specific gravities, it must be directly as the depths, that any body will sink in the fluid. The hydrometer is constructed with a hollow ball and a slender stem, with a weight screwed upon the bottom, that when it swims in any fluid lighter than water, it may sink to a certain depth, keeping the stem perpendicularly upwards, which by the graduations on it, indicates the specific gravities of the fluid in which it swims. For the part of the stem that sinks is so much greater, as the fluid is lighter or stronger. If it be made to sink to the middle of the stem in proof spirits, which are one half water, and the other half alcohol, or ardent spirits, such as being set on fire would leave gunpowder dry at the bottom, its sinking to a less or greater depth, will show by its graduations, how much any spirit is below or above proof.

8. Bodies specifically heavier than the fluid, in which they are immersed, lose so much of their weight, as is equal to the weight of an equal bulk of the fluid. That is, because a cubic foot of water weighs 1000 ounces, or 62.5 pounds, if a cubic foot of lead or iron be immersed in water, it will lose 1000 ounces of its weight, and sink down to the bottom by the remainder of its weight, and press upon it only with that remainder: or the hand that holds it suspended in the water will only feel this residue of weight, which is the difference of their specific gravities. The reason is plain from what we have proved before; that if the solid were of the same specific gravity with the fluid, it would be supported by it, and rest indifferently any where in it; so that it

would appear to have no weight, or to have lost all its weight, which was that of an equal bulk of the fluid. But if the solid be supposed heavier than the fluid, this excess of weight having nothing to support it, must carry it to the bottom, and be the only thing that is felt. Every body endeavours to descend in a fluid by its whole weight; but by Prop. 1. it is supported by a force, equivalent to the weight of an equal bulk of the fluid; and since these two forces act in contrary directions, the weight which the body retains is the difference between them. Therefore it loses the weight of an equal bulk of the fluid. But this weight is no otherwise lost, than as it is sustained by the action of a contrary force. It is for this reason that we do not perceive the weight of a bucket of water while it is in the water; not because the weight is destroyed, but because it is supported by the contrary pressure of the water upwards; not because fluids do not gravitate in fluids of the same specific gravity; but because there is an equal pressure contrary to the direction of gravity.

In the same sense too, we say that bodies specifically lighter than a fluid, lose as much weight, as is equal to the weight of an equal bulk of the fluid, which is more than what they have; when we only mean, that they are pressed upwards with a force, that is equal to the difference of their specific gravities. Thus a cubic foot of cork which weighs only 240 ounces in air, is said to lose 1000 ounces in water; by which we mean, that it is pressed upwards with a force equal to 760 ounces. So that in this sense the proposition holds true with respect to all bodies, whether they be heavier or lighter than the fluid; that they lose a weight, equal to that of the same bulk of the fluid.

9. The weight which is lost by a body in a fluid is

communicated to the fluid. If the solid be of the same specific gravity with the fluid, it loses its whole weight, when immersed in it; but if the whole fluid and solid be weighed together, they will weigh as much as when taken separately. Therefore the weight lost by the solid is, in this case, communicated to the fluid. But if the solid be heavier than the fluid, and suspended by a string, and thus dipped in a fluid, placed in the scale of a balance, the fluid will be found to have gained all the weight of the body, that is not supported by the string; that is, all that the body has lost; because it is supported by the pressure of the fluid upwards; and as action and reaction are equal and contrary, as much weight as the water takes off by its pressure, just so much does the body add to the water by its reaction. And the fluid will be found to weigh as much more, while the solid is suspended in it, than without it, as if a quantity of the fluid equal in bulk to the solid, had been poured into it, instead of the solid.

10. The weights which different bodies lose, in the same fluid, are proportional to their bulks. For they all lose the weight of the fluid, which they severally displace; and these weights are proportional to the quantities of the fluid displaced, or the bulks of the bodies immersed. A cubic foot of lead displaces a thousand ounces of water, or loses a thousand ounces of its weight, when immersed in water; and another body of twice the bulk displaces twice the quantity of water, and therefore loses 2000 ounces of its weight. So that the weights, which different bodies lose in the same fluid, are proportional to their bulks.

From hence we may easily deduce a rule, for finding the bulk of any irregular body, which cannot be measured by the common rules of geometry.

Weigh it first in air and then in water, and find how much of its weight it loses in water; then, since a cubic foot loses 1000 ounces in water, say as 1000 oz.: 1 foot :: the weight the irregular body loses in water : its bulk; because the weights lost are proportional to their bulks.

From this proposition, we see the reason, why a piece of gold, hanging in equilibrio, with a piece of copper, at the ends of a nice balance, in the open air, will in water preponderate against the copper, and appear heavier; because the gold being of less bulk than the copper, loses less of its weight in water, than the copper loses. For under water, each of them loses as much weight as the water can sustain, that is, in proportion to their bulks.

And as air is a fluid, as well as water, but of less specific gravity, this proposition holds universally true; that bodies which are in equilibrio in a lighter fluid, will lose their equilibrium in a denser fluid, the lesser body becoming heavier than the other. But if the equilibrium be made in the denser fluid, the greater body will preponderate in the lighter fluid. Upon this principle we account for the operation of the *barascope*, or a simple instrument, for ascertaining the different density of the air, at different times. Let a piece of cork and any heavy metal be suspended at the ends of a nice balance, in equilibrio, in a mean state of the air. Then if the density of the air increase, the lesser body will preponderate, but if it decrease, the larger body will preponderate: for this is the same, as weighing them in different fluids.

11. The weight, which a body loses in a fluid, is to its whole weight, as the specific gravity of the fluid is to that of the body. Because the weight, which the body loses in the fluid, is the weight of the fluid equal in bulk

with the solid; therefore the weight lost is to the whole weight of the body, as the weight of so much of the fluid as is equal in bulk with the solid is to the weight of the solid; that is, as the specific gravity of the fluid, to the specific gravity of the solid. For their specific gravities are the weights of equal bulks of both. Thus, if 19 ounces of gold, when weighed in air, weigh only 18 in water, losing thereby one; the specific gravity of water to that of gold, is as one to nineteen. And if 11 ounces of silver, weighed in air, weigh only 10 in water, losing one ounce of its weight; then as one is to eleven, so is the specific gravity of water to that of silver. Now, as rain water is commonly made the standard for comparing the specific gravities of bodies, we may call any quantity of it 1000, or 1. as a cubic foot of it weighs 1000 ounces avoirdupois; and then say, as the difference between the weight of any body in air and water, is to its weight in air; so is 1000, or 1. to the specific gravity of the body. Thus we may compare the specific gravities of any number of bodies, that will sink in water, with water, and dispose them in a table, making 1. the weight of the water, with which they are compared.

But there is some difficulty in finding how much weight is lost by a body which is specifically lighter than water, and consequently will not sink in it. This may be overcome by the following contrivance. I have a piece of copper, which weighs 45 grains in air, and only 40 in water, and I desire to find how many grains a piece of cork, which weighs 12 grains in air, would lose in water. I tie them together by a horse hair, which is nearly of the same specific gravity with water, and on trial, I find that the compound of 57 grains weight in air loses 55 grains in water, of which 5 are lost by the

and consequently fifty are considered as lost by
k. Therefore, I say, as 50 : 12 : : 1 : 0.24=the
gravity of cork, compared with water.
similar contrivance we find the specific gravities
es that would dissolve in water, by only casing
vith some substance whose specific gravity is
and will not dissolve in water, such as melted
ax.
to find the specific gravities of different fluids,
the following proposition.
The weight, which the same body loses, when
d in different fluids, is proportional to the specific
s of these fluids.
use the weight, which a body loses, in a fluid,
to the weight of an equal bulk of the fluid; so
quantity of each fluid, equal in bulk with the
veighs what the solid has lost in each respec-
and since the specific gravities of equal bulks of
fluids are as their weights, or as the weights
the solid, when weighed in each respectively,
eights must represent their respective specific
s: that is, as the weight lost in one fluid is to its
gravity, so is the weight lost in the other to its
gravity. $W=MS$, and if M is given $W=S$.
ample, a certain quantity of lead, when weighed
r loses 50 grains of its weight, and when weighed
rectified spirits of wine, it loses only 43; there-
bulk of water equal to the bulk of the lead,
50 grains, and the same bulk of spirits weighs
ns. Therefore, as 50 : 43 : : 1000=specific gra-
water : 0.86, the specific gravity of the spirits.
s by the two last propositions, may the specific
s of bodies, whether hard or fluid, be deter-
and be disposed in a proper table, making that

of rain water the common standard. And since a cubic foot of water weighs 1000 avoirdupois ounces; if the specific gravity of water be called 1000, the specific gravities of every other body, in the table, will show the weight of a cubic foot of each, in avoirdupois ounces. Hence, if the number of cubic feet in any body be known, and this number be multiplied by the weight of one foot, or by its specific gravity, the product will be its weight in ounces. Because $W=MS$. M, representing its bulk in cubic feet.

The advantages, arising from these principles of hydrostatical knowledge, are great and extensive to goldsmiths, jewellers, chemists, apothecaries, physicians, and indeed to every class and description of men; as will evidently appear by a few examples.

The adulteration of metals, which every man in a community is concerned to prevent, is quickly detected by their specific gravities. Suppose you had a compound of gold and silver weighing 30 ounces$=a$, whose specific gravity was to that of water as 15 to one$=6$; and knowing that the specific gravity of gold to water is 19 to $1=S$, and that of silver to water as 11 to $1=s$, the adulteration of the gold would be evident, at first sight, from the want of a sufficient specific gravity; and the quantity of gold in the compound may soon be found in the following manner.

Let $a=$the weight of the compound$=30$ oz.

And $x=$the quantity of gold sought.

Then $a-x=$the quantity of silver in the compound.

Now, because the weight of a body divided by its specific gravity is equal its bulk,

$\frac{a}{6}=$the bulk of the compound; and for the same reason

$\frac{x}{S}$=the bulk of the gold, and

$\frac{a-x}{s}$=the bulk of silver. And because the bulk of the compound is equal to the sum of the bulks of the gold and silver, we have $\frac{a}{6}=\frac{x}{S}+\frac{a-x}{s}$, which reduced, gives

$$x=a\times\frac{S\times\overline{6-s}}{6\times\overline{S-s}}=\frac{30\times19\times4}{15\times8}=19$$ ounces of gold in the compound. Hence this

RULE.

Multiply the specific gravity of the heavier ingredient, by the difference of the specific gravities of the compound and lighter ingredient, and divide the product by the rectangle of the specific gravity of the compound, and the difference between the specific gravities of the two ingredients; and the quotient, multiplied by the weight of the compound, will be the weight of the heavier ingredient.

In the same manner, we may discover the adulteration of any composition whatsoever, the purity and goodness of medicines, and the difference between stones and ores of any kind, and between genuine and factitious gems and precious stones: as the specific gravities of the adulterated and counterfeit compositions will always differ from those of the genuine.

The rule delivered above is accurate only when the two bodies are mixed together by apposition of their particles, without any diminution or increase of their bulks. For some bodies seem to penetrate each other's pores, when mixed. Thus, if gold be mixed with lead, zinc or bismuth; silver with tin, quicksilver, zinc or bismuth; copper with tin, iron, or zinc; lead with zinc or bismuth; or wax with ox tallow; the bulk of the compound is less than what it ought to be, by adding the bulk of one to that of the other.

On the other hand, the pores of some bodies are enlarged by the mixture. Thus, iron mixed with lead, tin, regulus of antimony, zinc, or bismuth; and mercury with tin, lead, zinc, or bismuth, are increased in bulk in a greater proportion, than what should arise from the addition of their bulks.

ible of specific gravities of different bodies; the thermometer at 50°, and barometer at 29.5 inches.

Rain water	1.000
Bees wax well refined	1.000
Salt water of the sea	1.680
Aqua fortis	1.300
Aqua regia	1.244
Oil of lintseed	.942
Oil of sassafras	1.099
Alcohol	.742
Spirit of wine	.876
Oporto wine	.993
Canary wine	1.043
Vinegar distilled	1 011
Cow's milk	1.040
Goat's milk	1.009
Mercury distilled	14.000
Sal gem	2.143
Allum	1.714
Nitre	1.900
Amber	1.050
Sulphur	1.800
Diamond	3.400
Agate	2.512
Rock chrystal	2.659
Iceland chrystal	2.720
Asbestos	2.943
Chalk	2.470
Talc	2.654
Fine gold	19.640
British standard gold	18.888
Platina from the mine	16.995
Select pieces of platina	27.500
Platina cupelled with lead	19.240
Fine silver	11.091
British standard silver	10.535
Pure lead	11.325
Copper	9.000
Iron	8.000
Steel	7.704
Tin	7.400
Lignum vitæ	1.367
Cork	.240
Lignum guiacum	1.337

ELASTIC FLUIDS.

HITHERTO, we have explained the nature and properties of incompressible and unelastic fluids, in a state of rest. But there are other fluids, which are elastic and compressible, such as the air, in which we breathe; and water, when heated to such a degree, as is sufficient to convert it into vapour, which is then found to be extremely elastic.

We must endeavour to explain the general properties of this kind of fluids, before we shall be able to understand the laws of their motions, or of bodies moving through them. As we always find a sufficient quantity of air in an elastic state, the branch of natural philosophy, which explains the nature and properties of air, is called *pneumatics*, as being the principal elastic fluid, with which we are acquainted. It may not, however, be improperly considered under the head of hydrostatics; especially when we reflect that it, as well as water, is sometimes found in a fixed, as well as in an elastic state. Whether ever it be found in that intermediate state, in which it can be no farther compressed, we cannot say with absolute certainty. Yet we know, that it may be so far compressed in a syringe closed at the end, that no force, with which we are yet acquainted, can urge the piston down to the bottom of the syringe.

It is often found in a fixed and unelastic state in the composition of natural bodies; as is evident from the great quantities that are disengaged by distillation, fermentation, putrefaction, explosions of gunpowder, and chemical solutions and mixtures. Half a drachm of the oil of carroway seed, poured upon a drachm of the compound spirit of nitre, produces such a quantity of air, as is sufficient to blow up an exhausted receiver.

with a force equal to 400 pounds. A strong acid poured upon chalk, disengages a great quantity of air, which before remained fixed and unelastic in the chalk. A cubic inch of pease, by distillation, yielded 396 inches of elastic air; of coarse sugar 126; of Newcastle coal 360 inches; and other substances yielded other quantities.*

That the air is unelastic in its fixed state, appears from this, that a cubic inch of oak by distillation produced 216 inches of elastic air, which is one fourth of the weight of the oak; and this quantity of air could not be compressed into the space of one inch by a force less than 19440 pounds; and would require a still greater force to reduce it to the size of one quarter of an inch. Now this force exerted upon every cubic inch in a tree, is more than sufficient to rend it in pieces. The air, therefore, in a tree, must be in an unelastic state.

By what secret operation of nature the air is thus deprived of its elasticity, we have not yet sufficient experiments to determine. We know that some bodies absorb it in very considerable quantities. A brimstone match has been found to absorb one fourth of the air in which it burned, while a tallow candle consumed but one eleventh part, and a rat, one thirteenth part of the air in which it breathed. Thus the operation of heat seems to fix the air in some bodies, while the heat of fermentation disengages it from others.† While the elastic vapour of water may again be reduced to an incompressible state by a small degree of cold, and by a still greater degree of cold, may be congealed into ice; air seems incongealable by any degree of cold whatsoever. Although air may be much affected with heat and cold, as we shall see presently, yet we know of no degree of cold, that can fix it into a solid body; and in this, it differs from every other elastic fluid, that we know, which, by a proper degree of cold, may be reduced to the incompressible fluid, from whence it was produced, or even to a fixed and permanent state.

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* These, and all other factitious and permanently elastic airs, are, by modern chemists, termed *gases;* and are considered as constituent parts of the bodies from which they are extracted.

† Atmospheric air is now well known to consist of two gases, in modern chemistry termed *oxygen gas* and *nitrogen gas*, or *azote*, with a very small portion of a third termed *carbonic acid*

After having made these observations, we shall proceed to consider the properties of an elastic fluid, which agree with, or distinguish it from, any other fluid, and which are such as these.

1. The particles of an elastic fluid, when compressed equally in every direction, cannot be at rest, unless they be all at equal distances from each other.

If the particles be nearer to each other in any one part of the fluid, than in another, they will be repelled most on that side, where the distances between them are the least; and consequently the particles which are farthest apart will move closer together, until they are all placed at equal distances from each other, and are equally pressed on every side, when there can be no more motion among them. This will make the density of the fluid the same in every part, as there will not be a greater number of particles squeezed together in any one part of the fluid, into a given space, than in any other part of the fluid.

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gas, all intimately united together. The burning of a candle, or any other inflammable body, or the respiration of an animal absorbs the oxygen gas, and renders the residuum unfit either for the support of flame or of animal life.

2. The elasticity or spring of a compressible fluid, whereby it endeavours to expand itself, is equal to the force by which it is compressed.

If it were either greater or less, the particles of the fluid would be placed either at a greater or less distance from each other. So that when any force has compressed an elastic fluid, so as neither to condense it more, nor to suffer it to expand more than a certain quantity, this compressing force is neither more nor less than its elasticity.

3. An elastic fluid, when equally compressed in all directions, presses in all directions with an equal force.

This has been proved before with respect to all fluids, that are incompressible; and a compressible fluid, which is pressed with a certain force, which is in equilibrio with its elasticity, is in the same situation for that time with a fluid that is incompressible, and therefore must be subject to the same laws. Therefore all fluids, whether compressible or incompressible, elastic or nonelastic, press equally in all directions. This is the reason of the spherical figure of a glass when blown at the furnace, while the glass is soft; and of the same figure of a blown bladder, which is equally strong on all sides.

4. The density of an elastic fluid, whose particles repel each other, with a force, that is inversely proportional to their distances, is proportional to the compressing force. The density of any body depends upon the number of particles crowded into the same space, and is proportional to that number; but as the number of particles, in any line, is inversely as the distance of one from another, and in a superficies, is inversely as the square of the said distance, and in a solid, inversely as the cube of the said distance of the particles from each other; it follows, that the density of any body, being as

the number of particles in that body, must be inversely as the cube of the distance of the particles from each other. But the compressing force must also be equal to the number of the particles, because it must be a counterbalance to the repulsive force of all together, as it does not barely bring a few particles nearer together, and leave others at their former distance, but squeezes them all equally near to each other. Now the repulsive force of all the particles, must be in proportion to their number, which number is proportional to the cube of the distance between any two particles, inversely; because by the supposition the repulsive force of any two particles is inversely as their distance. Therefore the compressing force which is a counterbalance to the repulsive force of the whole, must be inversely proportional to the cube of the distance of any two particles. But we before proved that the density of the fluid was in the same inverse proportion to the cube of the distance of any two particles; therefore the density of a compressible fluid, whose particles repel each other with a force that is inversely as their distances, is proportional to the compressing force.

Hence, a double compressing force produces a double density in the air, or any other elastic and compressible fluid, and a triple compressing force produces a triple density.

Cor. The space, which a given quantity of air fills, is inversely proportional to the compressing force. Because its density, which is proportional to the compressing force, is inversely as its bulk, that is, inversely as the space, which it fills.

5. The air has gravity sufficient to sustain the mercury in the barometer, from 28 to 31 inches high. That the air has weight, is abundantly evident from a thou-

sand experiments that are made with the air pump, some of which we shall presently mention; but what seems the most convincing, is the actual weighing of it in a balance; by which it appears, that a vessel, when full of air, weighs more, than when the air is removed. A gallon of air weighs nearly a drachm. The weight of the air may be very sensibly felt by the hand, on the end of an exhausted receiver. It is evident, from the fixing of two brass hemispheres together, by only extracting the internal air, and suffering the external air to press them together: from the fixing of the exhausted receiver over the hole of the air pump: from the breaking of a thin pane of glass, or a bladder tied over the mouth of the receiver: from the breaking of square bottles fixed over the hole of the air pump: from forcing water into hollow glass bubbles, by the pressure of the air, and thereby making them heavier than water, so that they shall sink in it: and from sustaining the mercury in the barometer.

If a glass tube, open at one end and closed at the other, be filled with mercury, and the open end be immersed in a bason of the same fluid, it becomes the common barometer; when the mercury will subside into the bason, until it stands at the altitude of 29.5 inches in the tube, above the surface of the mercury in the bason, in a mean state of the atmosphere, leaving a perfect vacuum at the sealed end of the tube, called the torricellian vacuum, from Torricelli, the inventor of the experiment. Now, that the mercury is supported in the tube by the pressure or weight of the atmosphere is evident from this consideration, viz. the mercury being a fluid, cannot rest, until every part of the surface is equally pressed. But there is nothing except the weight of the atmosphere to press upon the surface of the mercury to be a counterbalance to the weight of 29.5 inches of mercury pressing upon that part of the surface, that is under the tube. Were any part of the surface less pressed than any other, it would, from the nature of a fluid, rise higher, until the equality of pressure were restored; and every perpendicular column of the fluid pressed equally on the bottom. Therefore, as we find the mercury at rest, and one part of the surface, under the tube, is pressed with the weight of a column of mercury 29.5 inches high, we must conclude that every other equal part of the surface is pressed with an equal weight, and that the atmosphere alone furnishes this pressure, as nothing else is in contact with it. Hence a column of mercury, of any given base, and 29.5 inches high, weighs as much as a column of air of the same base, whose altitude reaches to the top of the atmosphere. But such a column of mercury of one square inch in the base would weigh 15 pounds; and therefore every square inch, near the surface of the earth, is pressed by the air with a weight of 15 pounds.

That the weight of the atmosphere alone is the true cause, why the mercury is supported in the tube, is farther evident from what happens when this is removed. When the air is removed from the surface of the mercury in the bason, the mercury in the tube subsides to the same level with it, and rises again to the former altitude upon the re-admission of the air to press on the surface of the mercury in the bason. And because the specific gravity of mercury, well sublimated, is 14 times as great as that of water, it is evident that the same pressure of the atmosphere is sufficient to sustain a column of water, fourteen times as high as that of mercury, that is, about 34 feet.

The weight of the air is exceedingly variable; sometimes sustaining the mercury at the altitude of 31 inches, and at others no more than 28, making the scale of variation three inches. This difference of weight is sensibly felt by the valetudinarian. It greatly affects the animal functions and the health. The asthmatical person, for instance, will find his disorder increase by the levity of the air. A pure, dense, and heavy air is no more than sufficient to distend his lungs, in respiration; but when the weight, and consequently the elasticity of the air is diminished, his lungs are not sufficiently distended, and he feels a difficulty in breathing, as the mercury begins to sink in the barometer. This lightness of the air arises from the atmosphere being filled with watry particles, now converted into an elastic fluid by evaporation, or from the quantity of it being lessened in any particular place, by high winds blowing steadily, for some time, in one direction.

People are generally mistaken in their judgments about a light and heavy state of the air; calling that light, which is in reality heavy, and the contrary. The reason of the mistake seems to be this; the greater weight of the air braces up the fibres and solids of the human body, and brings them to a proper tone, so that the blood vessels acting with full vigour, give a due velocity and necessary momentum to the blood, to overcome the obstructions continually forming in the fine capillaries. Hence, by a brisk circulation of the blood, and a proper compression of the solids, we find ourselves firm and alert, brisk and lively, and therefore imagine that the air is light, when it is in reality heavy, to produce this effect. And on the other hand, by the lessened weight and elasticity of the air, the fibres are relaxed, the force of the vessels is diminished, a languid circulation ensues, and viscidities and [illegible] formed; which produce agues and fevers, the renewal of old complaints and pains in some, and a sort of indolence, languor and lazy inactivity in all; from whence we derive the idea of heaviness in the air, when it is in reality lightest.

As many effects depend upon the different weights of the atmosphere, various contrivances have been tried to increase the scale of three inches variation in the barometer, so that the minuter alterations of gravity might be more easily discovered: such as the addition of a wheel, that shall be moved by the rising and sinking of the mercury in the bason, and carry an index over a graduated circle: and the throwing off the scale into a horizontal or diagonal situation; which have given rise to the names, of the rectangular, diagonal, and wheel barometers. But these are all attended with some inconvenience or other, so that they have generally given place to the division of the scale in the common barometer by a nonius or vernier scale, which is so contrived that the hundredth part of an inch may be distinctly perceived.

The barometer stands at the same perpendicular altitude, whether the tube be large or small; for although the base of one column be larger than the base of the other, yet it is supported by a column of air whose base is larger in the same proportion. Yet we find, upon a change of weather, the variation in the larger tube is greater than in the smaller; because in the larger tube, the weight of the mercury is so great, that the motion of it will not be much hindered by the attraction or repulsion of the tube: whereas in the lesser tube the action of the glass on the mercury is considerable in proportion to the weight of mercury.

If the barometer be filled with mercury adulterated with lead or tin, whose specific gravity is less than that of mercury, it will stand higher than a barometer filled with pure unadulterated mercury, at the same time and place. To make the barometer exact, the mercury ought to be well sublimated and freed from all heterogeneous mixture, before it be put into the tube, and then boiled in the tube, to purge it of all air and moisture.

The barometer will be generally low in rainy weather and in high stormy winds, but will suddenly rise when the storm is over. As a diminution of the gravity of the air, suffers the watery particles that float in it, when sufficiently condensed, to descend, and the same diminution of gravity suffers the mercury to fall in the barometer, these two effects will be frequently found together. Yet if the density of the air be diminished by heat, the same heat rarifies the vapour, and makes it more easily supported; so that the falling of the mercury in the barometer is not always accompanied with rain. And, on the other hand, rain may be produced without any alteration of the gravity of the atmosphere, when the vapours in a current of air are checked in their course, by mountains, and are thereby crowded together in large globules, which the atmosphere cannot sustain. And when the air moves horizontally with great velocity, as in violent storms, its pressure downwards will be diminished; as we shall hereafter find, when we come to speak of the motion of fluids. Hence the mercury in the barometer must necessarily fall, until this pressure downwards be restored again, by the ceasing of the storm.

6. As the air is an elastic compressible fluid, its density must be various at different altitudes from the surface of the earth. For the lower parts being pressed with the weight of the superior, they must be brought

nearer together, in proportion as the weight of the incumbent particles is greater. And hence we find, that the density of the atmosphere is greatest at the surface of the earth, and that it decreases upwards in a geometrical proportion to the altitudes taken in arithmetical progression, or as any series of logarithms, to their natural numbers. Now, as it is found that the air at the height of seven miles, is four times as rare as at the surface of the earth; if we take any series of distances upwards in arithmetical progression as, 7,14,21,28, the density will decrease, at these altitudes, in the geometrical ratio, 4,16,64,256, &c. Consequently, the density of the atmosphere must decrease upwards, until the gravity of the uppermost particle becomes a balance to its elasticity. How high this may be, no man can say, as we know not the least gravity of a particle of air.

Was the air of uniform gravity throughout, we might easily determine its greatest altitude by experiment. Let a tube open at both ends be sealed in the neck of a vial partly filled with water, and the lower end immersed in the water. By blowing a little air into the tube the spring of the air on the surface of the water will be increased, so as to sustain a column of water in the tube, equal to the spring of the air. When this instrument is carried upwards 72 feet, the column of water in the tube will be found to have risen one inch; therefore, 72 feet, or 864 inches of air, is equal to one inch of water, or their specific gravities are in this proportion: as 1 : 864. Now, if the air were of uniform gravity 12×72 feet of air would support a foot of water, but the whole height of the atmosphere supports a column of water 34 feet: therefore 12×72×34=29376 feet, (between five and six miles) is equal to the whole height of the atmosphere, if it were of uniform density. But as it continually decreases in density, we know that at the altitude of 45 or 50 miles, it is so rare as not to refract a ray of light, seeing the twilight ends, when the sun is about 18 degrees below the horizon.

From the knowledge of the ratio of decrease in the density of the air, we are enabled to measure any heights by the barometer. Observing that the mercury, when standing at the height of 30 inches, falls one tenth of an inch in ascending 85 feet, we may find the altitude at which it will fall any other quantity, as half an inch, by saying, as the $\log. \frac{1}{29.9} : 85 :: \log. \frac{1}{29.5} : 427.89$ feet, the altitude at which it falls half an inch. This, indeed, only gives the altitude grossly, and needs a correction on account of the change of density at the bottom and top of the mountain, whose altitude we would measure, arising from the effects of heat and cold. For this purpose, the use of the thermometer is also necessary. M. de Luc, from a great number of experiments, has deduced a very accurate rule for making this correction, which we shall deliver without troubling you with the mathematical investigation. A Fahrenheit's thermometer must be attached to the barometer, and the altitude of the mercury be observed in both, first at the bottom of the mountain and again at the top. Then take the difference between the tabular logarithms of the observed heights of the barometer at the two stations, considering the four first figures, exclusive of the index, as whole numbers, and the remaining figures as decimals, and add or subtract the .452 parts of the difference of the altitudes of the Fahrenheit's thermometer at the two stations, according as it was highest at the lower or upper station: thus you will have the height of the mountain in English fathoms nearly; to be corrected, as follows.

Take the mean altitude of the thermometer at the two stations, and say, as 449 is to the difference between that mean altitude and 40°, so is the height of the upper station found nearly, as above, to the correction of the same; which added or subtracted, according as the mean altitude of the thermometer was higher or lower than 40°, will give the true height of the upper station above the lower, in English fathoms, and this multiplied by 6 will give it in English feet.*

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* The height of a mountain may be easily obtained by the barometer and thermometer, with the help of the following table, which gives the height you must ascend to cause the mercury of the barometer to sink one tenth of an inch in any temperature of heat, from 32 to 80 degrees of Fahrenheit's thermometer, supposing that the barometer stood at 30 inches at the surface of the earth.

Temp. of Therm.	32°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	95°
Feet = $\frac{1}{10}$ Inch Bar.	85.86	87.49	88.54	89.60	90.66	91.72	92.77	93.82	94.88	95.93	96.99	98.04	99.10	100.15

When the barometer stands at any other height than 30 inches, the altitude answering to one tenth of an inch depression of the mercury in the barometer, will be in the inverse ratio of 30, to that height, in any given temperature of the air. Suppose in the temperature of 60° by Fahrenheit's thermometer, when the mercury would sink $\frac{1}{10}$ of an inch, by an ascent of 92.77 feet, while the barometer stands at 30 inches. I would know what altitude would sink the mercury $\frac{1}{10}$ of an inch, in the same temperature of the air, when the barometer stands at 29 inches.

Say, as $30 : 29 :: 92.77 : 89.68$ feet = to one tenth of an inch descent of the mercury.

7. The effects of the elasticity of the air are like those of its gravity; and included air acts by its spring, just as air not confined acts by its weight. The air which is loaded with the weight of the incumbent atmosphere, presses every way, from the nature of a fluid; and the force, which it exerts by its weight, no ways depends upon its elasticity. For this force cannot be altered, whether it be considered as elastic or not. But being elastic, as is evident from a thousand experiments, it is reduced, by the compressing force, into a less space;

The following rule will give the height of any mountain exactly, provided the mountain be not above 4000 or 5000 feet.

Let x=the height of the mountain.

A=the mean altitude of the barometer at the foot of the mountain, at the beginning and end of the operation.

a=the difference of the barometer at the foot and top of the mountain.

b=the number taken out of the above table, or found by it. Then

$$x=\frac{30ab}{A}=\text{the height of the mountain in feet.}$$

That is, multiply thirty times the difference of the altitudes of the barometer, taken in inches and tenths, by the number taken out of the table (or found by it, in the manner above required) in the lower line answering to $\frac{1}{10}$ of an inch sinking of the barometer, correspondent to the temperature of the air indicated by the thermometer; and divide the product by the mean altitude of the barometer, observed at the beginning and end of the operation, and the quotient will be the height of the mountain in feet.

N. B. If there be no barometer and thermometer left at the foot of the mountain while you ascend and descend, let the instruments carried up to the top, be placed again at the foot of the mountain, for 15 minutes, after your descent; and marking the difference between the altitudes of both, at the beginning and at the end of the operation, you must then find and take the mean of them, respectively, for the mean gravity and temperature of the air, during the operation.

and its elasticity increases or diminishes, with the distance of its particles from each other: and whether it be kept within a certain space, by the weight of the atmosphere, or by any other force, it equally endeavours to expand itself, and presses every way. If the air, therefore, near the surface of the earth, be included in a vessel, without altering its density, the elasticity of the included air will be equal to the weight of the whole atmosphere. And if that weight be removed, it will expand itself, with a force equal to that, with which it was compressed. The elasticity of the air appears from many experiments; such as, 1. The expansion of a small quantity of air in a bladder, in an egg, and in glass bubbles; 2. Bladders, when sunk in water, made buoyant by the expansion of the air in them; 3. Beer, water, and porous bodies, emitting great quantities of air in large globules, under the exhausted receiver; 4. Glass bubbles and images raised in water, by the expansion of air contained in them; so that the water, which caused them to sink, being protruded, they become specifically lighter than the water; 5. A small square bottle, broken by the spring of the included air; 6. Shrivelled apples made to look plump and fair, by the spring of the air; 7. And animals greatly swelled by the expansion of the air within them, when the external pressure of the atmosphere is removed.

As the elasticity of the air is proportional to the force that compresses it; when it is sufficiently condensed on the surface of water, under the receiver, to counterbalance the weight of the whole atmosphere, it produces a beautiful fountain or jet of water, springing up to a considerable height. Water boils with less heat, on removing the weight of the atmosphere.

The use of the diving-bell depends upon the spring of the included air, whereby it prevents the water from coming into the bell; so that persons may sit and breathe in it when sunk into the water to a very considerable depth, and for a long time, provided fresh air be conveyed to them; as the air frequently breathed, soon becomes unfit to preserve animal life. The air, which filled the bell, at the surface of the water, is compressed into half the space when the bell descends 34 feet; for then it is pressed with the weight of two atmospheres, as thirty four feet of water is equal in weight to a column of air of the same base, and an altitude equal to the height of the atmosphere. When the bell descends 68 feet, the air in it is pressed with the weight of three atmospheres, and consequently is two thirds full of water. The admission of fresh air into the bell will soon exclude this water by its spring and leave the bell full of condensed air. This condensed air gives a very sensible pain to the persons in the bell for some time, until the air in their bodies becomes equally dense with it, and counterbalances its pressure on them. Hence, the necessity of a slow and gradual descent of the bell, to give time to the air within the persons to become of equal density with that in the bell.

8. The elasticity of the air when confined in a vessel is greatly increased by heat; yet in the open atmosphere, it is neither increased by heat, nor diminished by cold.

The elasticity of the air is proportional to the compressing force; that is to the weight of the atmosphere, or the height of the mercury in the barometer, whether in summer or in winter, by day or by night; and therefore the elasticity of the air, in the open atmosphere, is greatest, when the barometer is highest, and least, when it is lowest, without any regard to heat or cold.

Yet if any quantity of air be included in a close vessel and do not communicate with the open air, its elasticity will be greatly increased by heat, as is evident from the swelling of a bladder partly filled with air, when laid before the fire for a short time. Or, if a thin glass vial, close corked, be set before the fire, the elasticity of the air in it increasing by the heat, will make the air expand so as to burst the vial. From hence, we may conclude, that when the particles of air are kept at the same distance from one another, heat will cause them to repel each other more forcibly, or make them more elastic. But if the heated air be at liberty to expand itself, and mix with the open atmosphere; then, as much more forcibly as the particles repel each other, so much farther will they recede from one another. Upon the whole, the heat will increase the elasticity or repulsive force of the air, and make it more rare, or set the particles at a greater distance; when they will repel each other with a less force, in proportion to the increased distance of the particles; so that the repulsion of the particles is as much lessened by their increased distance, as it is increased by the action of heat: and therefore the elasticity of the air in the open atmosphere is not altered by heat or cold.

From the increased elasticity of confined air, by the action of heat, we account for the additional degree of heat that water is capable of receiving, when heated air is confined on its surface, above what it could receive, when boiling in open air: so that in a digester, it will even dissolve bones and convert them into a jelly.

Yet, although a moderate degree of heat increases the elasticity of confined air, a very great degree of heat will totally destroy its salubrity, and render it unfit for animal life; as appears from the instant death of animals in adust air, after it has passed through the fire; when it produces the same effects with mephitic or fixed air.*

The effects of heat and cold in the air gave rise to the construction of *thermometers*, to ascertain their various degrees. Air being most sensibly affected with heat and cold, was first thought of for the thermometer, in the form I have already described. But thermometers of this kind are affected by the weight of the atmosphere, as well as by the spring of the confined air: and this renders them useless, except for extempore experiments, such as estimating the difference of heat in the rooms and cellars of houses. This therefore gave way to the use of other fluids, such as spirits of wine, linseed oil, and mercury, which have been tried with various success; some of them answering one purpose, and some another.

Thermometers made with these fluids generally consist, of a long tube or stem and a spherical or cylindrical bulb at the end. The bulb and part of the stem is filled with the fluid, and when heated to a proper degree, so as to fill the tube, the stem is hermetically sealed at the top, to keep off the pressure of the atmosphere, that the instrument may be affected only with heat and cold. As these fluids will rarify and expand with heat, and contract again with cold, the thermometer will stand highest in warm weather.

Thus far they may be said to vary alike, as their variations proceed from the same cause. Yet they seldom vary to the same degree, with the same degree of heat, when made of the same fluid. One may rise an inch, while another in the same place rises but half an inch.

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* Mere heat will not vitiate the air or destroy its salubrity; this takes place only when it suffers a decomposition by a burning body, or some other in which the oxygen, or vital principle of the air, is absorbed.—*Ed.*

The variation of two thermometers is directly as the capacities of their bulbs, and inversely as the bases of their stems. Suppose the diameter of the stems equal, but the capacities of the bulbs different: equal degrees of heat will swell and enlarge the bulks of the fluid; in proportion to their bulks: so that if the capacity of the one were double or triple that of the other, it would be enlarged twice or thrice as much; and if by swelling they rise in tubes of equal diameters, the largest must rise twice or thrice as high as the least. This is indeed upon the supposition, that the fluid is equally heated through in both. But on sudden changes of heat and cold, the case will be different, for the lesser ball will be sooner heated through, and be more expanded, in proportion to its bulk; unless the heat continues for some time, until it has penetrated the larger bulb also. So that on this supposition the rise of the fluid will be directly as the capacity of the bulb. Suppose, again, that the capacities of the bulbs were equal, but the diameters of the stems different. When the fluid is equally heated in both, equal quantities of it must rise in the stems, and therefore the heights to which they rise, must be greater in proportion as the diameters are less; that is, the heights will be inversely as the bases of the stems. Therefore when neither the capacities of the bulbs, nor the diameters of the stems are equal, the heights of the fluid in the thermometers will be directly as the capacities of the bulbs, and inversely as the bases of the stems.

Yet the variations of different thermometers may be compared together by some common scale, such as that of Fahrenheit; in which the freezing point is at 32, and the heat of boiling water at 212 degrees. If any other thermometer, however different in the diameter of the stem, or capacity of the bulb, have its scale divided in the same manner, the degrees of one will be proportional to the degrees of the other, and indicate the same degree of heat, at any particular time, however different the lengths of the scales may be.

The thermometer is thus made. The tube must be well heated to expel the air out of it, and when the end of it is immersed in the fluid, the pressure of the air on the surface of the fluid will force it into the bulb of the thermometer, until that be filled, and part of the stem, as the tube begins to cool. The tube thus filled is immersed in snow, just beginning to melt, or water beginning to freeze, and the point, at which the fluid stands in this situation, is marked 32, according to Fahrenheit's scale. It is then immersed in boiling water, if it be filled with mercury, and the point at which it then stands is marked 212 degrees, the intermediate space being divided into 180 parts called degrees. At 96, the heat of the blood is noted; and the part of the tube below the freezing point is divided in the same manner. If spirit of wine be used, it will not bear so great a degree of heat as boiling water, before it would boil and burst the tube; the thermometer is therefore immersed in water heated to such a degree, as to suffer melted wax poured on it to begin to coagulate, and the point at which the spirit then stands is to be marked 142 degrees. The fluid is then to be raised to the greatest height, that it can bear, and the tube sealed at that point, to exclude the pressure of the external air.

As spirit of wine boils and bursts the tube with a small degree of heat, such a thermometer cannot indicate the heat of boiling oils, or melted metals. But as linseed oil acquires four times as much heat before it boils, as spirit of wine, its use is more extensive, but it soon sullies the tube, and reaches no farther than the heat necessary for melting lead or tin. It therefore gave way to the mercurial thermometer, which sustains a much greater degree of heat and cold. But this also has its limits, as the mercury has been lately found fixed by a high degree of artificial cold, and rendered malleable, or capable of being stretched under the hammer.

The bulb of the thermometer should be made cylindrical rather than spherical, that the heat may sooner penetrate the fluid, and thereby the sooner show the variations of heat and cold. For the mercury has been found to sink in the tube upon a sudden application of heat, before it rose; which was owing to the expansion of the tube, before the heat reached the center of the mercury.

9. Air is necessary for the existence of flame and fire; as a candle and live coals are suddenly extinguished by withdrawing the air from around them: it is necessary to promote putrefaction and fermentation; as animal substances, such as flesh and eggs, are preserved from putrefaction for a considerable time in vacuo, and by covering them with lard or butter, or any other substance that would exclude the air from them: and I need hardly add that it is absolutely necessary for the preservation of animal life, when the foramen ovale is closed. Nature has provided a double mode of circulation of the blood, for those animals, that are to live in air and water alternately, one through the lungs, while they are in the air, and the other, through the foramen ovale, when they are in the water.

Having thus considered the nature and general properties of fluids, both incompressible and elastic, we now proceed to that part of hydrostatics, called *hydraulics*, which explains their properties and operations, when they are in motion.

HYDRAULICS.

Hydraulics is that branch of natural philosophy, which considers and explains the laws and operations of fluids in motion, whether they be compressible or incompressible; the construction of machines for putting them in motion, or for being moved by them; the nature of winds and sounds, of springs and tides.

The motion of a fluid arises, generally, either from its gravity, pressure, or elasticity; or from the action of heat, the force of machines, or the principle of gravitation.

The most natural order of treating this subject, is, first, to consider the motion of such fluids as are incompressible, and secondly, the motion of such as are elastic.

MOTION OF INCOMPRESSIBLE FLUIDS.

The first cause of motion in fluids, which we shall consider, arises from their gravity, by which the particles that stand highest, press upon those below them; whereby the whole surface becomes level, or rather a part of a sphere, whose radius is the same with that of the earth. Thus water in the reservoir will rise in a small duct or canal that communicates with it, to the same height, if the canal be close all round, and continued up to the altitude of the reservoir; otherwise it will run out in a continued stream, until the water in the reservoir sink down to the same level. The solution of this phenomenon depends upon this principle of hydrostatics,—that the pressure of fluids is in proportion to their perpendicular altitude, and not according to their quantity. Hence, water is often brought in canals under ground, from a reservoir in an elevated situation, to supply gardens, meadows, and cities, and distributed through various parts that are lower than the fountain. Hence also is the descent of waters in rivers and streams, from springs that are situated above them; and the breaking out of streams from the bottoms or sides of mountains, from cisterns in them, which are continually supplied by rains and vapours distilling through the earth. As a great quantity of the vapour raised from the surface of water returns to the earth in a silent and imperceptible manner, it was supposed to contribute but very little to the supply of rivers, until Dr. Halley instituted an experiment for estimating the quantity raised in the Mediterranean sea, supposing it to be 40 degrees long and 4 broad, on an average, in the course of a summer day. After salting some common water, and heating it moderately, until he brought it to the same degree of heat and specific gravity with the sea in a summer day, he suspended it to the end of a nice balance, and observed the quantity evaporated in a given time; from whence he determined the quantity evaporated from the surface of the whole Mediterranean sea, in a single day, to be, in round numbers, 5280 millions of tons. He next endeavoured to estimate the quantity of water furnished per day by all the large rivers, that empty themselves into this sea, supposing that each of them brought ten times as much as the river Thames; and found it to be only 1827 millions of tons; which is but about one third part of the quantity evaporated. The deficiency therefore must be made up by the rains and vapours, which fall again on the sea, and by that continual influx from the Atlantic ocean, which has been observed at the streights of Gibraltar; though, possibly, little may be owing to this latter cause, as subsequent observations have discovered contrary currents in the streights, which may nearly counterbalance one another. So that rivers are not wholly supplied by rains and melted snows, but principally by vapours. This may also be applied to the Caspian sea, which daily receives the waters of many large rivers, and never overflows, although without any known communication with any other sea. The vapour raised from it daily, and carried off by the winds, to other parts, may be a counterbalance to the quantity of water received from its rivers.

Those who suppose that water may be raised from the sea by subterranean canals, or columns of sand, whose bases communicate with the sea, are ignorant of the fundamental principles of hydrostatics. For, although columns of sand may raise and suspend the water of the sea, like capillary tubes, yet they can suspend it only to certain heights, and there they will retain it, without permitting it to flow off in a continued stream. This is evident enough from the ascent of water in a capillary tube. Should the diameter be so small, that the fluid could rise even to the top, yet it would not run off from it. And in a larger canal, the water could only rise to the surface of the sea, and therefore could not supply rivers, whose sources are much higher. It is indeed said, that springs have been found on the tops of mountains, but it is said by inaccurate observers; for the matter has been examined, and found otherwise, in the places, where such springs are said to be found. It is so contrary to the fundamental principles of hydrostatics, that we may safely affirm, that it never was, nor can be, fact; unless there be some land in the neighbourhood, in which the reservoir is placed, of greater elevation from whence the stream is supplied, by a bended canal under ground. Let such a canal descend ever so low beneath the bottom of the reservoir, and rising again from a valley, by the side of another mountain, the water would still rise in the said canal, till it arrived at the same height with the surface of the water in the reservoir.

As rivers run on an inclined plane, it may be asked, why their velocity is not continually accelerated. The reason arises from the friction and resistance of the sides and bottom of the canal, which, when it becomes equal to the accelerating force of gravity, must generate an equable and uniform motion. Hence, the water in rivers moves slowest near the sides and bottom, because of the greater resistance it meets with there.

In different parts of the same river, the velocity is greatest, where the bottom has the greatest declivity, on account of the greater accelerating force. But where the velocity is least, the depth or breadth of the river must be greatest, in order to admit the same quantity of water, to pass through any sections of the river, in equal times. For did not the same quantity pass through all the sections, whether deep or shallow, broad or narrow, in the same time, some part of the channel would at length be left dry. Wherever, therefore, the channel is narrow or shallow, the water must move with greater velocity, than where it is broad or deep. Hence, the momentum of the moving water of the whole river, at any particular section, is the same as at any other.

From the water's rising in a bended canal to the same height with the surface of the reservoir, we learn the ationale of artificial fountains and jets of water, for the upply of gardens, meadows and cities. The water is rought in hollow trunks or tubes under ground, from he reservoir in an elevated situation, to the place where t is needed; and upon turning a cock, it will spout up early to the same height with the surface of the water n the reservoir; but not altogether so high, on account f the resistance of the air, the friction against the sides f the hole from whence it issues, and the diminished elocity of the ascending fluid, by the action of graviation, together with the weight of the superior partiles, resting in some measure upon the particles, which mmediately follow them, with a greater velocity.

Water may not only be conveyed across a valley in ubes under ground, but also over a mountain, whose eight is not more than 34 feet above the surface of the eservoir, by means of a *syphon* laid across the hill; rovided the syphon be first filled with water. The *syhon* is a bended tube with legs of unequal lengths, sed in drawing off liquors from one vessel to another. The shorter leg being immersed in the fluid, if the ressure of the atmosphere be taken off from the surace of the fluid in the shorter leg of the syphon, the eight of the atmosphere will raise a column of water n the syphon 34 feet high, as it raises the mercury in he barometer. From thence descending by its own ravity along the longer leg of the syphon, as low as he surface of the water in the reservoir, it would by its weight become a counterbalance to the weight of the water in the other leg. And should the longer leg of the syphon be continued but a little lower, the gravity of that column of the fluid, would be greater than that of the other, and thereby cause it to run out. Should he legs of the syphon be less than 34 feet, yet still the water would run out of the longer leg; becaus pressure of the atmosphere on each end of the sy is equal, but the perpendicular column of water i longer leg being the greater, it will be more than cient to counterbalance the fluid in the shorter leg, therefore it will run out at the longer leg.

The ancients, from their ignorance of this hydr law of the ascent of fluids in a syphon, were oblig cut through mountains, and cast arches over valle their famous aqueducts; whereas, a tube laid acros valley under ground, or over the hill, would hav swered the purpose, with much less expense.

This also accounts for the phenomenon of the *talus-cup*, containing water until it be almost filled the surface of it reach the chin of the image of a talus fixed in the cup, with a syphon concealed i image, and then emptying itself by a hole in the tom. The syphon goes up one leg of the image, r to its chin, and then descends through the other end opening at the heel of the image in the cup, the other end passes through the bottom. Whi water is poured into the cup, it will rise in the sy until it come to the bend of it, which is nearly as as the chin of the image, and then, whatever mo poured into the cup will descend by its own gr pushing the air before it, until both legs of the sy be full; and because the leg that passes throug bottom is longer than the other, the whole fluid issue out of the bottom.

Sometimes the syphon is concealed in the han the cup, while the upper part is perforated, to ex the hydrostatic skill of the person who attempts to out of such a cup.

From the operation of the syphon, we are enabl account for reciprocating and intermitting springs. The reciprocating springs are furnished with a syphon under ground, which communicates with the bottom of the reservoir in the hill, and whose bend reaches as high as the surface of the water in the reservoir. While the water rises in the reservoir, being percolated through the earth, it will at the same time rise in the canal to the same altitude, until it begin to run over the bend of it, and then it will descend through the other leg of the syphon until the whole reservoir be emptied, when the stream must necessarily stop until the reservoir be filled again to the height of the bend of the canal or syphon. Thus the stream will run and stop alternately every 5 or 6 hours, or according as it requires more or less time to fill the reservoir.

But if a small stream from another reservoir communicate with this canal or syphon, above mentioned, the stream will continue to flow, with the water furnished from this last reservoir, while the other is in filling, and thereby exhibit the phenomenon of intermitting springs, which flow with great velocity for a few hours, and then run with a feeble stream, until the grand reservoir be again filled.

Upon the same principle of the pressure and weight of the air, we account for the rise of water in the common pump and syringes. The common pump is thus constructed. A valve opening upwards is fixed in the barrel of the pump, near the surface of the water in the well, and another in the piston, which works in the barrel, fitting it so exactly as not to suffer any air to escape by it. When the piston is drawn up from its lowest situation, it sustains the weight of the atmosphere above it; and the air on the surface of the water in the well, by its pressure, raises it into the barrel of the pump through the lower valve, until the spring of the attenuated air under the piston, added to the weight of the water raised through the lower valve, become equal to the weight of the external air pressing upon the surface of the water. As the piston descends for a second stroke, through the attenuated air below it, the weight of the water already raised shuts the lower valve, while the piston compresses the air below it, until its density and spring become greater than that of the air above the piston, when it will open the valve of the piston, which also opens upwards, and part of it will escape through it, and mix with the common air. When the piston is again elevated, it sustains the atmosphere above it as before, and attenuates the air below it, whereby the water will rise higher in the pump by the pressure of the external air, until the spring of the included quantity of air in the pump, now lessened by the escaping of some part of it through the valve of the piston, added to the weight of the column of water raised in the pump, become again a balance to the pressure of the external air. Thus you proceed, until all the air in the barrel of the pump escape through the piston, and the water following by the weight of the atmosphere pressing upon the surface of the water in the well, will spout out of the side of the pump, through a pipe inserted for the purpose.

Unless the piston reach within 34 feet of the surface of the water, in its lowest situation, no water can pass through its valve; because the weight of the atmosphere is sufficient to bear a column of water of no greater altitude than 34 feet, at its mean gravity, when the barometer stands at 29.5 inches. And if mercury were to be pumped up, the piston must descend within 29.5 inches of it; because a mercurial column of that altitude is equal to the weight of the atmosphere.

The forcing or lifting pump works by an inverted piston playing in the water of the well, having a valve opening upwards, through which the water rises while the piston descends, and when the piston is elevated, the weight of the water above it shuts this valve, and it is forced through the valve of the barrel, on which it is again sustained while the piston descends for a second stroke; whereby as high a column of water may be raised, as the operator is able to sustain: and in both these kinds of pumps, the pressure of the atmosphere sustains a column of water equal to 34 feet in altitude.

The pressure of a fluid against the sides and bottom of a vessel is increased by the motion of the fluid through a hole in the vessel. We have seen before, that the pressure of a fluid at rest on any point was proportional to its depth below the surface of the fluid. But as soon as the fluid begins to spout out of the side of the vessel, the pressure is greatly increased by the motion of the fluid; and experiments, made with a machine for the purpose, prove, that it is double to the pressure of the stagnant fluid. At a given depth, the pressure on all sides of the vessel is equal, by the known laws of hydrostatics; but when a hole is opened, and the fluid begins to spout through it, the pressure is doubled on the opposite side, and is in some measure increased round the circumference at that depth. Hence it may happen, that a vessel, that can contain any stagnant fluid, may suddenly burst, when it begins to spout out of the side. Because, not only that part of the fluid which is contiguous to the aperture, receives some motion, by which its momentum is increased; but also the whole mass of the fluid receives a certain degree of motion, and therefore acts with a much greater force, than it could exert, while the fluid was at rest. This may properly enough be denominated the *hydraulic paradox.*

The knowledge of this principle is not only necessary to explain many important phenomena of nature, but it may also be applied with good success to many mechanical and hydraulic purposes. The motion of spouting fluids is certainly a very powerful principle of action, and is sufficient for turning a mill of a very simple construction, without a single wheel, cog, or round. If a perpendicular cylinder be continually kept full of water, by a small stream flowing in at the upper end, while it revolves on an axis perpendicular to the horizon, and the axis carries round a stone for grinding corn or any other grain on its upper end, like the spindles of our common merchant-mills; and if four tubular arms, communicating with the cylinder, be fixed at right angles to it, to be carried round by the axis, which is therefore to be made fast to the lower end of the cylinder; motion might be given to the whole machine, by only suffering the water to spout out at the sides of these arms, in opposite directions, near their extremities. The reaction or pressure of the spouting water on the side of the arm, opposite to the aperture, having nothing to counterbalance it, must give motion to the arm in an opposite direction to that of the spouting water; and this motion, when once begun, will soon be greatly increased by the centrifugal force of the water in the circulating arm, which will occasion both a greater degree of pressure, and a greater discharge of water in a given time.

The celebrated Bernouilli imagined, from a computation he made of the force of this pressure, that a ship might be driven by it, at the rate of three or four miles per hour, without the help of sails or oars, by only pumping up water from the sea into a reservoir of considerable altitude, and letting it spout out of the stern of the ship. It is however to be doubted, whether the success of the project would answer his computation, however true the principle may be in theory. The mill above mentioned has been constructed and found to answer every expectation, because the centrifugal force of the water in the arms greatly increases the pressure and, consequently, the velocity of motion.*

While I am on the subject of the pressure of fluids in motion, you will permit me to remark, that the electrical fluid seems also to be possessed of this principle. When it is at liberty to escape from an electrified body by means of a sharp point, the pressure in a contrary direction will give motion to a wheel, in the radius of which it is inserted. Nay, if the axis of the wheel be laid on two parallel inclined planes, the wheel may be made to roll up the inclined planes, contrary to the known laws of gravity.

Having considered the pressure of stagnant fluids on the sides and bottoms of vessels, and the increase of pressure occasioned by their spouting through a simple perforation in the side of a vessel, without any tube inserted, we shall now consider their pressure on the sides of a horizontal canal, or tube inserted in the side of a vessel, while the fluid is in motion.

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* The powers of this kind of mill have been greatly overrated, by not duly considering the resistance arising from the *vis inertiæ* of the water in the horizontal arms, which is continually undergoing a change of direction from the time it enters till the time it escapes. The late Mr. Waring of Philadelphia has demonstrated, that the powers of this mill are no greater than those of an undershot, with the same head and quantity of water. See Vol. III. of the Transactions of the American Philosophical Society.—*Ed.*

If a vessel filled with water to any height communicate with a small horizontal tube inserted in the side, and in the same horizontal tube another of equal diameter be fixed perpendicular to the horizon; while the horizontal tube remains closed at the end, the water will stand in the perpendicular tube, at the same altitude as in the vessel, by the known laws of hydrostatics. But upon opening the end of the horizontal tube, the water will immediately subside in the perpendicular tube; provided the spouting water meets with no resistance from the roughness of the tube, the contraction of its orifice into a less space than its whole diameter, or its being bent into any angle or curve. From all these causes the spouting water will be retarded in different degrees; and therefore, when any of them operates, the water will not intirely subside in the perpendicular tube, but stand in it at a certain altitude. From hence we discover a remarkable difference between the pressures of stagnant and moving fluids; and conclude, that water moving with any velocity, in an even, smooth, horizontal canal, has no pressure upwards, although in a quiescent fluid, the pressure upwards and downwards is equal.

But if while the water was spouting out of the end of the horizontal tube, another orifice were opened in its side, of equal diameter with the tube itself, about 20 times as much water would issue from the end, as from the side of the tube, in a given time; which shows, that the lateral pressure was but about the twentieth part of the pressure in the horizontal direction.

And if an orifice were opened in the under side of the horizontal tube, while the water was flowing out of e end; about 16 times as much water would be disharged from the end, as from the under side, in the ame time; which shows, that the pressure on the botom is but about the sixteenth part of the horizontal npetus.

From these experiments, we conclude in general, at water moving in a horizontal direction, in smooth, gular, and even canals, exerts no remarkable force, xcept in the direction of its motion; that it is but little n the bottom, less on the sides, and none at all upards.

Hence we may infer, that a horizontal tube, which just sufficient to bear the pressure of the running tream of water, is immediately burst, upon shutting e orifice at the end, and confining the water at rest ithin it: whereas the vessel that was but just sufficient contain the stagnant fluid suddenly bursts, upon givg the fluid liberty to run out of a hole in its side.

What we have here advanced is on the supposition, at the spouting fluid meets with no retardation in its otion, either from the asperity of the horizontal tube, e contraction of its orifice at the end, or its inflection to any angle. While it remains straight, if the orifice e contracted, the fluid will begin to press upon the ube in all directions, with a force proportional to that ontraction. And while there is no contraction in the rifice, the pressure in all directions will be proportional the angle, by which the direction of the spouting uid is changed. An obtuse angle retards less than a ight angle, and this, less than one that is acute. If the etardation occasioned by one angle be added to that hich was produced by another, the sum will be found reater than that which arises from bending the horiontal tube in both these angles at different places at the same time. Yet the retardation occasioned by a change of direction in different angles, successively, continues exactly the same, let the order of the angles of change be what it will. Let a certain degree of retardation be occasioned, first by a right angle, then by an obtuse angle, and then by an acute angle; this retardation, and consequently the degree of pressure in all directions, upwards, downwards, and laterally, will continue the same, while the same changes of direction are effected by the same angles in any other order.

From the pressure of fluids, we learn to estimate their momenta, when issuing out of holes in the sides or bottoms of vessels. The momentum of force of any body in motion depends upon the quantity of matter it contains, and the velocity with which it moves. The momentum of a fluid issuing out of a given hole, either in the side or bottom of a vessel, is always proportional to the square of the velocity, or the square of the quantity. Because the quantity discharged, from a given hole, in a given time, must be as the velocity of its motion; and the momentum being as the rectangle of the quantity and velocity, we may substitute the one for the other, as they are proportional to each other; and then the momentum will be as the square of the velocity, or as the square of the quantity. Let M=the momentum, Q=the quantity of matter, and V=the velocity; then $M=QV=VV=QQ$.

The quantity of water discharged from a given hole in a vessel, in a given time, and the velocity with which it spouts out, are, each of them, proportional to the square root of the depth of the hole below the surface of the water in the vessel.

By the laws of hydrostatics, the pressure of a fluid is proportional to the depth below the surface, and this ssure being the cause of the momentum must be portional to it, as its effect; therefore the momentum as the depth of the hole below the surface; but the mentum is also proportional to the square of the ve- ity, and to the square of the quantity of matter. nce, the velocity and quantity of the fluid discharged, each proportional to the square root of the depth of hole, below the surface. Let D=the depth of the le, then, $M=D=VV=QQ$. Therefore $\sqrt{D}=V=Q$.

The momentum or pressure on the side or bottom the vessel, at a given depth, is as the area of the part inst which the fluid presses, by the principles of drostatics. Therefore if A represent the area of a hole the side or bottom of a vessel, $M=A$, or the momen- with which a fluid spouts through two unequal les, at the same depth, are as the areas of the holes.

The quantities of water spouting through unequal es, at the same depth, are proportional to the areas the holes. Because the momenta, which depend on the quantity and velocity, are as the areas, and ause the holes are supposed to be at the same depth, velocity of the fluid is the same in both holes, or is iven quantity; therefore $A=M=Q$. that is, the quan- es discharged through unequal holes, at the same pth, are as the areas of the holes.

The quantities of water spouting through unequal les, and at unequal depths, are proportional to the are roots of the depths and the areas of the holes. cause, when the holes are equal, $Q=\sqrt{D}$, or the ntity is as the square root of the depth; and when depths are equal, $Q=A$, or the quantity is propor- nal to the area of the holes; therefore, when neither equal, $Q=A\sqrt{D}$.

The quantities of water, spouting from unequal holes, and at unequal depths, will be as the velocity and area of the holes conjointly. $Q=A\sqrt{D}=AV$, for $\sqrt{D}=V$.

The momenta of a fluid running through unequal holes, at unequal depths, will be as the areas of the holes, and the squares of the velocities. For $M=QV$, and $Q=AV$. Therefore $M=AVV$.

The velocity with which any fluid spouts out of the side or bottom of a vessel is the same with the velocity which a heavy body would acquire in falling from the surface of the fluid to the depth of the hole. For the uppermost particle coming out of a hole at the surface, would have the same velocity, with which a heavy body begins to fall; and we have proved before that the velocity at any other depth increases with the square root of the depth of the hole, $V=\sqrt{D}$, just in the same manner that the velocities of falling bodies increase with the square roots of the distances. Therefore, at any depth, the velocity of the spouting fluid is the same that a heavy body would acquire in falling from the surface to the hole.

As the ratio between the velocities of the spouting fluid and the depths of the holes is the same with the ratio of the velocities to the spaces passed over by bodies falling freely by the force of gravity, it follows, that the spouting fluid observes the same laws with falling bodies.

A fluid runs out of a hole in the bottom of a cylindrical vessel with a velocity that is uniformly retarded, while the altitude of the fluid above the hole is continually growing less. The velocity of an ascending body being continually lessened by the constant action of gravity, is uniformly retarded, and the velocity of the pouting fluid observing the same laws, must also be niformly retarded. And the surface of the fluid in the essel will sink with a velocity uniformly retarded, beause the velocity of the spouting fluid is retarded in e same manner. Hence also it follows, that

The spaces described by the sinking surface of the uid, and the quantities discharged through a hole in e bottom of a cylindrical vessel, in any separate equal rtions of time, will be as the odd numbers, 1,3,5,7,9, c. taken backwards; or as the odd numbers, if the nes be taken backwards, from the emptying of the ssel to the beginning of the discharge. When the rface sinks with a uniformly retarded velocity, the aces described in equal times are as the odd num- rs, 1,3,5,7, &c. taken backwards, as in the case of dies falling freely or ascending perpendicularly: and the surface sinks by the quantity discharged, the antity issuing in equal portions of time must be in e same ratio as the odd numbers taken backwards.

For the same reason, the spaces described by the nking surface, and the quantities discharged by a hole the bottom, in unequal times, will be as the squares the times counted from the time when the vessel is npty, to the beginning of the discharge. This we have fore proved to be the proportion between the times d spaces described by falling bodies. And the times discharge have the same proportion to the spaces ssed over by the sinking surface of the fluid, and also the quantity of fluid discharged in the said times.

As a falling body acquires such a velocity at the end its fall, as would carry it uniformly through double e space fallen through, in the same time; it follows, at if the velocity of the spouting fluid should continue niform, double the contents of the vessel would be discharged through a given hole in the bottom, in the time that the vessel would empty itself, while the velocity is continually decreasing, with the decreasing altitude of the fluid.

The times in which vessels of equal heights would empty themselves through equal holes, are as the areas of their bottoms. If the area of one bottom were equal to any multiple of the other, as suppose ten times, there would be ten times as many columns, of the same base and altitude, in the one as in the other. You may suppose that the base of each of these columns was equal to the area of the hole; and then each of the columns would require an equal time to drop through the hole at the bottom, therefore the vessel, whose bottom was ten times as large as the other, would require ten times as much time to empty itself as the other would. Hence, the times for vessels of equal heights, but unequal bottoms, to empty themselves through equal holes in the bottoms, will be as the areas of the bottoms. If T=the time of discharge, and D=the diameter of the bottom, then $T=DD$.

The times of discharging vessels of equal heights and equal bottoms, through unequal holes, will be inversely as the areas of the holes. If the area of one hole be ten times the area of another, there will be ten times as many columns of the fluid in one as in the other, supposing each of the same base as the hole through which it must pass. But each column passes through its respective hole in the same time, as they are all of the same altitude. Hence, the times of emptying such vessels will be inversely as the areas of the holes. $T=\frac{1}{dd}$.

The times of emptying vessels of equal bottoms through equal holes, but of unequal altitudes, will be as the square roots of the altitudes. As the vessels have equal bottoms and equal holes, each will contain an equal number of perpendicular columns of the fluid of the same base, but of different altitudes. Now the time of any one of these columns dropping through the hole under it will be as the square root of its length, by the laws of falling bodies. The same is true of all the columns, and consequently of the sum of them. Therefore the times of emptying such vessels will be as the square roots of their altitudes. For if we multiply the terms of this ratio by the number of columns of the fluid, or any other given number, the ratio will still be the same. If there were ten columns of the fluid in each, of the same base with the area of the hole, ten times the time of one passing out, would be the time of emptying the vessel, and this is proportional to ten times the square root of the altitude, or proportional to the square root of the altitude : $T=\sqrt{A}$. Hence, all being different, $T=\frac{DD\sqrt{A}}{dd}$.

Hence it is easy to compute the time of emptying any vessel through a given hole, by first determining, from the laws of falling bodies, the time of discharging one perpendicular column of the fluid, of the same base with the area of the hole, and then multiplying this by the number of such columns in the vessel.

If a tube be inserted in the bottom of a vessel, it will empty the vessel in much less time, than a bare hole in the bottom could do; as the velocity of the spouting fluid, which is always proportionable to the square root of the altitude, is thereby greatly increased.

As both the velocities of a spouting fluid through a given hole in the side or bottom of a vessel, and the times of emptying cylindrical vessels of equal bottoms through a given hole in the bottom, are always proportional to the square roots of the altitudes of the fluid above the hole; which is the proportion of the ordinates of the parabola to their corresponding abscissæ; it is evident, that the ordinates of a parabola will express both the velocities of the spouting fluid, and the times of emptying such vessels, when the abscissæ of these ordinates express the altitudes of the fluid above the hole.

If a semicircle be described upon the perpendicular altitude of a vessel full of water, as a diameter, and an ordinate be drawn parallel to the horizon from a hole in the side of the vessel, from which the water spouts horizontally, the said ordinate will be equal to half the distance to which the water will spout. Because the space passed over by a body in motion is equal to the rectangle of the velocity and the time, when the motion is uniform, as the horizontal velocity of the spouting water is. But the velocity of the water, being the same that a heavy body would acquire in falling from the surface of the water to the hole, is proportional to the square root of the said distance; and as the acquired velocity would carry it over twice the said distance in the times of falling, the velocity is equal to twice the square root of the said distance from the surface to the hole; and the time of flight, until the water reaches the horizon, is the same that a heavy body would take to fall from the hole to the horizon, or through the remainder of the diameter of the circle below the hole, and therefore is proportional to the square root of the said remainder of the diameter. Now the horizontal distance to which the water spouts, being equal to the rectangle of the velocity and time, is equal to twice the square root of the depth of the hole, multiplied by the square

ot of the altitude of the hole above the horizon,
hich rectangle is equal to twice the said ordinate, per
E. 6.

Hence, as the greatest ordinate is drawn from the
nter, and two others can be drawn equal to each
her, at equal distances above and below the center; it
llows, that the water will spout to the greatest hori-
ontal distance from a hole in the middle, and to the
me distance from two holes, at equal distances above
d below the middle.

The motion of a spouting fluid is every way similar
that of a projectile, and the curve in which it moves
a parabola; because it is acted upon by two forces,
z. gravity, and the pressure of the water in the vessel,
hich gives the spouting water a uniform horizontal
locity. The height of the water above the hole is the
npetus of the projection, which is always equal to one
urth of the latus rectum of the parabola described;
d may be easily found by dividing the square of the
stance to which the water spouts by the altitude of
e hole above the horizon: the quotient is the latus
ctum, or four times the impetus or height of the wa-
r above the hole. But it is needless here to repeat what
e have demonstrated already concerning the motion of
rojectiles.

By the properties of fluids in motion already deliver-
d, we are able to estimate the force that circulates
e fluids in the human body, which is probably the
ost compounded hydraulic machine in nature; such
the force of resilition, in the blood spouting from the
eins and arteries in bleeding; the force, that carries the
hyle up the thoracic duct to the left subclavian vein, or
e force by which the blood ascends from the feet to
e heart. To give only one example: let a principal

artery in a living animal be cut, and the velocity of t blood moving in it is shewn by the perpendicular heig to which it would rise in a tube inserted in the arte In short, there is nothing in the circulation of the flui in the animal economy, that can be understood witho an acquaintance with the laws of motion in fluids.

In the foregoing propositions, we have considered t cases of spouting fluids, only where the force that dri them out is the pressure of the fluid contained in t vessel, from whence they spout. A continued stream much greater force may also be produced by the el ticity of air compressed upon the surface of the flu as in fire-engines, constructed for the purposes of exti guishing fire in a city. The water is forced by a pist into a close vessel filled with air, until the air is su ciently condensed on its surface; and when the air is liberty to expand itself by its elastic spring, it will pr ject the water in a continued stream to the tops of t highest houses. The common fire-engines are co structed with two forcing pumps, each communicati with an air-barrel fixed between them, at which plac there are two valves opening inwards in the air-vess at the bottoms of the pump-barrels are also two valv opening upwards, as in the common pumps, near t surface of the water in the trough. The pistons of t pumps, without any valves in them, descend down these valves, and when they are raised, the water fo lows them by the pressure of the external air, to fill t vacuum made by the piston, while the spring of the a in the air-vessel shuts the valve that is in it, and pr vents the air from following the piston. When the pi ton descends, it shuts the valve in the pump-barrel an opens that in the air-vessel, so that the water that wa raised in the pump, is thereby forced into the air-barr and condenses the air on its surface. The pistons are fixed to opposite ends of a lever, worked by men, so that while one is ascending the other is descending, and the water is continually pressed into the air-barrel. Through the top of the air-vessel a pipe passes down to the water in it, and the water by the spring of the air on its surface, is forced up through this pipe, and by means of proper joints in it, is thrown in any direction, at pleasure.

There is also another fire-engine, so called from its being worked by fire, for the purpose of working a number of common pumps, to raise water out of mines, &c. The immediate cause of motion in this machine is the spring of elastic vapour, and the weight of the atmosphere, acting alternately. The machine is thus constructed. Over the mouth of a large copper boiler is fixed a large hollow cylinder, in which a piston plays, without any valve. This piston is fixed to one end of a large lever, while a number of pistons of common pumps is fixed at the other end of it. The piston over the boiler is raised by the force of the elastic steam rising from the boiler, as soon as its elasticity is more than sufficient to counterbalance the weight of the atmosphere above it, and the pistons of the pumps descend. But when the piston over the boiler rises to a sufficient height, it opens a cock to let into the cylinder a small jet of cold water, to convert the vapour into water again; and then the weight of the atmosphere presses the piston over the boiler down again, while the other end of the lever raises the pistons of the pumps.*

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* Many important improvements have of late years been made or steam-engine; particularly by Messrs. Boulton and

OF THE TIDES.

Another cause of motion in water is the attractive force of the sun and moon, producing that alternate swell and depression of the surface of the sea and rivers, which we call the tides of flood and of ebb, and which are next to be explained.

The tides principally depend on the attraction of the moon, though in some measure upon the attraction of the sun. The waters of the ocean being attracted by the moon, with a force, which is inversely as the squares of the distances from the moon, they must have their gravity towards the earth in some measure diminished on the side next to the moon; and as this diminution of gravity must be greatest on the side next to the moon, the waters must stand highest from the center of the earth on that side, and thereby make what is called a tide of flood, or high water: and at 90° distance all round this protuberance, the water will be lowest, and thereby constitute the tide of ebb, or low water, because there the attraction of the moon in some measure conspires, with the attraction of the earth, and thereby brings them nearer to the earth's center. On the opposite side

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Watts in England. Their principal improvement consists in condensing the steam in a vessel appropriated for that purpose, into which the steam passes, by means of a tube, from the principal cylinder. This condensing vessel is furnished with two small pumps; one for the purpose of extracting the air, and the other for that of extracting the water produced from the condensed steam. In some of their steam-engines, the steam acts only on the under side of the piston, the upper side being exposed to the counteraction of the atmosphere; in others, the steam acts, alternately, on both sides of the piston, the action of the atmosphere being entirely excluded by an air-tight stuffing round the piston-rod.—*Ed.*

of the earth, there is also a tide of flood, at the same time. As the whole globe of earth and water is attracted by the moon with different forces, decreasing as the squares of the distances increase, the water on the side of the earth opposite to the moon, will be less attracted than the center of the earth, on account of its greater distance, and therefore may be considered as left behind, to stand highest from the center; and this occasions another tide of flood opposite to the former. By this means, the waters, which surround the earth, will put on the figure of a prolate spheriod, whose greater axis points towards the moon; and as the earth revolves round her axis, so as to bring any place under the moon once in every 24¾ hours, it follows, that we must have two tides of flood and two of ebb in that space of time.

The greatest height of the water is not exactly when the moon is on the meridian of a place, but about three hours after; as the longer axis of the spheroidal figure of the water does not point exactly to the moon, but to the eastward of it. The reason is this: although the force of the moon to raise the water is greatest when she is in the zenith or nadir; yet her force begins to act before she comes to these points, and continues to act after she has passed them; and as long as this force continues, the water will continue to rise, which will be about three hours afterwards. And for the same reason, the tide of ebb, which is six hours after the flood, will happen when the moon has been set about three hours. As the moon comes to the meridian of any place, about three quarters of an hour later on any day, than on the preceding day, the tides must be so much later also.

The greatest swell of water following the path of the moon, when she is in the northern signs, the tide of flood, which comes with the moon to the meridian of any place in northern latitudes, will be greater than the tide which comes to the same place 12 hours afterwards: because the greatest altitude of this latter tide will be in south latitude, directly opposite to the moon. The contrary happens when she is in any of the southern signs, with respect to any places that are north of the equator; for the latter tide will then be the greatest. This is evident from a bare inspection of a globe.

If to the force of the moon we add that of the sun, the tides must be greatest at the conjunctions and oppositions of the sun and moon, as their attractions conspire at that time to raise the water, and these are called the spring tides; but when the sun and moon are in the quadratures, the tides are least, as their forces are then contrary to each other, the effect of the one being to raise, while that of the other depresses the water. These are called neap tides.

The spring tides are therefore produced by the sum of the attractive forces of the sun and moon, but the neap tides by the difference of their attractions. Were their forces equal, there would be no neap tides, because the sun would depress the waters, as much as they are raised by the action of the moon. But we find that the neap tides rise in the open ocean about 7 feet, while the spring tides rise about 11 feet: consequently, their forces to raise the tides are to one another as two numbers, whose sum is 11, and whose difference is 7; and these two numbers are 9 and 2; and as all the tides follow the moon, its power must be greater than that of the sun in the proportions of 9 to 2.

Although both the sun and moon raise the water by their attractive force, they will not produce four separate tides of flood in each day, but only two; for at the conjunctions and oppositions they would raise the water in the same place, and at the same time; and the same will be the effect of their contrary attractions at the quadratures, where the sun's attraction would make low water, while the moon's would make a tide of flood in the same place, and therefore the sun only prevents the flood from being as high as it would otherwise be by the single attraction of the moon. Nor will they produce separate tides at any intermediate distance between the syzygies and quadratures; for although they would by their separate forces raise tides in different places at the same time; yet their joint effect will be only to hasten or retard the time of high water, according as the sun or moon passes the meridian first, or by opposite meridians, in the following manner.

When the moon is passing from conjunction or opposition to the quadrature, she passes the meridian after the sun, and the time of high water will be sooner than three hours after the moon's passage over the meridian: but when she is passing from the quadrature to the conjunction or opposition she passes the meridian before the sun, and the time of high water will be retarded, and be more than three hours after her passing the meridian. Their joint effect will produce one common tide between the places, where each of them would produce high water by their separate forces, which occasions the acceleration and retardation of the times of high water, bringing on the tides of flood sometimes sooner, and sometimes later, than three hours after the moon's passage over the meridian. The greatest tides are not at the time of the conjunction or opposition, but about three tides afterwards. Although the sun and moon act with the greatest force at the syzygies, and therefore should raise the highest tides at these times; yet we must remember that any motion communicated to water will continue for some time after the moving force ceases to act; as the waves of the sea raised by a storm continue after the storm is over. And if at the sizygy both should cease to act, the tide would not then instantly cease, but the water, subsiding by its own gravity, would acquire such a velocity as would carry it below its proper level, and thereby cause it to rise again above it; and the force of the sun and moon, in their subsequent returns to the meridian, conspiring with this tendency of the water to rise, will occasion a greater swell in the second or third tide after the syzygy. Hence, the greatest tide will be some time after the conjunction and opposition; but it cannot be long after, because the moon passing towards the quadratures begins to counteract the force of the sun, and thereby to lessen the tides.

Two very great spring tides never follow each other at the conjunction and subsequent opposition. For should the moon be in her perigee or nearest distance to the earth at any conjunction with the sun, when her force to raise the water is greatest, at her next opposition she will be at her apogee, where her influence is least; and therefore the spring tide at that opposition will not be so high as at the preceding conjunction.

In winter, the spring tides are greater, and the neap tides are less, than in summer. Because the sun is nearer to the earth in winter than in summer, and his influence to raise the tides is greater in proportion to his nearness to the earth. Now, as the spring tides are produced by the joint influence of the sun and moon, they must be greatest, when the force of the sun is greatest, the moon's force being supposed to be the same. And, as the neap tides are produced by the difference of their orces, they must be less, as the force of the sun inreases.

The force of the sun or moon to raise a tide is greatst, when they are in the equator, and decreases graually as they approach towards the poles, where neiher of them could produce any tide at all.

The spring tides are the greatest and the neap tides re the least about the equinoxes. At the equinoxes, the un is in the equator, where his force is greatest, and he moon either in it or not far from it, as she is never nore than about 5⅓ degrees from the ecliptic; therefore he spring tides are then greatest. But the neap tides eing produced by the difference of their forces at the uadratures, when the sun is in the equator the moon vill be at or near to one of the tropics, where her inluence is least. Therefore the neap tides at this time of he year ought to be least, especially when we reflect hat the moon has much the greatest influence in proucing the tides.

The highest spring tides and the lowest neap tides re not precisely at the equinoxes, but sometime before he vernal equinox, and sometime after the autumnal quinox. They ought to be highest about the winter olstice; as the sun's approach towards the earth makes his force to raise the tides greater at that time than at ny other, so far as it depends upon his distance from he earth. But they also ought to be highest, when both uminaries are in or near to the equator, at the time of he equinoxes. But as they cannot be highest at both hese times, they must attain to their greatest altitude t some intermediate time between the winter solstice nd the equinox; that is, before the vernal, and after the utumnal equinox. And for the same reason, the neap ides, which accompany these tides of flood, will also be lowest about these times; because they are produced by the difference of force in the sun and moon; and as the sun's force, which is the lesser of the two, increases at these times, both on account of his nearness to the earth and his being in the equator, the difference between their forces will then be least, and so produce the least neap tides.

The tide of flood, which is raised under the sun or moon, will be something greater than that which is raised at the same time on the opposite side of the earth, on account of the greater attraction of the water nearest to these luminaries.

This is the general theory of the tides in the open ocean; but the situation of lands and rivers will make very considerable alterations, both in the times and altitudes of the tides at particular places. As when the tide has to pass along a large river, which receives a great quantity of water at its mouth, but afterwards grows narrow and confined towards its source; it will rise to a greater altitude than in the open sea, and the time of high water will be later; as in the bay of Funday and the Bristol channel, where the tide is said to rise 70 feet, and the time of high water will be later as the length of the river or bay is greater.

Should two tides come from different seas, to any port, at the distance of six hours after one another; when it should be high water by one, it will be low by the other. There will be therefore no rising or sinking of the surface of the water, because one tide will supply the water as fast as the other carries it off. But as there will be more water brought, when the moon is above the horizon, than when she is under it, there will be a difference of altitude in the water every twelve hours, which will make one tide of flood and one of ebb, every twenty-four hours. It is upon the same principles that we account for the tides and half tides, as they are called, in different places, where a tide from one part of the sea is brought to any place, when the tide from another part is half spent, arising from the distance from which the water must come to raise a small tide in the middle of the ebb.

MOTION OF ELASTIC FLUIDS.

WHEN water is converted into steam by heat it becomes extremely elastic, and if confined will exert a prodigious force, as is evident from the eolopile. This is a small vessel made with an incurvated neck, that it may direct the steam, issuing from it through a narrow orifice, to the fire which converts the water it contains into vapour, and blows it like a pair of bellows. In using this instrument, great care must be taken to keep the orifice open, for should it happen to be closed, the vessel would burst with a loud explosion, and possibly prove fatal.

We have already taken notice of the advantage that may be derived from this elastic fluid, in the working of fire or steam-engines, for raising water from mines, &c. and future experience may derive many other advantages from it.

Air is also an elastic fluid much more beneficial to mankind, whose motion exhibits various phenomena, the principal of which we must now explain.

Upon the elasticity of the air depends the operation of the air-pump, by which it may be almost totally extracted from any vessel. Its construction is on the same principles with those of a common pump, which we have already explained.

If the barrel of the air pump communicate with a close vessel, when the piston is raised up from the lower valve, the elastic force of the air in the vessel will open this valve, and part escaping through it will follow the piston and fill the barrel of the pump; and as it occupies a much larger space than before, it must be considerably rarefied. As the piston descends, it condenses the air in the barrel, and thereby shuts the lower valve, which prevents its return to the vessel, and when it becomes more dense than the external air, it will open the valve of the piston and escape through it, and mix with the common air. Thus by the working of the pump any assigned degree of rarefaction may be produced in the air in the vessel, which is commonly called the *receiver*. But the whole of the air in the receiver can never be extracted, so as to constitute an absolute vacuum; because the expulsion of the air, at every stroke of the pump, is effected by the spring or elasticity of what remains in the receiver.

If the valves of the air pump be made to open the contrary way, it becomes the *condenser*, by which an additional quantity of air may be pressed into any vessel, until the increased spring of the condensed air be at last sufficient to burst the vessel. In this instrument, the ascent of the piston closes the lower valve, and prevents the air in the vessel from following the piston, while the weight of the atmosphere opens the valve of the piston, and presses the air through it into the barrel of the condenser, which by the descent of the piston is pressed into the vessel.

If air be sufficiently condensed by such an instrument in a tight vessel, which may communicate at pleasure with the barrel of a gun, by means of a trigger, it will, by its elastic force, when at liberty to expand, project a ball with great force, and answer the purpose of powder. This is called the air-gun.

WIND.

WIND is a stream of air; or air in motion. The natural state of this fluid, as well as of all others, is rest; which it endeavours to keep, or to retrieve, when lost, by a universal equilibrium. When this natural equilibrium is lost, by any cause whatever, there must of consequence be a motion of all the circumjacent air towards the point where it is destroyed, to restore it again. And this motion is what we mean by wind.

Various are the causes of particular winds: but the most common and principal causes are heat and cold; the first, by rarefying the air and making it lighter; and the last, by condensing and making it heavier. When the atmosphere is more rarefied by heat in one place than in another, the heated air will ascend, and the colder and more condensed air will rush in from every other quarter. When the air in a room is heated, the external air, which is cooler, will force itself into the room through any aperture, and thereby produce a stream.* The pressure on the outside will be greater than on the inside; and the air, being a fluid, will move in the direction of the greatest pressure, until the equilibrium be restored. Therefore, when any part of the atmosphere is more rarefied than another, the denser air will rush in to that part, as it will press more in that direction than in any other, and thereby produce a wind. The current of air through a broken pane of glass, into a heated room, is a sufficient proof of this position. It may, however, be rendered still more evident by an easy experiment. If a water-plate be filled with warm water, and placed in a tub of cold water; and if a lighted candle be blown out, and the smoking wick be held over the cold water, the smoke will move towards the heated air over the plate and ascend there. But you will see a contrary motion of the smoke, if the experiment be reversed, by filling the plate with cold water, and placing it in a tub of warm water.

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* When the air is heated in a room without a chimney, its elasticity being increased, will produce a current outwards, through any aperture or crevice; but when heated in a room having a chimney, the heated air will ascend, and its place will be supplied from the external air.—*Ed.*

The whole torrid zone, which bounds the apparent annual course of the sun, will have the air in it considerably rarefied at different seasons of the year, according as the sun is north or south of the equator, by receiving a greater quantity of his direct rays, than any other part of the atmosphere. The point of greatest rarefaction will not be directly under the sun, but about three hours to the eastward of him, and in his diurnal path; because every body requires some time to be heated, and does not instantaneously receive the greatest degree of heat it is capable of sustaining. This point of greatest rarefaction following the sun in his daily motion, will continually shift its place towards the westward, heating the western air, and leaving the eastern air to cool. To this point, wherever it is, the colder air will continually rush in, to restore the equilibrium of the atmosphere, from the north and south, the east and west. But the current from the south will be overborne by the current from the north, while the sun is north of the equator; as the air is more heated between the tropics and south of the point of greatest rarefaction, than on the north side of the tropics. The contrary happens, when the sun is on the south side of the equator, and for the same reason; the current of air from the south prevailing over that from the north; so that from the vernal to the autumnal equinox there will be a north wind, and from the autumnal to the vernal, there will be a south wind towards the point of greatest rarefaction. And as this point follows the apparent motion of the sun from east to west, while the sun gradually heats the western air and suffers the eastern air to cool, it follows, that the eastern current will prevail over the western, and thereby generate an east wind. But the north and east winds, when compounded together, will produce a northeast wind, while the sun is on the north side of the equator, which will blow for six months. In like manner, the currents of air from the south and east, when compounded together, while the sun is south of the equator, will generate a southeast wind for the other six months of the year. These are called trade winds, from their great utility in trade and navigation.

This theory we find verified in the Atlantic, Pacific, and Ethiopic oceans. In these seas, we find a wind blowing either from the northeast or southeast through the year, which extends to about 30 degrees on each side of the equator. We cannot expect, however, that these tropical winds will be always regular all over the world, unless the whole surface of the globe were uniformly covered with water; so that there might not be any extraordinary rarefaction produced in any place by any other cause than the direct heat of the sun. The land is susceptible of a greater degree of heat than the water, and will be generally more heated, as it constantly presents the same surface to the rays of the sun, while the heated surface of the water is, from its continual motion, constantly changing. As the sun, therefore passes over islands and continents, the atmosphere over them will be more heated, than that over the open ocean, and consequently the trade winds will vary their direction, and set in upon the land; as we find on the coast of Guinea, where the trade wind blows from the westward for a considerable distance in the Atlantic ocean, and sets in upon the coast. And, indeed, in every tropical island, we find the sea breezes setting in upon the land in all directions, as soon as the morning sun has produced a greater rarefaction in the air over the land, than over the water. And, in general, the winds are more variable in all places, and at all times, near the shores, than in the open seas, on the same account.

As some variations in the direction of the wind may arise from the land in general, so the particular nature of the soil may make these variations more considerable.

A flat sandy country is generally more heated by the sun than any other, and therefore will occasion a stronger current of air towards it.

Great ranges of mountains, in any country, may reflect the wind and alter its direction; and the beds of large and extensive rivers open a channel in which the wind will generally blow: as we find the river St. Laurence, which runs upwards of two thousand miles from its source to the sea, and filling many large and extensive lakes in its course, opens an extensive channel for the wind in an eastwardly direction; while the Mississippi, rising from the same country, flows southward through a no less extent of land, and thereby opens a southern channel for the wind; and from these two directions compounded together, we account for the general northwest winds, which prevail in North America.

When the snows cover the tops of mountains, for a considerable time before they begin to melt, the air will be greatly condensed, and afterwards it will be rarefied, and thereby the wind may blow in one direction while the snow continues, and in a contrary direction when it melts.

From these, and a thousand other causes, which cannot be particularly enumerated, the air must be subject to various perturbations, and have a tendency to motion in different directions, depending on the situation of countries, the degrees of heat and cold in the climate, the nature of the soil, the degree of cultivation, together with many other causes producing fermentations, eruptions of volcanos, the extrication of fixed and subterraneous air, and the like; by which means we must be subject to variable winds almost perpetually. Yet near the equator, where the cause which produces the general trade winds operates with the greatest force, these variations will be less frequent and more regular, especially in the open sea.

In the Atlantic ocean, there is more land on the north side of the equator, than on the south, especially on the African side: and the air over it will therefore be more rarefied than at the equator, and consequently the limit between the northeast and southeast winds will not be the equator, but some parallel to the northward of it: and we find in fact that the limit between them is about four degrees north of the equator.

The place where the eastwardly and westwardly winds commence in the Atlantic ocean, near the coast of Guinea, is called by the seamen, the calms, rains, and tornados. The reason is, because the atmosphere in that region is divided between the two currents, which blow from [illegible] upon the African coast, and the other to [illegible] ing the trade wind;

and being in some measure carried off by them, it becomes too light to bear the clouds, which are brought from every quarter by the air, rushing in, to restore the equilibrium of the atmosphere; so that they descend there in perpetual rains, with the most terrible thunder and lightning. The air about that place being acted upon, by these contrary forces, in all directions, will therefore be in equilibrio, and will not move in any direction, so that there will be little or no wind in that place, which is for that reason denominated the calms.

If the air be carried off from this place by opposite currents, it may be asked, how fresh air is supplied? But the answer is easy,—from opposite currents above the place which bring the clouds and produce the perpetual rains.

By the evacuation of the air, its pressure there will be diminished, and consequently the neighbouring air will rush in, from all quarters above, which will form a kind of eddy where the opposite currents meet, and thereby occasion those violent storms, which are called *tornados.*

On the American side of the Atlantic ocean, the trade winds will become more and more easterly as they approach the American continent. Because, as they are compounded of a north and east wind, or a south and east wind, in different seasons of the year; instead of blowing from the northeast or southeast, they will verge towards the east; the eastern current will be stronger than either the north or south current; because the land of America will be more heated than the water, and the eastern current will gradually prevail over that from the north or south.

In the Atlantic and Ethiopic seas, the trade wind will be more easterly in north latitudes, and more southerly in south latitudes, when the sun is near to the tropic of cancer; and when he is in the tropic of capricorn, the trade winds in north latitudes will be more northerly, and in south latitudes more easterly than usual. When the sun is near the tropic of cancer, the air under it will be as much rarefied as about the equator, which will diminish the force of the current from the north, and strengthen the current from the south, towards the point of greatest rarefaction; by which means the easterly current will prevail in north latitudes, and the southerly current will prevail in south latitudes; so that on the north of the equator, the trade winds will blow more easterly than usual, and in south latitudes, more southerly, as being in a direction nearer to the stronger current. But when the sun is near the tropic of capricorn, similar changes must take place. For as the air is considerably rarefied from the equator to the tropic, there will be scarcely any southern wind in southern latitudes, and consequently an easterly wind will prevail in these latitudes, while the northern current in north latitudes will, from the coldness of the air on the north side of the equator, prevail over the eastern, and cause the trade winds, on that side, to be more northerly than what they would otherwise be.

In the Indian ocean, from Madagascar to New Holland, between the latitudes of ten and thirty degrees south, there are the same trade winds that prevail in the same latitudes in the Atlantic and Pacific oceans. And besides these, we have periodical winds called *monsoons*, which blow for six months in one direction, and in the other six months, from the opposite point of the compass. In the northern parts of the Indian ocean the monsoon is northeast from October till April, and from April till October it is southwest. The first is the regular trade wind accounted for already. And the southwest wind, in the summer months, is occasioned in the same manner, as the same wind is produced on the coast of Guinea, by the great rarefaction of the air on land.

The air over the continent of India and China being greatly heated, ascends, and the colder air from the southwest rushes in to restore the equilibrium. There is, indeed, this difference, that the land on the coast of Guinea lies under the equator, and therefore remains heated through the whole year, so that the southwest wind blows there continually. But the land of India and China lies north of the equator, and there is no continent to the south, so that the southwest wind will blow in the Indian ocean only for half of the year, or while the sun is over the land. The position of the islands in this ocean, with respect to each other, may occasion either one or other of the currents, of which these monsoons are compounded, to be stronger than the other, and consequently cause the general direction of the wind to vary a little from the regular course. As we find to the east of Sumatra and Java, the northeast wind in winter is more northerly, and the southwest wind in summer is more southerly, than in the other monsoons to the west.

In the southern parts of the Indian ocean, from three to ten degrees south, the regular south trade wind blows from the end of May to the beginning of December, and a northwest wind blows for the other six months; which is probably occasioned by the position of New Holland or Zealand, over which the air being rarefied, may cause a current from the northwest to set in towards it.

These are the general causes and affections of the trade winds and monsoons, which we find to be most regular in the great Pacific ocean; however, none of them are without exception, arising from different circumstances of heat and cold, land and water, situation and direction.

The velocity of wind is very variable, from one to fifty or sixty miles per hour. Various machines have been invented to determine this velocity; but without any of them, it may be found with sufficient precision, by observing how far a light feather is carried by it in a second. From many experiments it is found that even in violent storms and hurricanes, it seldom moves with greater velocity than sixty miles per hour.

If a current of air strike against any obstacle, such as the sail of a ship or of a mill, the force of the stroke will be proportional to the area of the obstacle and the velocity of the wind. And as the velocity of the wind may be measured by the space that a heavy body must fall to acquire that velocity, the force of the stroke will be proportional to the weight of a column of air whose base is the area of the obstacle, and whose altitude is the distance from which a heavy body falls to acquire the velocity with which the wind blows. This is also true of water in motion.

When the area of the sail is given, the force will be proportional to the square of the velocity. The force is as the quantity of matter and velocity conjointly; but the quantity of matter is as the velocity; for the quicker or slower that the wind moves, the more or fewer particles strike against a given surface in a given time. Therefore, the force will be as the square of the velocity. This is true of any fluid whether compressible or incompressible; and so is the last proposition.

Hence, it will be [illegible] icult to swim or row against a stream, that flows with double the velocity of another; and a wind that blows with double the velocity of another will give four times the momentum to a given sail, to move the ship. Now, if the obliquity of the sail to the wind be given, its perpendicular area is also given, and the force of the wind to move the sail will therefore be as the square of the velocity.

The momentum of a current of wind to move a sail, when it blows with a given velocity, will always be as the square of the cosine of the angle of incidence, or the angle between the direction of the wind and the perpendicular to the surface of the sail.

The momentum of a stroke from a given body will always be in proportion to the cosine of the angle of incidence, as we have seen in mechanics. But the momentum of a stroke is also proportional to the quantity of matter in the striking body: and the quantity of matter in a stream of wind against a given sail will increase as the cosine of the angle of incidence increases; therefore, the momentum will be as the cosine of the angle of incidence, on a double account; first, because an oblique stroke is to a direct one in this proportion, and secondly, because the quantity of air, which strikes the oblique sail, is to the quantity, which strikes a direct one, in the same proportion. Consequently, the momentum of the wind, to move a given sail, will be as the square of the cosine of the angle of incidence, when the wind blows with a given velocity.

OF SOUNDS.

It is the business of metaphysicians to contemplate the idea of sound in the mind, and of anatomists to describe the construction of the ear, and the manner of conveying the impressions made upon it, by the motion of air, to the brain; by which means we acquire the idea of sound. Our business is only to inquire what motions in the bodies around us, and particularly in the air, are necessary to produce in us the idea of sound.

From the experiment of the bell's being very faintly heard, or not heard at all, under the exhausted receiver, we conclude, that sound has a necessary dependence upon air. The tremors of a sounding body are impressed upon the contiguous air, and are communicated by the elasticity of the air to the tympanum of the ear, and by it to the air included in the internal cavities, until the auditory nerve receive the impression, and thereby excite in us the idea of *sound*.

It is out of my line to dwell upon the structure of the ear, and its admirable contrivance for collecting even the languid vibrations of the air, and for propagating them with force and inconceivable distinctness to the common sensory in the brain.

This is only to be effected by an anatomical lecture, where you have an opportunity of examining every part of this complicated organ, without which no description can give you any tolerable idea of its mechanism and uses. By attending to a single lecture on this subject, you will see how the pulses of air, entering the meatus auditorius, are condensed by various reflections, in their passage to the membrana tympani, which, being rendered more or less concave, lax or tense, by the action of the malleus and its proper muscle, can thereby compress or rarefy the air in the labyrinth, according as the eustachian tube is open or shut. This membrane appears to be contrived by a curious apparatus of parts to be relaxed or stretched in various degrees, so as to be adapted to receive the tremors from a sounding body, of every degree of intenseness, and to communicate them through the included air to the nerves that are expanded over the surface of the cochlea. For the fibers of the septum transversale are evidently contrived like the strings of a harpsichord, of various decreasing lengths, and possibly of different tensions and thickness; so that some of them may be in concord with the sounding body, and tremble with the same vibrations: by which means, every distinct sound is communicated to the brain without confusion, and the mind perceives the infinite distinctions of harmonious and discordant tones.

From this construction of the ear, we see the reason why deafness ensues upon a rupture of the drum of the ear, or the obstruction of the eustachian tube: why the ear is affected with acute pain, in descending into the sea in the diving bell, when the external air is more condensed on the outside of the drum of the ear than on the inside, whereby it is immoderately stretched: and why a person, listening attentively, insensibly opens his mouth; as there is a communication by the mouth with the auditory nerve.

The particles of a sounding body, when put in motion by percussion, vibrate backwards and forwards, through very small spaces, by their elastic force. This is evident to sense in the strings of a violin, and in the motion of a bell. The eye may see and the hand may feel the trembling of both. When an elastic string is bent out of its right-lined direction by any force, it will return again by its elasticity, and acquire such a velocity, when it becomes straight, as will carry it to as great a distance on the other side, if its motion be not impeded by any external cause, such as friction, or the resistance of the medium in which it moves. In this situation, the elasticity of the string counterbalances its velocity, and brings it back again. And thus it vibrates, backwards and forwards, through small spaces, until its motion be destroyed by friction and the resistance of the medium. In like manner, when the circular edge of the bell is struck with a hammer, the particles struck move forward, while those at 90 degrees distance move outward, and the circular figure of the bell is changed into an ellipsis, having the shortest diameter in the direction of the stroke; but, in the return of the particles by their elasticity, their acquired velocity will carry them beyond the circle, and the longer axis of the ellipsis will now be in the direction of the stroke. Thus will the diameters of the ellipsis change directions in every vibration, till the motion be destroyed.

The same stroke, which makes the string or bell vibrate, makes it sound also, and as the vibrations decay, the sound grows weaker; and the air must convey these vibrations to the ear. Were there no air in contact with the sounding body, no sound could be heard. As the air is exhausted by the air-pump, the sound of the bell under the receiver grows weaker; and, could all the air be extracted, it could not be heard at all. On the other hand, if condensed air were in contact with the sounding body, the intensity of the sound would be greatly increased. And hence, persons in the diving-bell, seem to each other to speak much louder than usual, as they sink deeper in the water, or as the air in the bell becomes more condensed.

When an elastic body vibrates, the air which lies before it, or in contact with it, is propelled or driven forward, and by that means it will be condensed. When this condensed air expands itself by its elasticity, it will expand in all directions; not only returning with the elastic body, but also moving forwards, and condensing the air, which lies beyond it. While this condensed air will produce the same effect, condensing the air that lies still farther forwards. And thus the motion communicated to the air, by the sounding body, will be continually carried forwards, until it reach the ear.

These condensations of the air, which are produced by the sounding body, and propagated, through it, to the ear, are called *pulses*. And to make the theory of sound as plain as possible, we must endeavour to trace it to its first principles, in the motion of an elastic string.

1. An elastic string, when bent out of its right-lined direction by any force, endeavours to return with a force proportional to the *distance* over which it must move to attain its right-lined direction again, which is called the *sagitta*.

It is found, by experiment, that if a string be bent by a certain weight to a certain distance, it will be bent to double that distance, by double that weight, and to triple that distance by three times that weight. Now these weights must be an exact counterbalance to the elasticity of the string, by which it endeavours to return to its proper place. Therefore, the elasticity of the string, or its tendency to return, being proportional to the inflecting weights, and these to the distances or sagittæ, must be proportional to the sagittæ.

Hence it will follow, that the motion of the string backwards will be accelerated, as it receives an impulse from its elasticity every instant; but, as this impulse is not equal in every instant, but decreases as the string approaches to a right-lined direction, the motion will not be equably or uniformly accelerated, but accelerated with a force proportionable to the sagittæ. The same reasoning may be applied to the retardation of the string's ity after it has passed its right-lined direction. So hese strings are accelerated and retarded by the laws as a pendulum vibrating in a cycloid.

All the vibrations of the same strings are perform- the same times.

the last proposition, the elasticity of the string, accelerating force by which it returns to its rec- l direction, is proportional to the distance that ring has to pass over, and therefore the velocity ated by this force must also be proportional to the distance. But, if the velocity be proportional to istance, the time must be given. Therefore, the of the vibrations of the same string must be all , when the space over which it passes, is propor- to the velocity. For, if it move with double or velocity, when it has to move over double or the distance, all the times of vibrations must be

nce, the motion of an elastic string is like the mo- f a pendulum moving in a cycloid; where the ve- is always proportional to the distance the body o acquire the velocity; and where all the vibra- are performed in the same time.

e times of vibrations in different strings are as the ters and lengths of the strings directly, and as the roots of the tending forces inversely.

: T=the time of vibration, D=the diameter of the , L=its length, and F=the tending force or weight ich it is stretched. Then $T=\frac{DL}{\sqrt{F}}$.

ACB* be a string stretched by the force F, while drawn into the direction ADF, by the inflecting E, which is equal to the elasticity of the string and a counterbalance to it. Now, as the tending force acts in the direction BD, it may be represented by BD, which may be resolved into two, viz. BC and CD, whereof CD represents the restituent force, which is equal to the string's elasticity$=E$, and the other acts in the horizontal direction, and may be considered as equal to BD, because of the smallness of DC, or equal to the tending force F. Therefore $F : E :: BC : CE :: L : S$; putting $S=$the space over which the string passes in half a vibration. Hence, $E=\frac{FS}{L}$. Therefore in the same string, where the tending force and length are given, the elasticity is proportional to the sagittæ$=S$, as we have seen before.

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* See Plate 4, Fig. 4.

But since the sagitta or space passed over is equal to the time and velocity of the vibration, $E=\frac{FTV}{L}$. And as the elastic force may be considered as uniform for the small time it acts, the momentum or quantity of motion generated will be $ET=\frac{FVT^2}{L}$. But this momentum will be as the velocity and quantity of matter that is $V\times LD^2$. Therefore, $\frac{FVT^2}{L}=VLD^2$, or $FT^2= L^2D^2$ and $T=\frac{LD}{\surd F}$. Q.E.D.

Hence, if D and F be given, or if there be two strings of the same diameter and tended with equal weights, but of different lengths, the times of vibration will be directly as their lengths. Thus, half the monochord vibrates in half of the time, in which the whole chord vibrates; two thirds of it, in two thirds of it; and three fourths of it, in three fourths of the time: the first is called an octave, where the coincidence is at every second vibration of the shorter; the second is called a fifth, where the coincidence is at every third vibration of the shorter; and the third is called a fourth, where the coincidence is at every fourth vibration of the shorter, &c.

If one of these strings, both of equal tension and diameter, but of different lengths, be struck, the other will vibrate with it in aliquot parts, the points of division between the parts remaining quiescent; as is evident to the sight, by pieces of paper remaining at rest on these points, while similar pieces laid on any other points will be shaken off by the vibrations. If one string be twice as long as the other, it will vibrate by halves, the middle-point remaining quiescent; and if three times as long, there will be two points at rest, while the string is vibrating in the three different lengths of it. The reason is, because the same vibrations of the shorter string are communicated to the contiguous air, and by the air to the quiescent string, which, being of equal tension and diameter, cannot vibrate in unison with the shorter, or with the contiguous air, in any other manner, than by vibrating in lengths, each being equal to the length of the shorter string. And hence, we see the reason of the construction of the double-stringed violin, whose untouched strings vibrate in unison with the others, and thereby greatly heighten the melody and harmony of the music.

Hence, also, if F and L be given, T is proportional to D; that is, if the tending force and lengths of two strings be the same, but their diameters different, the times of their vibrations will be as their diameters. If the diameters be as 2 : 1, the string, whose diameter is greatest, will vibrate slowest in the same proportion, and their coincidence will be at every second vibration of the least; and they will therefore sound an octave.

Lastly, if D and L be given, T will be as $\sqrt{F}$; that is, if the diameters and lengths of two strings be the same, the times of their vibrations will be as the square roots of the tending forces, inversely.

Now, as the tone of a string depends entirely on the time of the vibration, it is easy to understand, that whatever the sounding body be, or how many soever there may be of them, if they perform the same number of vibrations in the same time, they will all sound the same note and be in unison. And if the vibrations be performed in unequal times, the coincidence will be after certain intervals, and the shorter that these intervals are, so much the more agreeable is the consonance to the ear. Hence, when one string vibrates twice, while the other vibrates once, the coincidence will be most frequent, and therefore it is called the most perfect concord, as it is most agreeable to the ear. When the times of their vibrations are as two to three, the coincidence will be at every third of the quickest, and therefore, this, which is called a fifth, is in the next degree of perfection.

There are but seven whole notes or tones in the scale; for when you come to the eighth, it is no other than the octave to the first, and the octave above and below is only a replication of the same sound. And a skilful artist, in the construction of musical instruments, will compound the various proportions of length, diameter, and tension of strings, in such a manner as to produce the most agreeable consonance to the ear.

If, instead of strings of the same diameter and tension, but different in lengths, one string be taken and stopped at different places while it is vibrating, it will give different sounds, according to the length of the string that is suffered to vibrate without interruption. The whole length of the string, whatever that may be, is called the monochord, and the sound produced by it is called the key-note. If half the string be suffered to vibrate, the sound produced is called the octave to the first note or sound; because musicians have particularly six different sounds distinguished between them, which are excited by stopping the string at certain places between these two points, while the remainder is suffered to vibrate. The proportion of length for making these sounds will be assigned presently.

The highest sound of the octave is expressed by half the line, and if this half be divided again in the same manner, a higher octave is produced by half of it, or the one-fourth of the original line; and so on, as far as we please. So that if the divisions of the first octave be once ascertained for each particular note, we may take one-half, one-quarter, one-eighth, &c. of their lengths, for the divisions of the next and succeeding octaves.

The fifth of the first or key-note, is found by taking the $\frac{2}{3}$ of the whole string, and if this note be considered as the key-note, its fifth may be found in like manner, by taking two-thirds of its length, and so on, for any succession of fifths as far as you please.

Now, if you carry on the succession of octaves as far as seven, and of fifths, as far as the twelfth, they will be found nearly to coincide; but if the succession be carried farther, they will diverge farther, so that no octave agrees more nearly with a fifth, than the seventh with the twelfth fifth; and because an imperfection in the octaves is intolerable to the ear, but an imperfection in the others does not produce so disagreeable an effect, we must make use of the twelfth fifth for the seventh octave, and divide the imperfection that would thereby be introduced into the twelfth fifth, among all the rest, which will be thereby rendered inconsiderable and almost insensible.

And if we find all the octaves of these twelve divisions of the string, we shall have twelve distinct notes in half the string, or in the first octave, to which, if we add the sound of the whole string, we shall have thirteen distinct sounds in the octave; all derived from this succession of fifths and their octaves; and this shows the reason, why no more notes are introduced into the octave.

If the whole length of the string be supposed to be 100,000, the succession of octaves and fifths will be

octaves of 100,000		fifths of 100,000	
1st,	50,000	1st,	66,666
	25,000		44,444
	12,500		29,629.6
	6,250		18,753
	3,125		12,402
	1,562.5		8,268
7th,	781.25		5,512
			3,674.6
			2,450
			1,633.3
			1,088.8
		12th,	725.866

The eight notes or sounds used in the common scale of music, are determined by the lengths of the string which gives these sounds. The whole length of the string, when vibrating, gives the first note or sound, which musicians generally call the key or first note, or C; $\frac{8}{9}$ of the length sounds the second note called D, but between this and the first they have interposed another sound, made by $\frac{15}{16}$ of the string, which they call the second minor. The third note, generally called the third major, and marked E is sounded with $\frac{4}{5}$ of the string; but between this and the second they have interposed another semitone called the third minor, which is sounded with $\frac{5}{6}$ of the string. The fourth note called F s sounded with $\frac{3}{4}$ of the string. The fifth note called G s sounded with $\frac{2}{3}$ of the string, but between the fourth and fifth they have interposed a semitone called the fourth major, which is sounded by $\frac{32}{45}$ of the string. The sixth note, called A, or the sixth major, is sounded with $\frac{3}{5}$ of the string, but between this and the fifth they have nterposed a semitone called the sixth minor, which is sounded with $\frac{5}{8}$ of the string. The seventh note, called B, or seventh major, is sounded with $\frac{8}{15}$ of the string, but between this and the sixth they have interposed another semitone called the seventh minor, which is sounded with $\frac{5}{9}$ of the string. The eighth is the octave, which is sounded with half the length of the string; and s the first note of the succeeding octave or seven notes; for the sounding of which seven notes, the remaining half-part of the string is to be divided in the same ratio, as the whole string was divided for sounding the first seven whole notes. Consequently, as the notes of the second octave have the same ratio as the notes of the first, among themselves, they will be sounded with half he length of the string that was necessary to sound the correspondent notes of the preceding octave: and so on, d infinitum. Hence, when the divisions of the string are obtained for sounding the first seven notes of the first octave, the divisions of the remaining part of the tring for sounding the notes of the next octave, are easily found by taking one half of the foregoing lengths for the corresponding notes of that octave, and so on, for as many octaves as you will.

Now these proportions of the lengths of similar strings for sounding these different notes, were not assumed in an arbitrary manner, but from the constant experience of the agreeable or disagreeable sensations raised by hearing two of these notes sounded together. The most agreeable sensation is, when two strings are sounded together, whose lengths are as 2 to 1: the next is when they are as 3 to 2; after these come the proportions of 3 to 4, 4 to 5, 3 to 5, 5 to 6, and 5 to 8; which are all called concords, from the agreeable sensations excited by them. The other proportions are disagreeable in a greater or less degree, and are therefore called discords, unless they be the double, quadruple, octuple, of the above, viz. their octaves, or double octaves, &c.

It is necessary next to show, why only seven principal notes are admitted in the octave, together with five other half notes, usually called flats and sharps. Next after the octave, the fifth is the principal concord, and gives the greatest pleasure, and all the other notes of the octave are derived from a series of successive fifths.

For let the first note of the octave be denominated by any of the first seven letters of the alphabet, as for instance by C, then its fifth is G, being sounded by $\frac{2}{3}$ of the string; the fifth of this is D, being sounded by $\frac{2}{3}$ of the length, which sounds G, and the octave of this note D, is found in the first octave and called by the same name, although it be in reality found in the second octave. Now, the fifth of this note, viz. D, is A, whose octave is found in the first octave; and the fifth of this note is E, whose double octave is found also in the first octave; and the fifth of E is B, whose octave is also found in the first octave; and the fifth of B is F, whose octave is also found in like manner in the first octave, and is the note of the same name. And finally, the fifth of this note is C, which is an octave or double octave of the original key-note, assumed above. So that the successive fifths of any key-note, will stand in this order, C, G, D, A, E, B, F, and C, which are the seven notes of the octave. So that the seven notes, sounded by the abovementioned proportions of the string, have their octaves or double octaves, &c. found in the first half or octave of the string; and the last, coinciding with the first, when only eight are used, shows that the first and eighth are only octaves of each other, and that there are only seven whole notes or tones, together with the five semitones interposed between them.

In whatever key a piece of music is performed, its fifth note is the most predominant of its concords; and as the notes must be so ordered that for the sake of modulation any note may be taken for the first or key-note, its fifth is found by taking two thirds of its length; and again, two thirds of that length for its fifth, and so on, until we find a fifth whose length nearly coincides with some octave; which is found in the seventh octave and the twelfth fifth, so that the seventh octave will answer for the twelfth fifth. As the number of fifths in that series is twelve, by finding all the octaves between them, we shall have twelve distinct notes within half the string, to which, if we add the sound of the whole length of the string, we shall have thirteen distinct sounds in the octave, viz. seven principal notes, and five semitones, with the octave; which shows that there can be neither more nor less than seven whole notes in the octave.

Now, as the whole string and the twelve successive fifths, each being $\frac{2}{3}$ of the preceding, form a geometrical series of thirteen numbers, whose common ratio is $\frac{2}{3}$, and the extremes any number and its half, as the twelfth fifth must be taken equal to the seventh octave, for an imperfect octave is intolerable, while a small degree of imperfection in the fifths may be borne; the imperfection of the last fifth may be easily divided among them all, so as to produce an equal temperament, let which note soever be taken as the key. This is easily done, by finding a ratio of the series, by which the eleven mean proportionals may be interposed between the two extremes; this will temper all the fifths, and consequently all the notes of the octave, equally. Now, as the least extreme is to the greatest, or as one is to two, so is unity to the root of the ratio, less by one than the number of terms; that is, to $2^{\frac{1}{12}}=1.05947$. Now, if the length of the string be divided by this ratio, the quotient will be the length of the second tempered note, which again divided by the ratio gives the third, and so on, through the octave: or, multiply the ratio by half the string and you have the seventh, and so on, backwards.

Thus, if the length of the whole string be supposed to be 100,000, the length of the notes will be

	Tempered.		Real.	Names.
I.	100,000	C=1.	=100,000	The key-note
	94,387	$=\frac{15}{16}$	= 93,750	The second minor
II.	89,090	D= $\frac{8}{9}$	= 88,889	Second
	84,090	$=\frac{5}{6}$	= 83,333+	Third minor
III.	79,370	E= $\frac{4}{5}$	= 80,000	Third major
IV.	74,915	F= $\frac{3}{4}$	= 75,000	Fourth
	70,710	$=\frac{32}{45}$	= 71,111	Fourth major
V.	66,743	G= $\frac{2}{3}$	= 66,666+	Fifth
	62,997	$=\frac{5}{8}$	= 62,506	Sixth minor
VI.	59,462	A= $\frac{3}{5}$	= 60,000	Sixth major
	56,123	$=\frac{5}{9}$	= 55,555+	Seventh minor
VII.	52,173	B= $\frac{8}{15}$	= 53,333+	Seventh major
VIII.	50,000	C= $\frac{1}{2}$	= 50,000	Octave.

Thus, if the lengths of the notes be determined by this means, there will be an equal harmony throughout, let whatever note be taken as the key-note, as they will be found to differ very little from the true lengths given in the fractional numbers annexed.

From what has been already said concerning the tremors and vibrations of a sounding body, and the known elasticity of the air, we can easily see, that whatever motion is generated in any particle, is communicated to all around it as from a center; and consequently, the tremors from a sounding body will be propagated in hollow concentric shells of air, which are called aerial pulses, or waves, being analogous to the waves excited in the surface of water, when a stone is thrown into it; the elasticity of the air effecting in one case, what the gravity of the water performs in the other. The spaces, where the air is condensed by the vibrations of a sounding body, are called *pulses*, and the spaces, where it is rarefied, are called *intervals* of the pulses; and as these are produced by the alternate vibrations of the sounding body, backwards and forwards, they necessarily succeed one another alternately. The space into which the air is condensed is called the thickness of the pulse, which continues the same at all distances from the sounding body; as all the vibrations which raise these pulses are made in the same time, and if the elastic string continued to move longer, in raising a pulse, it would be so much the thicker.

Having paved the way, by the foregoing observations, we shall next consider the principal properties of these pulses or waves of elastic air.

1. They are propagated, from the sounding body, in hollow concentric shells, in all directions, until they meet with some obstacle, from whence they will again

begin to diffuse themselves in the same manner as from the first center. Hence, we see the reason, why several persons hear the same sound in different places. If the sound pass through a window into a room, it will be heard in any part of the room, where the ear may be placed, and not solely in the direction in which it comes. For, when the pulse of condensed air passes by any obstacle, or through a hole in it, the motion of the air will be communicated in all directions around it. And for this reason, sound is as well heard through a bent, as through a straight tube; whereas light cannot be propagated through any tube, but such as is perfectly straight, because it does not depend upon any pressure or elasticity of any medium whatever.

2. The density of these aerial pulses decreases as the squares of the distances from the place of the sounding body increase. As the force of motion is the same which is exerted on each concentric shell of air, however distant it may be from the sounding body, it must decrease as the number of particles in each shell increases; and this number of particles increases with the squares of the distances; therefore the density of the pulses, which depends upon the force exerted upon them, is proportional to the squares of the distances inversely, from the place of the sound.

3. Hence the intensity or strength of any sound decreases also as the squares of the distances from the sounding body increase; because the intensity or strength of any sound depends upon the degree of force, with which the air is condensed and strikes upon the ear; and this force we see decreases with the squares of the distances inversely. By the intensity of sounds we do not mean the tone or note; in which respect, they are distinguished into grave and acute, according

as the impression on the ear continues a longer or shorter time, and does not depend upon its distance from the sounding body. But the intensity of a sound depends upon the force with which the ear is struck, which is greater or less, according as the ear is nearer or farther off. The tone of a sound is the same at all distances, but the ear may be at such a distance from the sounding body as to occasion the sound to be denominated loud or low, or not to be heard at all. As the vibrations of the sounding body grow weaker at any place, the intensity of the sound gradually decreases.

4. The intensity of sound is greatly increased by the peaking-trumpet, and by otacaustics, which are no other than speaking-trumpets inserted in the ear, to ollect and condense the languid pulses of the air, and hereby assist the hearing. As sound is always louder, n proportion as the air is denser, it must follow, that he sound passing through a trumpet must be greatly ugmented, by the continual reflections of the air from he sides of the trumpet, and the air being thus greatly ondensed, must act with much more force upon the xternal air, at its exit from the tube, and consequently propagate the sound to a much greater distance, and ffect the ear with much greater force, than it could have done without such an instrument. A person speakng at the smaller end of a speaking-trumpet exerts the whole force of his voice upon the included air, whereby he density of the pulses is increased, and their force or momentum, to move the external air, will be as much greater, than without the tube, as the whole surface of sphere, whose radius is the length of the tube, is greater than the larger end of the trumpet. Because, he force of voice, that would be spent, without the ube, upon all the particles in the surface of that sphere,

is now confined to that part of the sphere only, which is included in the orifice of the larger end of the trumpet. And as the intensity of sound is increased in an inverse ratio to the number of particles to be moved, the advantage of the speaking-trumpet must be in the same proportion.

From this it is plain, that the effect of these instruments principally depends upon their length; yet some advantage may also be derived from the figure of the side, which is greatest when the elastic portions of air included in the transverse sections of the tube, in equal distances along the axis, are in a geometrical ratio to each other, as we have seen in the collision of elastic bodies. For when any motion is communicated through elastic bodies, which are in this proportion, the momentum of the last is the greatest possible. Now this will be the proportion between the transverse sections of air taken at equal distances along the axis of the trumpet, when it is made by the revolution of the hyperbola, or logarithmic curve round its assymptote. This therefore is the best form of a trumpet.

As the loudness or intensity of sound is promoted by the density of the air, we see the reason, why a sound can be heard much farther under water than in the open air, as water is 860 times denser than air, the continuity of the particles of water effecting the same as the elasticity of the air effects, but in a higher degree. You see also the reason why a low sound is easily communicated through the whole length of a tree, when it is sound, and the particles are continuous. Ship-builders and carpenters thus estimate the soundness of various pieces of timber, whose inside they cannot see.

5. We find by repeated experiments, that sound moves with the same velocity at all distances from the

sounding body, and that it travels at the rate of 1142 feet per second. Although the velocity, with which the particles of the sounding body move, gradually decreases with the sagittæ or spaces over which they move; yet they perform all their vibrations in equal times, and as they raise a pulse in every vibration of the same thickness, and in the same time, these pulses must succeed each other at equal intervals, and consequently must move forward with equal velocity, while there are any excited by the sounding body. Therefore, the velocity of sound must continue the same, from the first pulse that is excited to the last.

Nor is the velocity of sound altered by the difference of tone; a grave tone moves equally fast with one that is acute. The fact is evident, in a ring of bells, where we hear the different notes from the different bells in the same order, in which they are excited, at all distances, to which they can be heard. Did the grave tones move either slower or quicker than the acute, their order would be changed in their progress, and the note, which at one distance would come first to the ear, would at another distance arrive last. A difference of time, in the vibrations of sounding bodies, is that which constitutes the difference of notes; and hence the same sounding body always produces the same note, as all its vibrations are performed in the same time. But, if another sounding body vibrate twice as quick, it will raise two pulses in the same time, that the first raised only one; yet these two pulses will preserve their proper interval invariably through the whole of their progress, and the space occupied by both will be the same with the space occupied by the single pulse raised by the first body in the same time. Therefore, the velocity of the sound, as it passes over equal distances in the same time, must continue the same, let the tone be grave or acute.

From the equable velocity of all kinds of sounds, we may estimate distances, by observing the time, which any sound spends in going from one place to another. Thus, the distance of a cloud, from whence lightning issues, is known by only observing the number of seconds between the flash, and the explosion reaching the ear; for in every second the sound passes over 1142 feet.

6. As the times of vibration in the same body are all equal, the intervals of the pulses produced by it must also be equal. If an open tube, like the pipe of an organ, of 5½ feet long, sound in unison with a string, that vibrates 100 times in a second, the interval of the pulses raised by such a pipe must be about 11 feet, or double the length of the tube. The vibrations of the string produce a hundred pulses in a second, while the first pulse has passed over 1142 feet; therefore there must be in that space a hundred pulses, and each distant from the other 1142 feet; but the vibrations of the air produced by the pipe are the same, as it sounds in unison with the string, and the number and intervals of the pulses must also be the same; so that the intervals of the pulses are nearly twice the length of the tube. And, from various experiments, this is found to be the fact.

From this property of the pulses of air we have the distinction of notes or tones in music. As the tone of a sound depends upon the duration of the stroke on the drum of the ear, and as this duration is always proportional to the intervals between the pulses, which move equally fast; the tone will be more grave or acute, as the impression on the ear continues longer or shorter;

or as the interval of the pulses is greater or less. So that hose bodies, which vibrate slowest, and thereby make he greatest intervals between the pulses, have the gravest and deepest tone, as the impression on the ear continues longest; and those bodies which vibrate quickest have the sharpest and shrillest tone. As we have seen already that the times of vibrations, and consequently he tones from musical strings, depend on the lengths, diameters, and tension of the strings; so the same variations will take place in the tones of musical pipes in an organ, arising from the diameter and length of the pipe, and the intensity of the blast, which condenses the air.

7. The velocity and intensity of sound will be altered by the direction of the wind. When the atmosphere s still, the pulses are propagated in all directions from he sounding body. But as the wind is a current of air moving with a certain force, the pulses excited by the sounding body will partake in some measure of its motion. The pulses therefore, which move in the direction of the wind's motion, will be carried to a greater distance on that side, and thereby increase the intensity of he sound in that quarter: so that a person on that side to which the wind carries the sound, will hear it farther, and louder, at a given distance, than he could hear it, in the opposite direction.

From hence it is evident, that if the wind moved with the same velocity that sound does, the pulses, as soon as they were produced, would be all carried off in the direction of the wind, and the sound would not be heard at all against the wind. But the wind, even in storms and hurricanes, seldom moves more than 70 or 80 feet per second, and sound moves at the rate of 1142 feet in the same time. Therefore, as sound moves with a much greater velocity than wind, the pulses will be propagated, and the sound will be heard, against the wind; though it is weaker and reaches to a smaller distance in this direction, than it does on the other side of the sounding body where it has a fair wind for its conveyance.

Now, if the current of air move the same way with the sound, the sound will partake of its motion, and be accelerated in its velocity by it; but if the sound move in a contrary direction to the wind, it will be retarded by the action of the wind, and therefore will move against the wind with a less velocity than that with which it moves with the wind.

8. The pulses are propagated from the same body, and from different bodies at the same time, without disturbance or confusion. But the capacity of distinguishing different sounds, when the pulses strike the ear in quick succession, depends upon a good ear, perfected by long experience and habit.

9. The particles of air striking against any obstacle, will be reflected back from it, under an angle equal to the angle of incidence; and consequently, new pulses will begin to diffuse themselves every way from that place; and, arriving at the ear, excite in us the idea of a sound, originating from the place of reflection; which secondary sound is called an *echo*. As the place of the sound is always supposed to be, where the waves begin to diffuse themselves, in all directions; a person, speaking at one end of a tube, is heard, as speaking from the other. As the phonocamptic object is more or less distant from the first place of the sound, the echo will be less or more distinct and audible; as the pulses were more or less languid before they

ere reflected. If the sound be reflected from various bjects, at different distances from the ear, it may be ften repeated, and thereby constitute the *prattling cho*. Hence the fragor and rumbling noise of thunder nd cannons: hence, also, a number of the last syllables f a sentence will be distinctly heard; as the sound will e returned to the ear, from different objects, at different distances.

Should all the pulses, excited at any place, be collected together in a single point, after various reflecons, by any particular contrivance, the sound will be s distinctly heard in this point, as at the place from hence the waves were first diffused. Hence we acount for the effect of the whispering gallery in the dome f St. Paul's in London. This dome is a hollow hemiphere, with a gallery in the inside, in which every pulse xcited by the lowest whisper is collected at the oppote point of the gallery, where all the circles of the phere intersect each other; and there the lowest whiser is as distinctly heard, as at the mouth of the speaker.

From this property of sound, we may estimate the readth of rivers, or the distance of any object that eflects the sound, as easily as from the direct sound; s the reflected sound travels at the same rate with the irect; its velocity being the same both before and after eflection, viz. 1142 feet per second. By this property f sound and the laws of descent in falling bodies, we an also estimate the depths of wells, and the height of eeples, &c. by observing how many seconds elapse, etween the time when the heavy body begins to decend, and the time when the reflected sound reaches he ear. Thus,

Let x=the time, in seconds, of the descent of a stone om the top to the bottom.

Let s=the seconds elapsed from the beginning of the descent till the return of the sound.

Then s—x=the seconds of the sound's ascent.

Let a=16.12 feet, the space a heavy body falls in a second.

And b=1142 feet, the space that sound moves over in a second.

Then as $1''^2 : a :: x^2 : ax^2$=the height of the steeple or depth of the well.

And as $1'' : b :: s-x : bs-bx$=the height of the steeple, &c.

Then $ax^2=bs-bx$.

$$x^2+\frac{b}{a}x=\frac{bs}{a}.$$

Therefore $x=\sqrt{\frac{b}{a}\times s+\frac{b}{4a}}-\frac{b}{2a}$

OPTICS.

Optics is that branch of natural philosophy, which explains the nature of light and colours, the cause of vision, and the construction of optical instruments.

Optics is generally divided, by writers on this subject, into two parts, called dioptrics and catoptrics; by the former is meant the vision occasioned by light transmitted through any transparent substance, and by the latter, the vision by the rays of reflected light.

Before we enter upon this science, it will be necessary to define and explain a few terms, which do not often occur in common life, and are almost peculiar to this branch of philosophy.

A *ray* of light is a stream of particles issuing from the sun, or any other luminous body, in one and the same direction.

A *beam* or *pencil* of rays is a number of rays proceeding parallel to each other from the same luminous body.

The *axis of a beam* or pencil of light is the middle ray.

A *medium*, in optics, is any thing that is transparent and affords a passage for the rays of light; and, in this sense, a vacuum through which the rays pass is denominated a medium.

The *right course* of a ray of light is a straight line, for all rays proceed in right lines from a luminous body; as is evident from the similarity of the shadows, which are projected from opake bodies, to the bodies themselves, being terminated by right lines, from the luminous point, passing through the various corners and sides of the bodies.

The *inflexion* of a ray of light is its being turned out of its right-lined direction, while it is passing on in the same medium, by the attractive force of any body, near which it passes. If a pencil of rays pass by the edges of two knives set together within a tenth of an inch, they will be so much bent out of their right-lined direction, by the attraction of the knives, that they will be dispersed in the shadows of the knives, which will therefore be in contact.

The *reflexion* of a ray of light is its being so much turned out of its right course as to be sent back into the same medium again.

The *refraction* of a ray of light is its being bent out of its course into another direction, when it passes out of one medium into another.

The *angle of incidence* is that contained between the incident ray, and a perpendicular to the surface of the medium, at the point of incidence.

The *angles of reflexion* and of *refraction* are those contained between the said perpendiculars and the reflected or refracted rays.

Rays are said to be parallel, converging, or diverging, according as the lines they describe are always at the same distance from each other, approach nearer together, or recede from each other in their progress.

The point from which rays proceed, is called the *radiant point*.

The point to which they converge is denominated the *focus*.

The imaginary radiant point, is the point from whence the rays seem to diverge, when they have been reflected or refracted.

The imaginary focus, is the point to which the rays tended before refraction or reflexion, which turned them off before they arrived at it.

The divergency and convergency of rays is measured by the small angles, contained between the lines which they describe.

If rays proceed from a radiant point, or tend to a focus at an infinite distance, their divergency or convergency is considered as nothing, and they are said to be parallel.

A *lens* is a thin round piece of polished glass, which has both its sides spherical, or one side spherical and the other side plane. Of these, there are five sorts, viz. the plano-convex, double convex, plano-concave, double concave, and meniscus, or concavo-convex.

The *axis of a lens* is a line perpendicular to both its surfaces.

If the axis of a ray coincide with the axis of a lens, it is said to fall directly upon the lens, but if otherwise, obliquely.

A *mirror* is a polished surface of glass or metal, either convex, concave, or plane.

After premising these definitions, we are next to consider the nature and properties of the rays of light.

Light consists of very small particles of matter, very probably of different magnitudes, emitted in all directions from a luminous point, in right lines, with amazing velocity, and decreasing in density as the squares of the distances from the radiant point increase.

That light is propagated in right lines is evident, from what we have observed already concerning the shadows of bodies, being similar to the bodies which project them. And, indeed, were not this the case, they would bend round them, like the circular pulses of air, which convey sound, and prevent them from casting any shades at all: and it is equally evident, from the impossibility of seeing any object through a bended tube. The same consideration proves, that it consists of particles of matter, and that it is not an impulse of the luminous body communicated to a subtile elastic fluid; for, upon that supposition, it would pass as easily through a bended as through a straight tube; and, bending round an opposing obstacle, it would render the luminous body as visible as if nothing stood in the way: in the same manner that sound is heard, by the vibrations of the elastic air. And as there is no middle opinion, but light must be either a real body, or an impulse upon some subtile fluid; and as it cannot be the latter, being continually found to disagree with all the laws of such a motion, we conclude that it must consist of real particles of matter; especially when we reflect farther, that it is acted upon by the attractive force of matter, as well as other bodies. This is evident from the inflexion and refraction of a ray of light passing by the edge of a knife, or any other body, and its change of direction upon entering a medium of different density from the medium in which it was moving.

That light consists of separate particles is evident, from the reflexion of some of its rays, while others are transmitted by any object, on which they fall. And our being able to see all the objects that are before us, through the smallest pin-hole in a card, by means of the rays that come from every point of every one of the objects, through that hole, evidently proves the inconceivable smallness of the particles of light. If a particle of light weighed but the two hundredth part of a grain, it would strike the eye with a momentum equal to that of a cannon ball, of ten pounds' weight, when first discharged from the cannon. And if so small a particle would have such a prodigious momentum, on account of the velocity of the rays of light, to lessen this momentum, they must be inconceivably small indeed, for the tender coats of the eye to bear their continual impulses. You will wonder less at the extreme minuteness of the particles of light, when you reflect upon what we have said, concerning the infinite divisibility of matter, and the almost infinite number of luminous particles emitted from a lighted candle in a single instant, which are sufficient to render it visible in every point of a sphere of eight miles in diameter.

That the rays of light spend some time in their progress from one place to another, and are not propagated instantaneously, is a subtle discovery of the celebrated Reaumur, derived from his observations of the eclipses of the satellites of Jupiter. If from the known period of one of his moons, the time of an eclipse were calculated for a spectator in the center of the sun, or at the earth at her mean distance from Jupiter; it would be found, as he found it, that when the earth was nearest to Jupiter, that is, in his oppositions, this eclipse would happen between seven and eight minutes before the calculated time; and that when the earth was at the greatest distance from Jupiter, or when the sun and Jupiter were in conjunction, the eclipse would happen as much later than the calculated time; which phenomenon is to be accounted for only on the supposition, that the rays of light employ between seven and eight minutes, in passing over the semidiameter of the earth's orbit, which is about ninety-five millions of miles. From hence it is plain, that the velocity of light is nearly twelve millions of miles in every minute.

That rays are propagated in all directions from every point of a visible body appears from the impossibility of placing an eye in any position, from whence right lines may be drawn to any part of the body, but a sufficient number of rays from it will reach the eye to make it visible: provided the distance be not so great, as to give them room to diverge so far as not to be dense enough to occasion vision, when they fall upon the eye. For,

The rays of light decrease in density, in an inverse ratio to the squares of the distances from the radiant point. The same number of particles that fall upon any given surface at a given distance must, from their right-lined direction, be disseminated over four times that surface at double the distance, and over nine times that surface at triple the distance; and therefore can have but one fourth or one ninth of their former density at these distances: as is evident from 19 E

Notwithstanding the inconceivable minuteness of the particles of light, yet we shall presently see, when we consider their separation by the prism, that they probably have different magnitudes among themselves.

If the rays of light be sufficiently condensed in the focus of a lens, or mirror, they will burn with an intensity or force, which is proportional to the area of the lens or mirror directly, and to the area of the circular spot inversely, into which they are collected, at the focus: that is, directly as the square of the diameter of the lens or mirror, and inversely as the square of the focal distance. Because, the larger that the area of the glass is, so many more rays are transmitted or reflected by it, and their power of burning must be proportional to their number: and farther, their power of burning will also be increased by their density, because in that condition, there are more rays collected in a given space, and therefore the less the circular spot is, into which a given number of rays is collected at the focus, the greater will be their density and power of burning. Therefore, as the power of burning depends upon both these circumstances, it will be directly as the area of the lens or mirror, or the square of its diameter, and inversely as the area of the spot into which the rays are collected. Now this circular spot is the image of the sun inverted, when the rays proceed from the sun. For the rays which proceed from one limb, cross those that proceed from the opposite limb, in the center of the glass, and paint the image of the sun, in the focus, in an inverted position. Therefore the image and the object will always appear under the same angle from the vertex of the glass. Hence the image of the sun, being always seen under the same angle from the vertex, will be greater or less, in proportion to the square of its distance from the vertex. For the area of the image is proportional to the square of its diameter, by 19. E. 6. and this is proportional to the square of the focal distance, by 2. E. 6. Therefore as the intensity of force in the rays of light to burn any object is as the area of the glass directly, and as the area of the solar image inversely, it will also be as the square of the diameter of the glass directly, and as the square of its focal distance inversely.

Cor. 1. Therefore in two lenses or mirrors of the same diameter, the power of burning will be inversely as the squares of their focal distances.

Cor. 2. And in two lenses or mirrors of the same focal distance, the power of burning will be as the squares of their diameters.

The heat of a common wood-fire is about thirty-five times the heat of the rays of the sun in their common state; as it is found by experiment to raise the mercury in the thermometer about thirty-five times as high as the heat of the sun can raise it. Would we therefore make a glass of a given diameter, that should burn with the intensity of a wood-fire; the area of the solar spot in it, must be only one thirty-fifth part of the bigness of the sun: the diameter of the glass being equal to the apparent diameter of the sun.

A mirror designed either for the purposes of burning, or the copious reflexion of light, should be made of the whitest, and at the same time of the hardest metal we can procure. Because the harder the metal is, the more perfect polish will it receive; and the whiter the metal is, the more copiously will it reflect the rays of light; since the rays of the sun, when thrown promiscuously upon any body in their natur[illegible] e idea of white: and therefore a white body will reflect more of them than a body of any other colour, all other circumstances being equal. If a mirror were made of black marble, though of a considerable diameter, it would scarcely condense the rays of the sun to such a degree as to produce any sensible heat. Villette made a mirror of about four feet diameter, and near three feet focus, which condensed the rays of the sun 1750 times as much as in their common state, and therefore burned with an intensity 490 times more than a wood fire. But to make the rays from the moon burn, was an effect 7000 times greater than what it could produce.

OPACITY AND TRANSPARENCY.

When the rays of light fall upon any body, some of them are reflected from the surface to the eye, which render that surface visible: if any of them after entering the pores of the body should be reflected to the eye by any of the internal particles, these particles will also become visible: and should a sufficient number be reflected to the eye from all the internal particles of any body, the whole would become visible, and the body would be denominated *transparent* or *diaphonous*. But if the rays of light, after entering the pores of the body, be variously reflected from particle to particle in different directions, so that they are suffocated and lost, and few or none of them come to the eye from the internal parts of the body, to render them visible, the body is said to be *opake;* and nothing but the external surface of such a body can be seen by us, as the internal particles return no rays to the eye, by which they might be seen.

Although the whole body may be opake, yet the minute parts, of which it is composed, may transmit the rays of light, and become transparent; as is evident in the thin plates of an oyster-shell.

The transparency of a body depends upon the smallness of its pores. Because if they were large, there would be an irregular reflexion of the light, from the air which fills them. Although paper is opake, yet when its pores are filled with a substance of nearly the same density with itself, such as oil or water, it becomes transparent. If a number of plates of polished glass be laid together, so as to admit a thin plate of air between each of them, the rays of light, in passing through them, will be variously reflected by the air, but few of them will be transmitted through the whole, and it will thereby become opake. But if the plates of air be removed, and water substituted in their place, or if the polished surfaces of the glass be brought into close contact, the rays of light will be transmitted, and the whole will be transparent. A piece of transparent glass is rendered opake, by being reduced to a powder, and having its pores greatly enlarged; so is water or beer, when agitated into froth. Therefore the pores of bodies must not be less than of a certain assignable dimension, to render them opake; for by making them less, or filling them with a body denser than air, they become transparent. We need not be much surprised at this, when we reflect upon the extreme minuteness of the particles of light, and the possibility of having the solid parts of a body in any assignable proportion to the pores, and yet none of these pores exceeding any given diameter or magnitude.

REFRACTION OF LIGHT.

WHEN a ray of light passes out of one medium, into another of different density, in a direction perpendicular to the surface of the second medium, it is accelerated or retarded in its motion, according as the second medium into which the ray enters, is of greater or less density than the other; but it suffers no change in its direction. As there are more particles in a given space, in the surface of the denser medium, than in the rarer, to attract the ray of light, while it is approaching the denser medium, and the attractive force is exerted in the direction in which the ray moves, its motion will be accelerated for a small space before and after its entrance into the medium; until the attraction in contrary directions become equal. But when it is moving out of a denser into a rarer medium, its motion for the same reason will be retarded.

If a ray of light fall obliquely on a denser medium, while it is moving through a rarer, it will not only have its velocity increased, but will also have its direction altered into another, nearer to the perpendicular to the surface of the denser medium at the point of incidence. For it is then acted on by two forces; by one in the direction of its own motion, and by the attractive force of the denser medium in a direction perpendicular to its surface. The ray must therefore move in the diagonal of a parallelogram, whose sides are in the directions of these forces; that is, it must move in a direction nearer to the perpendicular, making the angle of refraction less than the angle of incidence. But if the ray move out of a denser into a rarer medium, it will be refracted from the perpendicular, making the angle of refraction greater than the angle of incidence: so that what was before the angle of incidence, now becomes the angle of refraction, and vice versa.

This may be illustrated by a well known experiment. Let a dollar be placed in the bottom of a bason on the floor, and if you recede from it, until a ray from the farther edge of it, just passing by the edge of the bason, reach the eye, so that the whole dollar becomes invisible; in this position of the eye, if the bason be filled with water, the whole dollar will become visible; as the ray, which now comes from the nearer edge of the dollar, is refracted from the perpendicular, upon its passage out of the water into the air, and thereby is brought low enough to enter the eye.

The velocity of the ray of light, after it enters a denser medium, is to its velocity before it enters it, as the sine of the angle of incidence to the sine of the angle of refraction. For the greater that the attractive force of the denser medium is, the more will the ray be bent towards the perpendicular, and the less will be the sine of the refracted angle. The attractive force of the denser medium acts in right lines perpendicular to its surface, whereas the direction of the incident ray is supposed to be oblique to the surface of the denser medium. The diagonal, therefore, which the refracted ray will describe by the joint influence of these two forces, must lie nearer to the stronger of these forces, and we say in the proportion of the sines inversely. Let RO* be a ray of light moving through a rarer medium ATB, and incident on the denser medium BGA in the point O; instead of going on in the same direction ROG, it will be refracted into the line OF, and have its velocity increased in the proportion of RC to FD, or the sine of the angle of inci-

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* See Plate 4, fig. 5,

dence to the sine of the angle of refraction: that
velocity in the denser medium will be as much
er than that in the rarer medium as the sine
angle of incidence is greater than the sine of th
of refraction. Let its velocity in the rarer medi
represented by RO, which may be resolved in
and CO or RH, whereof the first is parallel
surface of the attracting denser medium, and th
is perpendicular to it. Now the force of attr
acting in directions perpendicular to the surface
not at all alter the force or velocity in parallel
tions. And therefore the ray proceeding wit
motion will in equal times go through those
parallel intervals, that lie between RH and CO
between OK and IS. Produce the refracted r
it meet IS in the point S. Now as the ray
through the denser medium by the joint effic
the two forces OI and IS in the diagonal of a
lelogram, whose sides are in proportion to th
will pass over the diagonal OS, in the time t
would pass over the space OI, or HO, or RC,
parallel motion alone, that is in the time of desc
RO. Now make OF=RO, and SO will repr
the velocity of the ray in the denser medium, an
or RO will be its velocity, as before, while it was
rarer medium. But FO : SO :: FD : SK=OI=
RC; that is, the velocity of the ray of light i
rarer medium is to its velocity in the denser me
as the sine of the angle of refraction is to the si
the angle of incidence. Q. E. D.

Cor. As the difference between the velocitie
ray, moving in any two given mediums, must al
be the same; let the obliquity of the ray be wh
will; it follows, that the sines of the angles of
dence and refraction will always be in a constan
tio to each other, in all obliquities of the incident

because these sines are as the velocities in the different mediums inversely, and these velocities do not depend upon the obliquity of the ray, but on the attractive force of the mediums.

The refraction of a ray of light is not made, all at once, upon its passing from one medium into another, but gradually, and partly in one medium and partly in the other; and as it is acted upon by two forces, viz. its own projectile force, by which it passes over spaces proportional to the times, and by the attractive force of the denser medium, which is as the squares of the times; it will move in the curve of a parabola, by what we have demonstrated in the doctrine of projectiles.

It is found by experiment, that when a ray passes out of air into water, the sine of the angle of incidence is to the sine of the angle of refraction, as 4 to 3; and when out of air into glass, as 3 to 2, or rather as 17 to 11, more nearly. And if the passage of the ray be the contrary way, the proportion is inverted, as the angle of incidence now becomes the angle of refraction, and vice versa. It is also a general observation, that the denser any medium is, the greater is its refractive power, as more particles attract the ray in a given space; and consequently the greater will be the disproportion between the sines; yet oils and sulphureous bodies refract the rays of light more than other bodies of equal density.

As the eye is not sensible of any alteration in the direction of a ray of light, by its refraction, it always views the object, from whence the ray proceeds, in the direction of the ray when it enters the eye. Hence the bottoms of vessels filled with water appear elevated above their real situation; hence also a stick, partly in water and partly in air, appears bent at the place where the air and water meet; as the part that is under water is elevated above its real situation. Hence also we account for the elliptic figure of the horizontal sun or moon. As the rays proceeding from the lower limb are more refracted by passing through the atmosphere, than the rays which come from the upper limb, and therefore through a part of the atmosphere that is less dense. Hence the upper and lower limbs will appear to be brought nearer together, than their real distance, while the eastern and western limbs will be equally elevated, and therefore kept at the same distance from each other. The vertical diameter therefore will appear to be shortened, while the horizontal diameter continues of the same dimensions. Hence also the sun, moon, and stars, when near the horizon, appear elevated above their real positions; because as the rays from them pass through a vacuum into the air, they are refracted towards the perpendicular, and coming to the eye in an angle of greater elevation above the horizon, will represent the object in a higher situation than what is real. So that the sun or a star may be seen before they are actually risen above the horizon.

If a ray of light move through a denser medium with such obliquity, that when it should emerge into a rarer, the angle of refraction, according to the proportion stated above, should be more than a right angle, the ray will not be refracted into the rarer medium, but will be returned through the denser, making the angle of reflexion equal to the angle of incidence. Because when it approaches near to the surface of the rarer medium, it will move in the curve of a parabola, for a small space, from what has been already observed, and when it is ready to emerge into the rarer medium, the angle of refraction should be either a right angle or greater. If it emerged into the rarer medium, when the angle of refraction would be a right angle, it would move along the surface of either medium, where they were in contact; but this it cannot do, because it is more attracted towards the denser medium, than towards the rarer, and consequently it will return through the denser medium in a similar curve of a parabola, whose principal vertex is at the point of reflexion, and which will make the same angle with every parallel plane of the denser medium. Now if you conceive the number of the parallel planes into which the denser medium is divided to be infinite, and their distance exceedingly small, so that the action of the attractive force may be perpetual, the angle of reflexion being always equal to the angle of incidence, at any one of these planes, they will be equal at last; and the ray will emerge at last from the denser medium on the same side, on which it was incident, and in the same angle. This will happen in glass, when the angle of incidence is about 40° 20′, and air is in contact with its second surface; for then, none of the rays that are incident upon it, in a greater angle, will be transmitted into the air, but will all be reflected in an angle equal to the angle of reflexion.

What we have hitherto said concerning the reflexion and refraction of a ray of light, is upon the supposition of an equal degree of reflexibility and refrangibility in the rays of light: but this will not be found to be strictly true. For the rays of light are differently refrangible, and as differently reflexible.

If the rays of light be transmitted through a prism, they will be refracted in different directions, and separated into the seven original colours, in the following order: *red*, *orange*, *yellow*, *green*, *blue*, *indigo*, and *violet*. The red rays being the least refrangible, the violet the most, and the intermediate rays susceptible of intermediate degrees of refraction. From this we conclude, that the particles of light are, very probably, of different magnitudes; as some of them are more powerfully attracted by the prism than others. The red rays must consist of the largest particles; hence, moving with the same velocity as the rest, they will have a greater momentum, on account of their greater magnitude, to overcome the force of attraction, and therefore will be less turned out of their former direction by the prism; and the violet rays, consisting of the least particles, will, of consequence, be most refracted.

Nor are all the rays of the same colour equally refrangible; for, consisting of particles of different magnitudes among themselves, they are refracted to different places, and therefore cover a considerable space. Nay some of the red rays are more refrangible than some of the orange, and some of the orange than some of the yellow, &c. so that the rays of different colours are not refracted into separate spaces, but are mixed together, towards the verges of the different colours; and hence, it is difficult to determine where one colour ends and another begins. Were these rays equally refrangible, the sun's image formed by the prism would be round like the sun. But there are as many images formed, as there are rays of different colours, one above another, and overlapping one another when the sun's rays pass through a prism; or one beyond another when they pass through a lens.

That colour is an inherent property of the rays of light, and that it does not depend upon the refractive

ower of the prism, is evident from this experiment.
f the rays of the sun fall upon a prism, and be there-
y refracted into their different colours, and the rays
f any particular colour, as red or blue, be separately
ransmitted through a second prism, they are no far-
her separable. They are indeed refrangible, and may
e refracted in a different direction; but the colour is
ever changed. If the colour depended upon any ope-
ation of the prism, and were not inherent in the rays
f light, the second prism would separate the rays as
vell as the first. This, however, never happens: the red
ays continue red, and the blue rays, blue, after trans-
nission through the second prism. But we shall say
nore on this subject, when we come to consider the
olours of natural bodies.

If the solar rays be equally refracted by two prisms
f different kinds of glass, the dispersion of the rays
f any colour, effected by the one may be much great-
r than that by the other, and the coloured spectrum or
mage of the sun formed by the one will be much
onger than that formed by the other. It is found, that
n equal angles of mean refractions, a prism of white
lint glass, in the composition of which there is a con-
iderable quantity of lead, will disperse the component
olours of light, much more than a prism which
bounds with alkaline salts.

REFLEXION OF LIGHT.

When the rays of light fall upon any body, instead
f being transmitted through its pores, they are co-
iously reflected back; not by impinging on the solid
arts of the body and rebounding again like an elastic
ball, but by some power that is equally diffused over
the surface of the body, and very probably of a re-

pulsive kind; as we know, that where the attraction of cohesion ends a power of repulsion begins. And many of them are reflected from the second surface of the glass, after having entered its pores. There are many reasons, which induce us to believe, that light is reflected in the manner above mentioned. If it be incident upon glass in an obliquity of 41° or 42°, when air is in contact with the second surface, it is wholly reflected. But in this degree of obliquity, if water be in contact with the second surface of the glass, the rays will be transmitted through it, until the obliquity be increased to 45° or 46°. Now it cannot be supposed, that in one degree of obliquity, the rays of light should meet with nothing but pores to transmit them, and in another degree, with nothing but solid particles to reflect them: nor that they should meet with more particles in air than in water to reflect them. We find also that the reflexion from the first surface is less copious than that from the second; and in general, the rarer that any transparent medium is, that is in contact with the second surface of glass, the more copious is the reflexion. If glass, water, air, and a vacuum be successively applied to the second surface of glass, the rays of light will be more copiously reflected in each succeeding experiment than in the former. Now it is absurd to suppose that light should meet with more solid particles to reflect it, the rarer the medium is, which causes the reflection; and that it should meet with more solid particles in a vacuum, where there are none at all to reflect it, than in glass or water: and yet the reflexion from it is most copious: and from air more copious than from water, and from water, than from glass.

Besides, Sir Isaac Newton found that the rays of light were as differently reflexible as they were refrangible; and that those that were most refrangible, were also most reflexible. Hence it is, that the atmosphere appears to be blue. When all the rays of other colours are transmitted through the atmosphere near to the top of it, where it is exceedingly rare, the blue rays alone, being most easily reflected, are brought to the eye. Blue rays will be reflected when others are transmitted.

If the object-glass of a refracting telescope, of considerable focal length, were laid upon a plane glass, the light falling upon the thin plate of air between the glasses will be alternately transmitted and reflected, at different distances from the center, or point of contact. In the center, where the lens and glass are in contact, the light will be wholly transmitted, and thereby cause the central spot to appear black. At a small distance from thence the light will be reflected in various coloured rings, and so on to a considerable distance from the center. Should these distances be taken in arithmetical progression, as 0, 1, 2, 3, 4, 5, &c. then at the distances 0, 2, 4, 6, 8, &c. the light will be transmitted, and at the distances 1, 3, 5, 7, 9, &c. it will be reflected. The same appearances are observed in bubbles of water, rendered tenacious by a little soap. But in these, the concentric rings of coloured light reflected, continually expand, as the bubble grows thinner by the continual subsiding of the water, until the bubble breaks.

These phenomena prove, not only that some rays may be transmitted while others are reflected; but also, that the reflexion depends upon the thickness of the bubble, and of the plate of air between the glasses.

But that which proves, beyond all contradiction, that the rays are differently reflexible, according to their refrangibility, is the following experiment.

After the rays of the sun are separated by a prism, let them be received by another, which, by its revolution on its axis, will begin to reflect the violet rays, while it transmits the rays of other colours, and reflects the red rays last of all.

This may be adequately accounted for, upon the supposition, that the violet rays are of the least magnitude, and therefore, coming with the least momentum, are most easily reflected by the repulsive force of the medium on which they are incident.

But that the transmission of rays of one colour, at the same place, where others of a different colour were reflected, should depend upon the different thickness of the medium through which they pass, or of that which reflects them, is not so easily accounted for. Sir Isaac Newton, to whom we are indebted for all that we know of light, supposes that we have sufficient foundation from this phenomenon to conclude, that the ray's of light, in their passage from the sun, are, at certain intervals, disposed to be alternately transmitted and reflected. After all the experiments which he made upon light, that prince of philosophers confessed, that he was not able to account for the mode of operation which nature uses in producing these phenomena. Yet, even the hypotheses of so great a genius are not to be lightly rejected. His solution of this appearance is consonant to that simplicity and regularity, which we discover in all the works of nature. His opinion was, that if the rays of light were incident on any medium, in a fit of easy transmission, they were accordingly transmitted; but if in a fit of easy reflexion, they were reflected.

upon a piece of green cloth, as it can reflect none else, it must appear red; and so for any other colour.

3. If rays of any particular colour be more copiously reflected from any body, it will appear of that colour whose rays it reflects most copiously. Yet the colour will not be so vivid, as if no heterogeneous rays were reflected at the same time. It will be in some measure diluted and tinged with the colours of the other rays.

4. If all the rays of the sun were equally and promiscuously reflected from a body, it would appear white, a little inclining to yellow, which is the colour of the sun's rays when blended together. If the rays of the sun, when once separated by the prism, were again collected in the focus of a large lens, and thrown together in confusion, they would appear white. There is a condition relating to sight, which suggested the method of ascertaining this matter beyond contradiction; which is, that the impressions of light remain for some short space of time upon the eye. As when a burning coal is whirled about in a circle; if the motion be very quick, the eye will not be able to distinguish the coal in any particular part of the circle, but will see a complete circle of fire. The impression made by the coal in any one position is not entirely worn out, until the coal returns again to the same place and renews the impression. This led sir Isaac Newton to try whether the rays of light would not appear white, when thrown upon the eye in such quick succession, as that it should not distinguish any particular colour. He therefore contrived an instrument like a comb, which, when applied behind the prism or lens, would intercept some of the colours of the refracted ray, while it transmitted the rest, and found that while it was moved slowly, the sun's image changed colour every moment, according as this or that particu-

And the reflected rays, being all of one sort, or of different sorts in different proportions, give us the ideas of different colours.

The colours of natural bodies are not inherent in them, but are dependent on the rays of light. Were all the rays of the sun of the same colour, there would be no other in the world. If red rays, for instance, were incident on any body, and none else, the body would appear red, whatever might be its natural colour. Natural bodies may have particles in their surfaces, of such a determinate magnitude, as to be adapted to the reflexion of the rays of light of a particular colour, more copiously, than those of any other colour, and thereby appear of that particular colour; which has occasioned their being denominated blue, or green, or red, &c. Yet if the red rays of light alone fall upon a body denominated green, as it can reflect no other kind of rays, it must necessarily appear red. Hence, we conclude,

1. If the rays of light be wholly transmitted or nearly so, the body, which transmits them will appear black, which is the absence of all coloured light, as all bodies appear black in the dark. We have indeed not any bodies perfectly black, since then they would be invisible; for the blackest bodies, with which we are acquainted, always reflect some light, by which they are seen, although not in such quantity, as to excite in us an idea of any particular colour. This appears from the blackness of the central spot in the lens, and the bubble, where the rays are almost all transmitted.

2. If rays of any particular colour alone be reflected by a body, it will appear of that colour and of no other, whatever may be its appearance when it reflects the rays of every colour promiscuously. If red rays alone fall

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l and tinged with the colours of the other rays.
f all the rays of the sun were equally and promis-
ly reflected from a body, it would appear white,
inclining to yellow, which is the colour of the
ays when blended together. If the rays of the sun,
once separated by the prism, were again collected
focus of a large lens, and thrown together in
ion, they would appear white. There is a condi-
elating to sight, which suggested the method of
ining this matter beyond contradiction; which is,
e impressions of light remain for some short space
e upon the eye. As when a burning coal is whirled
in a circle; if the motion be very quick, the eye
ot be able to distinguish the coal in any particu-
t of the circle, but will see a complete circle of
he impression made by the coal in any one posi-
not entirely worn out, until the coal returns again
same place and renews the impression. This led
ac Newton to try whether the rays of light would
pear white, when thrown upon the eye in such
succession, as that it should not distinguish any
lar colour. He therefore contrived an instrument
comb, which, when applied behind the prism or
vould intercept some of the colours of the refract-
, while it transmitted the rest, and found that
it was moved slowly, the sun's image changed
every moment, according as this or that particu-

lar colour was transmitted; but that when the motion was very quick, the eye lost all perception of the distinct colours, which came to it from time to time, and a perfect whiteness resulted from a mixture of all the impressions of the different colours on the eye. The same may be proved by painting the circumference of a wheel, with the seven original colours, in proportion to their breadths when they are separated by the prism; for if this wheel be turned quickly on its axis, the whole painted circumference will appear white.

From what we have said, it appears that the colours of natural bodies are not inherent in them, but depend solely upon the rays of light, which they reflect; and that they appear of this or that particular colour, only from the aptitude of their surfaces to reflect one species of coloured light rather than another, or in a greater quantity; and that this aptitude arises from nothing but the different sizes of the particles and pores in their surfaces.

The minute parts of all opake bodies, except metals, are transparent, as is evident from the transmission of light through thin plates of them; and even metals become transparent, when dissolved in a strong acid. And between the solid parts of opake and coloured bodies there must be many spaces or pores, either empty or replenished with other mediums of different densities, as water between the solid particles, which give it any particular tinge, or air between the aqueous globules, that constitute mists and clouds: and possibly there may be spaces void of both air and water, and yet filled with some other more subtile substance. For we find, that there are various reflexions of light made from the internal parts of bodies, and that these reflexions are made only when the ray, in its progress, meets with a medium of different density, and most copiously when this difference is greatest. As the thin plate of air between the convex and plane glasses, mentioned before, transmitted or reflected the rays of different colours, according to its different thicknesses: and as the concentric rings of colours appearing in the bubble of water, continually varied with the changing thickness of the shell of water: this shows that the particular colour of any ring depends upon the particular thickness of the plate of air or shell of water, where the incident light is reflected to the eye. In the same manner, the transparent parts of bodies, according to their several sizes, reflect rays of one colour and transmit those of another; on the same grounds as the thin plates of air or water transmit or reflect them: and this seems to be the foundation of their different colours. For, if a transparent substance, of the thickness proper to produce any colour, should be cut into slender threads, or broken into fragments, we can see no reason why every fragment should not keep its own particular colour, and a heap of such fragments constitute a body of that particular colour. So that the different sizes of the particles, of which bodies are composed, must be the reason of their appearing to be of different colours. This is farther confirmed by the analogy between the colours of natural bodies and the colours of the rings exhibited in the bubbles of water and the glasses above mentioned. These plates of air or water do not appear of the same colour at the same place, when viewed obliquely, as when seen directly; for they will be observed to expand and grow larger with the increasing obliquity, with which they are viewed. This shows, that the transparent substance between the glasses does not exhibit the same colour, at the same thickness, in all situations of the eye: just as we observe that the colours of the same part of a peacock's tail vary with the position in which it is seen. The colours of variable silks, and many other substances, change in like manner with the position of the eye. Besides, we find the colours of many substances growing faint and diluted, by being wetted with water or oil, which can intimately penetrate their pores, and recovering their former lustre upon being dried; as we find the colours reflected from thin pieces of Muscovy glass grow faint, by wetting the glass with water, and grow vivid again, when dry.

To all this it may be added, that many bodies undergo a change of colour from trituration and a comminution of their parts. Thus the colours, which painters use, will be a little changed by being ground elaborately, which is doubtless owing to their having their particles broken and lessened. Mercury, by being variously tortured in the fire by the chemists, may have the size of its particles so changed as to appear sometimes red, or yellow, or white; and lead, by a similar process, sometimes appears red and sometimes white.

Copper, in the mass, appears red; but when dissolved by a strong acid, appears blue. Mineral waters are tried by the change of colour they effect in the syrup of violets, which is naturally blue: if an acid salt prevail in the water, it will change its colour into red, by intimately incorporating with the mineral, and coalescing into larger globules, which always reflect the red rays of the sun more copiously, than particles of a lesser size: but if an alkali prevail in the water, it will not so closely cohere to the particles of the syrup, and the particles of the compound will be of a lesser size, such as are adapted to the reflexion of a green colour; and therefore the mixture will appear green. If the fumes of a strong acid, which are its most subtile and energetic particles, reach a green cloth, they will break and lessen the particles in its surface to such a degree, as to make it appear blue; whereas, if it reach the cloth in substance it will operate less violently upon it, and change it into a yellow colour.

Upon the same principles, we may account for that surprising change of colour produced by the mixture of various fluids, where the particles of the one, by their action upon the other, in different degrees, either make them greater or less, so as to dispose them to reflect the rays of this or that particular colour more copiously than the rest, and thereby cause the body to appear of a particular colour. Hence it is, that for the production of black, the particles must be smaller than for the exhibition of any colour, viz. of a size, answering to the thickness of the plate of air, where, by reflecting little or no light, it became dark and opake. Yet the particles must not be too small, for that will make them transparent through a deficiency of reflexion in the internal parts. They must be of a size bordering on that which is disposed to reflect a blue, as we see that all blacks partake a little of that colour. Hence, we see the reason why bodies dissolved by fire, that most subtile dissolvent, generally become black; why the same follows upon putrefaction; why, in setting a razor upon a hone, or in grinding glasses upon plates of copper, the particles of the metal, glass, or sand, usually become very black; and also, why black substances so easily communicate their hue to others: as the particles of such bodies, from their extreme subtileness and minuteness, readily overspread any surface and fill up the inequalities arising from their roughness.

On the same principle, we account for the different tints of the clouds, which they receive from the rising or setting sun, together with many other phenomena observable in the atmosphere; such as that vapours, when first raised, do not destroy the transparency of the air, being divided into particles too small to cause any reflection from their superficies. But when, in order to compose drops of rain, they begin to coalesce into larger globules, of size sufficient to reflect some rays, and to transmit others, they may form clouds of different colours, according to their sizes.

Hence, we see the reason, why two transparent and colourless fluids may, upon being mixed together, become opake; as the mixture of the spirits of wine and the spirits of hartshorn produces an opake coagulated substance, called the offa alba: as the mixture is not so intimate, as to leave the pores small enough for the transmission of light. Two coloured fluids also, one red, and the other blue, the first transmitting the red rays of light only, while the other transmits the blue, will, when mixed together, become opake; or when one is held behind the other, no object can be seen through both: because the rays which pass through the one are stopped by the other. Thus, also, leaf-gold appears yellow by reflected light, but of a bluish tint by transmitted light. There are also some sorts of liquors, which reflect one sort of rays, while they transmit others, as a tincture of lignum nephriticum, and thereby will appear of different colours: while other fluids appear of the same colour, both by transmitted and reflected light; as the same rays, which pass through the fluid, are reflected again from the farther surface.

THE RAINBOW.

ave now paved the way for the explication of
oured arch which we often see in the cloud,
ated the rainbow; when the sun, on the one
ines bright on the drops of descending rain, on
r.

phenomenon, though observed in the heavens
e the deluge, remained inexplicable, until the
refrangibility of the rays of light was discovered
saac Newton. Antonio de Dominis, archbishop
to, and after him Descartes, expressly showed
rainbow was formed by the reflexion of the
s from the drops of falling rain, but neither of
uld account for the diversity of its colours.
uld indeed easily confirm their opinion, that
bow was formed by the reflexion of the sun's
n the drops of rain, by a well known experi-
hanging up a glass globe filled with water op-
the rays of the sun; for by varying the posi-
he eye, or by elevating or depressing the globe,
us colours of the rainbow would successively
the globe; but they had no idea of these co-
ing an inherent property of the rays of light,
parated by refraction. So that the cause of the
colours remained an inexplicable secret, until
Newton unravelled the web of light, and ac-
for every circumstance.

are two rainbows generally appearing at the
e, both of which depend upon the same cause,
are formed in a different manner.

interior or primary bow is formed by two re-
and one intermediate reflexion of the rays of
the drops of falling rain: but the exterior bow

is produced by two refractions and two intermediate reflexions of the rays of light.

The primary bow is formed in the following manner. Let BNFG* represent a drop of falling rain, AN, a ray of light from the sun, falling upon it in the point N, and refracted to the point F, in the opposite side of the drop, from whence it will be reflected to G, making the angle of incidence at F equal to the angle of reflexion. At G, part of the beam will be reflected again, but the greater part will be refracted into the air in the direction GR, from the perpendicular IG, and be separated into its various colours, the red lying nearest to the perpendicular IG, and the violet farthest from it, according to their different refrangibility.

Now, as the least refrangible or red rays are refracted lowest, and the other colours above them, it is evident, that those will first come to the eye of the spectator, as the drop of rain is descending, and that the rays of the other colours will, in succession, reach the eye, according to the degree of their refrangibility; until the drop has fallen so low, that the violet rays alone can come to the eye, while all the rest are refracted below it. But as there are drops of descending rain at all these intermediate altitudes, each exhibiting the colour peculiar to its altitude at the same time, the spectator will see all the colours at once, in different arches, lying one above the other, the red being uppermost, and the inner verge of the bow being violet. These altitudes depend upon the altitude of the sun at the time, as there is a constant angle made by the incident and refracted ray of each particular colour, and when the sun is in the horizon, the altitude of the bow is greatest, being then a complete semicircle above the horizon: but the angular radius of this semicircle, being equal to the angle made by the incident and refracted ray, is constantly the same for each particular colour.

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* See Plate 5, Fig. 2.

From the constant ratio that subsists between the sines of the angles of incidence and refraction, it is found, by computation, that the angle made by the incident ray and the least refrangible of the red rays is 42° 2′, and the angle made by the most refrangible of the violet rays and the incident ray is 40° 17′, and these are the semidiameters of those arches respectively; and their difference, viz. 1° 45′ is the breadth of that bow, were the sun but a point. But as his diameter subtends an angle of 32′, by so much must the breadth of this bow be increased; so that it will be found to be 2° 17′. Hence the radius of the outer verge of the red arch, being increased by half this angle of the sun's diameter, will be 42° 18′; and the radius of the inner or violet arch, being diminished by the same angle, will be reduced to 40° 1′. Because, the sun's diameter being 32′, a ray from the upper limb of the sun proceeding parallel to a ray from the lower limb, may be refracted and reflected to the eye under the same angle between the incident and emergent rays, from a drop of rain situated 32′ above another drop, from which a ray of the same colour might come to the eye, making the same angle with the incident ray, which had issued from the lower limb. And this would be the breadth of the bow, if all the rays of light from the sun were but of one colour. And if a beam of light from a single point of the sun, as for instance, from the lower limb, be dispersed by its different refrangibility over a space of 1° 45′ in the heavens, a beam from the upper limb must be so dispersed, that the verge of it must be 32′ distant from the verge of the other.

Yet still it must be remembered, that the bow is not formed by all the rays of light that fall upon that side of the drop which is next to the spectator, above its axis, but only by such as are insensibly near to a certain point N*, and are so refracted within the drop as to unite in the same point F, on the farther side of it. These rays coming parallel from the sun will have a certain inclination to the surface of the drop, near the point N, and being refracted to the point F, will be reflected from it, under the same inclination to each other, to the point G; because the angle of incidence at F will be equal to the angle of reflexion. And as these rays return from the point F, inclined in the same manner to each other as they were before their incidence upon it, they will make the same angles with the surface of the drop at G, as they made at N. Wherefore, when they are refracted into the air at G, they will proceed from G, in lines parallel to each other, as well as when they were incident on the drop at first. And as these emerging rays are parallel to each other, they will not diverge from each other in their passage, and therefore will enter the eye, conveniently situated to receive them, in sufficient plenty to excite a sensation of colour. Whereas all other rays, as SB, TD, whether nearer to or farther from the axis of the drop PQ, will be refracted to some other point or points as E, nearer to the axis than the point F, because the arch QF will be a maximum. If the rays TD and SB be refracted to the same point E, they will be reflected from it under the same inclination to each other, which they had before their reflexion, and therefore will be refracted into the air at the points H and I, in lines parallel to each other; but they will not be efficacious enough to excite the idea of colour, as they are then deprived of all the contiguous and intermediate rays that fell on the drop between the points B and D. And if they be refracted to different points on the farther side of the drop, they will be reflected from them, so that they cannot fall upon the nearer surface in the same angle which they made with the surface of the drop, at their entrance into it, so that they will not be refracted into the air parallel to each other, and therefore cannot come to the eye in sufficient quantities to excite any distinct sensation of colour. For this reason, these rays are denominated *inefficacious.*

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* See Plate 7. fig. 1.

It may be asked, why the rays that enter the drop at the point N, or near to it, are refracted to the eye, more copiously than any others that enter the drop at any other place; and why they alone can excite in us the idea of colour? The reason is this: as the rays from the sun are incident upon the drop, farther and farther from its axis PQ, they will be refracted farther and farther from the point Q, on the farther side of the drop, until you come to the point N, from whence they will be refracted to the point F: but if they be incident upon the drop, beyond the point N, they will not be refracted to a point above F, but below it; so that all the points of reflexion on the farther side of the drop will lie between the points F and Q, making the arch FQ a maximum. Therefore, the rays that are incident on the drop near to the point N, both above and below it, will be reflected at the point N, or very near to it. So that about this point N alone, such rays can enter the drop, as after two refractions and one reflexion can enter the eye with sufficient density to excite in us the distinct sensation of colour. Therefore these rays alone are denominated efficacious, in distinction from those, which entering the drop, either above or below that point, are so refracted into the air, as to proceed diverging from each other, or deprived of their intermediate rays, so that they cannot enter the eye in sufficient quantities to give us the idea of colours. For there is an absolute necessity, that several rays should be refracted together at the point N, or near to it, that, being reflected insensibly near to the point F, they may be reflected together to the point G, from whence they may come to the eye, after refraction at G, into the air, not only parallel to each other, but also in sufficient quantities, to affect the organ of vision in a vivid and sensible manner.

If a line be drawn from the sun through the eye of the spectator, it will be parallel to the line described by the incident efficacious ray, and therefore will make an angle with the ray after two refractions and one reflexion, equal to the angle formed by the said refracted and incident ray; that is the angle AXR= XRY, as they are alternate angles. Hence if the line XR should revolve round RY, like the legs of a pair of compasses, the line RX would meet with many drops of rain in its revolution above the horizon, which might refract a ray of light to the eye of the spectator, under the same angle, that is necessary for the refraction of the rays of any particular colour to the eye of the spectator. Therefore all those drops, that are any where situated in the circumference of the cone, described by the line RX, will exhibit the same colour, with what is refracted in the direction

XR. And this being true of all the colours of the bow, we hence see the reason of its circular form.

The exterior or secondary bow is made by two refractions and two intermediate reflexions of the rays of light, by the drops of descending rain. If a ray of light from the sun, as AN*, fall upon the lower side of the drop at N, and entering it there be refracted to F, and from thence be reflected to D, and again reflected to G, and there refracted into the air; it will be separated into its various colours, having the violet or most refrangible rays bent farthest from the perpendicular IG, and the red rays lying nearest to it. As soon as the drop has descended so low, that any of the refracted colours can enter the eye of the spectator, the violet will first reach it, as these rays are most refracted from the perpendicular, and all the others are refracted above them; and consequently as the drop descends, the different colours will come to the eye successively, from the violet to the red, according to the degree of their refrangibility, having the order of the colours inverted with respect to the primary bow. But as there are drops enough at all altitudes to refract all the colours of light at the same time to the eye, the spectator will perceive various coloured arches in the cloud, one above another, the outer verge of the bow being violet, and the inner verge red.

In this, as well as in the formation of the primary bow, we are to remember, that all the rays that fall upon the nearer side of the drop below the axis, are not concerned in the production of the exterior bow; but only such as are efficacious, that is, such, as falling parallel upon the drop, are so refracted through it, as to be reflected within it, parallel to each other;

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* See Plate 7. fig. 2.

because, when they are finally refracted into the air, they will proceed parallel to the eye, and enter it in such quantities, as will sufficiently excite in us the idea of colour. We have seen already, that no rays can be efficacious in the formation of the bow, unless they are contiguous and parallel. Those rays, which are parallel to each other, after they have been once refracted, and once reflected, will be also parallel and efficacious after two refractions and two reflexions. Because, whatever inclination the reflected ray has at one point of reflexion, with the concave surface of the drop, the same it must have at the other point of reflexion, and therefore must emerge from the drop parallel to any ray to which it was parallel between the points of reflexion. In order to this, they must cross each other, within the drop, after the first refraction, at a certain distance from the first point of reflexion, and again at an equal distance from the second. Now this point of intersection not being the same with the point of reflexion, as it was in the formation of the primary bow, where they coincided, it follows, that the point of incidence must also be different, and farther from the axis of the drop. Thus if the rays AN and BC*, coming parallel to each other from the sun, fall on the drop, in the points N and C, they will be so refracted, as to cross each other in the point S, and proceeding diverging from thence to F and H respectively, they will be reflected to the points D and K, parallel to each other, whence they will be again reflected to the points G and L, crossing each other in the point T, which is equally distant from the points of reflexion D and K, as the point S is from F and H. Therefore these rays, in passing from the point S, will fall upon it in the same angle as the same rays will make with the surface LG, after diverging from the point T. Consequently these rays, making with the surface of the drop, the same angles both at the points of incidence and emergence, will be parallel after they are refracted into the air, as before their entrance into the drop. By these means, those rays will become efficacious, and enter the eye of the spectator dense enough to give him the idea of colour. But all other rays, whether they be incident on the drop nearer to, or farther from the axis of it, will not be refracted, so as afterwards to be reflected parallel to the rays before mentioned, and consequently upon emersion from the drop will neither be parallel to those rays nor to one another, and therefore contribute nothing to the formation of the bow; being deprived of their intermediate rays, and refracted into the air diverging, so as to make no sufficient impression on the eye, to produce the idea of distinct colours.

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* See Plate 9. fig. 1.

If a line were drawn from the sun through the eye of the spectator parallel to the incident ray, it would make an angle with the refracted ray, equal to the angle comprehended between the incident and refracted rays; and if the refracted ray were supposed to revolve round the said line, always preserving the same angle, its extremity would pass by many drops, in the revolution, all of which would exhibit the same colour to the eye; because the incident and emergent rays would still make the same angle: as we have already observed concerning the circular form of the primary bow. Hence, the drops which exhibit the different colours in this bow, must be disposed in the circumference of a circle, as they were in the other. But a different angle between the incident and emergent rays is required in this bow, from what is required in the other, for the exhibition of the same colour. It is found by computation, that the least angle, under which the red rays can come to the eye after two refractions and two reflexions, is 50° 57′, and the least angle under which the violet rays may come to the eye is 54° 7′. Whence all the colours of the exterior bow will be formed between these, viz. in the space of 3° 10′; if the sun were but a point. But as his diameter subtends an angle of 32′, the breadth of this bow must be increased by this quantity, which will make it 3° 42′: and by the same quantity must the distance between the bows be diminished for the same reason. So that the distance between the bows will be thereby reduced to 8° 23′.

As the exterior bow is formed by two refractions and two intermediate reflexions, it must be much fainter than the first; because a considerable part of every beam of light is lost by transmission through the drop at every reflexion: and as there are more reflexions in the production of the exterior bow than in the other, the colours must of consequence be fainter.

From the principles already laid down, we are able to account for the phenomena observable in the bows, the chief of which are such as follow:

1. That each is variegated with all the prismatic colours. This is a natural consequence of the different refrangibility of the rays of light. Were they all equally refrangible, they would all come to the eye without separation.

2. That two bows generally appear at the same time above one another in the heavens. These are formed by different drops at different altitudes; the first by the rays of light being incident on the upper

part of the drops, and being once reflected within them, and the other by rays which are incident upon the lower part of the drop, and suffer two reflexions within it, before they are refracted into the air. And as the angle between the incident and refracted rays is greater after two reflexions than after one, the bow formed after two reflexions must appear higher in the heavens than the other.

3. The inverse order of the colours of the bows is another phenomenon, which arises from the manner in which the rays are incident on the drops of falling rain, and the different parts of the drop from which they emerge. When the rays are incident upon the lower part of the drop, and emerge from the upper, the violet colour will appear in the upper part of the bow, as is the case of the exterior bow, because the violet rays being most refrangible, will be first refracted to the eye; but when they are incident on the upper part of the drop, and emerge from the lower, the red rays, as being least refrangible, will be first refracted to the eye, and the outer verge of this bow must appear red.

4. The diameters of the bows, their different breadths, and distance from each other, all depend upon the angles made by the incident and emergent rays, after one or more reflexions.

5. The altitude of the bow above the horizon, depends not only on the same angles, but also on the altitude of the sun at the same time. For if the center of the sun were 54° 23′ high, the outer verge of the exterior bow would be in the horizon, and if above that altitude, the bow would be entirely below the horizon, and at lesser altitudes, above it.

The bow will appear to rise higher and higher above the horizon as the sun descends, until it come to the horizon, when the bow will be a complete semicircle. So that the altitude of the bow is always inversely as the altitude of the sun; and no bow can appear either after the sun is set, or while he is higher than the semi-diameter of the bow.

6. There can no bow appear, but when the sun is shining, and at the same time it is raining on the opposite side of the spectator.

7. The bow generally begins to vanish at the middle of the arch, as the highest drops of rain in the cloud form that part of the bow, and in their descent, when there remain no more drops of sufficient altitude for that purpose, the bow must there vanish, while there are drops enough yet, on each side, to form the ends of the arch. Sometimes the cloud is not of sufficient extent to form both the horizontal ends of the arch at the same time, and therefore, but a part of the bow will appear.

It may be asked, why did not the bow appear in the cloud before the deluge? Did it never rain before that catastrophe? It might have rained when the sun was set, or when he was more than 54 degrees high, when no bow could be seen, and the rain might continue between the spectator and the sun, until the cloud were expended; or in any other direction but that of an opposition to the sun. And that superintending Providence, which designed to constitute the rainbow as a pledge of the assurance that he gave, that the world should never more be destroyed by water, might have prevented the concurrence of such circumstances in the time of rain, as were essentially necessary for the formation of a bow.

Having now explained the principal phenomena of the bows, we shall next lay down the mathematical principles, upon which the computation depends, for finding the dimensions of the bows, or the angle made by the incident and emergent rays, after any number of reflections within the drop.

PROPOSITION I.

The difference between the angle of incidence, and any multiple of the angle of refraction, that is greater by one, than the number of reflexions within the drop of rain, is equal to half the angle between the incident and emergent rays, when the number of reflexions is odd, or to half its supplement, when the number of reflexions is even.

Let ANFG* represent a drop of rain, SN a ray of light from the sun, incident on the drop at N, and refracted to F, and from thence reflected to G, where let it be either refracted into the air in the direction GE, or reflected to A, where let it be refracted in the direction AB, crossing the incident ray in R, after two reflexions, or in K, if it emerge after one reflexion, the incident and emergent rays being produced when necessary. When the number of reflexions is odd, a line drawn through the middle point of reflexion and the center, or through the center and middle of the middle chord, when the number of reflexions is even, will bisect the angle between the incident and emergent rays. Because the number of reflexions and refractions on each side of this line are equal. Draw a diameter parallel to the incident ray, viz. PCQ, parallel to SN.

When there is but one reflexion, at F, half the angle SKE, between the incident and emergent rays, viz.

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* See Plate 10, Fig. 1.

the angle SKC=PCM=NCM—NCP=CNF+CFN—NCP=twice the angle of refraction lessened by the angle of incidence NCP. Therefore, when there is but one reflexion, as in the formation of the interior bow, half the angle between the incident and emergent rays is equal to the difference between twice the angle of refraction and the angle of incidence.

Where there are two reflexions, at F and G, and the incident and emergent rays cross each other in R, making the angle ARN=BRS; let RV be drawn perpendicular to RCT, which bisects the angle between the incident and emergent rays, and it will be parallel to FG, and SRV=half the supplement of SRB=RVQ=VQT=CFQ+FCQ=CFQ+CKN=the angle of refraction CFQ+twice the angle of refraction—the angle of incidence=three times the angle of refraction, less the angle of incidence. Because NKC=FCQ=twice the angle of refraction—NCP, the angle of incidence; as we have already proved, in the former part of the proposition, where there is but one reflexion: That is, half the supplement of the angle made by the incident and emergent rays, after two reflexions, is equal to three times the angle of refraction—the angle of incidence. Q.E.D.

The same otherwise.

Produce RA* to E. Now SRE, the supplement of the angle between the incident and refracted rays, is equal to both the internal angles at E and K; but we before proved that the half of the angle at K was equal to twice the angle of refraction—the angle of incidence, and therefore the whole angle at K is equal to four times

* See Plate 10, Fig. 1.

the angle of refraction—twice the angle of incidence, and the angle at E is equal to the angle AGK—the angle EAG or FGK, that is equal to AGF, or twice the angle of refraction. Therefore, the supplement of the angle between the incident and emergent rays is equal to six times the angle of refraction less by twice the angle of incidence, and therefore half the supplement of the angle between the incident and emergent rays is equal to three times the angle of refraction less the angle of incidence.

In the same manner might it be demonstrated, that, after any number of reflexions, any multiple of the angle of refraction, which exceeds the number of reflexions by unity, when lessened by the angle of incidence, will be equal to half the angle, or half the supplement of the angle, between the incident and emergent rays, according as the number of reflexions is odd or even.

PROPOSITION II.

The nascent increments or fluxions of the angles of incidence and refraction are to one another as the number of reflexions increased by unity, to unity.

When there is but one reflexion, let SN* be a ray from the sun refracted to F, and from thence reflected to G: let also another ray AB infinitely near to SN, be refracted to the same point F, and reflected to H. The angle NCP, of incidence, is thereby enlarged by the small increment NCB, which, by 20. E. 3. is double of NFB, the increment of the angle of refraction CNF; which increments are generated in the same time.

Again, when there are two reflexions of the ray of light within the drop: let SN† be a ray from the sun

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[*] See Plate 10, Fig. 2.

[†] See Plate 10, Fig. 3.

twice reflected at F and G, and AB another ray twice reflected at L and H, and infinitely near to the former. This ray, because of its obliquity, must cross the former and be reflected parallel to it within the drop, as has been already explained. Now, the nascent increment of the angle of incidence NCP is NCB, which is three times as much as FCL, the increment of the angle of refraction CFN or CNF, generated in the same time. For, since the arches NF,FG are equal, and the arches BL,LH are equal also, from the nature of a circle, it follows, that NF—BL = NB—FL = FG—LH = 2FL. Therefore NB=3FL. That is, the cotemporary increment of the angle of incidence is three times as much as the increment of the angle of refraction, when there are two reflexions. Q.E.D.

PROPOSITION III.

The nascent increment or fluxion of an arch is to the nascent increment or fluxion of its sine, as radius is to its cosine.

Let there be an arch GD,* which is increased by DF, while the sine AD is increased by EF. Now, from the similarity of the triangles DFE and DCA, DF : EF : : DC : CA, that is, the nascent increment or fluxion of an arch is equal to the fluxion of its sine multiplied by radius, and divided by its cosine. Q.E.D.

By these three propositions we are able to compute the dimensions of the bows, by a very easy process, from the known ratio of the sines of the angles of incidence and refraction out of air into water, which are found for the different rays, to be as follow; putting I for the sine of the angle of incidence, and R for that of the angle of refraction.

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* See Plate 10, Fig. 4.

I : R : : 108 : 81. Extreme red
I : R : : $108\frac{1}{8}$: 81. Orange beginning
I : R : : $108\frac{1}{5}$: 81. Yellow beginning
I : R : : $108\frac{1}{3}$: 81. Green beginning
I : R : : $108\frac{1}{2}$: 81. Blue beginning
I : R : : $108\frac{2}{3}$: 81. Indigo beginning
I : R : : $108\frac{7}{9}$: 81. Violet beginning
I : R : : 109 : 81. Extreme violet.

PROBLEM.

To determine the apparent semidiameter of the bows from the given ratio of the sines of the angles of incidence and refraction, supposing the sun but a point.

Let x=the sine of the angle of incidence, and therefore $\sqrt{1-xx}$=its cosine, $r=1$; put the ratio of the sines of the angles of incidence and refraction, as $1 : p$, then $1 : p : : x : px$=the sine of the angle of refraction; and $\sqrt{1-p^2x^2}$= its cosine.

The fluxions of the arches or angles of incidence and refraction are, by Prop. 3,

$\frac{\dot{x}}{\sqrt{1-xx}}$, and $\frac{p\dot{x}}{\sqrt{1-p^2x^2}}$ respectively; but these fluxions, by Prop. 2, are to one another, as the number of reflexions more $1=n+1=m$ to 1, that is,

$$\frac{\dot{x}}{\sqrt{1-xx}} : \frac{p\dot{x}}{\sqrt{1-ppxx}} : : m : 1 : : \frac{1}{\sqrt{1-xx}} : \frac{p}{\sqrt{1-ppxx}};$$

and by squaring the terms we have $m^2 : 1^2 : : 1-ppxx : \overline{1-xx} \times p^2$ therefore $m^2p^2-m^2p^2xx=1-ppxx$; hence by transposition, $m^2p^2xx-ppxx=m^2p^2-1$. Therefore

$xx=\frac{m^2pp-1}{pp\times\overline{m^2-1}}$: that is, $xx=\frac{m^2-\frac{1}{p2}}{m^2-1}$ and $xx=\frac{4pp-1}{3p^2}$, for the primary bow, where $m=n+1=2$, and for the extreme red. $xx=\frac{9pp-1}{8pp}$ for the exterior bow, where $m=n+1=3$, and for the extreme red.

Now, when the sine of the angle of incidence is found, the sine of the angle of refraction is also given from their given ratio; and by Prop. 1, the difference between twice the angle of refraction and the angle of incidence, is equal to half the angle between the incident and emergent rays, or the semidiameter of the primary bow; and the difference between three times the angle of refraction and the angle of incidence is equal to half the supplement of the angle of incidence and emergence, in the exterior bow, or the semidiameter of the second bow. If we operate according to these rules for the rays of the different colours, we shall find the result agreeable to the dimensions of the bows already mentioned.

Cor. The tangent of the angle of incidence is to the tangent of the angle of refraction, as any number of reflexions in the drop of rain increased by unity is to unity; that is, in the same proportion as their fluxions.

By Prop. 2. $\frac{\dot{x}}{\sqrt{1-xx}} : \frac{p\dot{x}}{\sqrt{1-ppxx}} :: n+1 : 1$, and multiplying by x and dividing by $\dot{x}$, we have $\frac{x}{\sqrt{1-xx}} : \frac{px}{\sqrt{1-ppxx}} :: n+1 : 1$. But the sine divided by the cosine is equal to the tangent, when the radius is unity. Because the cosine is to the sine as radius is to the tangent. Therefore $\frac{x}{\sqrt{1-xx}}$ is the tangent of the angle of incidence, and $\frac{px}{\sqrt{1-ppxx}}$ is the tangent of the angle of refraction. Hence, the tangent of the angle of incidence is to the tangent of the angle of refraction as any number of reflexions in the drop of rain increased by unity is to unity. Q.E.D.*

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* This problem may be more elegantly resolved by using the tangent instead of the sine of the angle of incidence, in the following manner.

Let t=the tangent of the angle of incidence, and m=the number of reflexions+1. rad.=1.

As $m : 1 :: t : \frac{t}{m}$=the tangent of the angle of refraction. Then

$\frac{t}{\sqrt{1+tt}}$=sine of angle of incidence, and $\frac{\frac{t}{m}}{\sqrt{1+\frac{tt}{m^2}}}$=the sine of angle of refraction. These sines are as $1 : p$, as per substitution.

$$\frac{t}{\sqrt{1+tt}} : \frac{\frac{t}{m}}{\sqrt{1+\frac{tt}{m^2}}} :: 1 : p :: \frac{1}{\sqrt{1+tt}} : \left[\frac{1}{m\sqrt{1+\frac{tt}{m^2}}} = \right] \frac{1}{\sqrt{m^2+tt}}.$$

Hence $1 : p^2 : m^2+tt : 1+tt$, and $1+tt=p^2m^2+p^2tt$; $tt-p^2tt=p^2m^2-1$, and $tt=\frac{p^2m^2-1}{1-p^2}$=the square of the tangent of the angle of incidence.

In like manner, if we would operate for the secant of the angle of incidence, we should find $p\sqrt{\frac{m^2-1}{1-p^2}}=s$=the secant of the angle of incidence. If s=the secant of the angle, then $ss=1+\frac{p^2m^2-1}{1-p^2}s^2=\frac{p^2m^2-p^2}{1-p^2}$, and $s=p\sqrt{\frac{m^2-1}{1-p^2}}$,=secant of 59°, 23′, for the red rays of the primary bow.

PROPOSITION IV.

As the angle of incidence increases from nothing to 90°, the angle made by the incident and emergent rays will first increase and then decrease, and become greatest, when, after any given number of reflexions, called n, the tangent of the angle of incidence is to the tangent of the angle of refraction, as $n+1=m$ to 1, that is, as the number of reflexions increased by unity is to unity.

The angle between the incident and emergent rays is equal to m times the angle of refraction—the angle of incidence, and this will be a maximum, when its fluxion is equal to nothing; and retaining the notation in the foregoing problem, we have $\frac{mp\dot{x}}{\sqrt{1-ppxx}}-\frac{\dot{x}}{\sqrt{1-xx}}=o$, and by transposition, and multiplying

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That the increments of the angles of incidence and refraction are as the tangents of these angles, may be demonstrated, without fluxions, in the following manner.

Let ACD* be the angle of incidence, and ABD the angle of refraction, whose tangents are DE and DA, respectively, to the radius BD; BE being parallel to AC. Now, if we suppose the line AC to move about the point A, the end C will move the line BD into the situation BG, whereby the angle of refraction ABD will be increased by the small increment DBG, while the angle of incidence ACD is increased by the cotemporary increment DFG. Now, in such small angles, the angles are proportional to their sines: therefore DFG or its supplement BFD : DBG : : (BD : DF or DC : :) DE : DA, that is, the cotemporary increments of the angles of incidence and refraction are as their tangents respectively. Q.E.D.

Cor. Hence these tangents are as the number of reflexions increased by unity, to unity, by prop. 2.

* See Plate 11, fig. 1.

by x, and dividing by $\dot{x}$, we shall have, $\frac{x}{\sqrt{1-xx}} = \frac{mpx}{\sqrt{1-ppxx}}$, which converted into a proportion is, $\frac{x}{\sqrt{1-xx}}$ =the tangent of the angle of incidence is to $\frac{px}{\sqrt{1-ppxx}}$=the tangent of the angle of refraction, as $m=n+1$=the number of reflexions increased by unity, to unity.

Cor. As this equation is the same with what we had for computing the dimensions of the bows, it follows, that the efficacious rays alone make the angle between the incident and emergent rays a maximum.

The inefficacious rays, which do not make this angle a maximum, are not concerned in the production of the bows, for reasons before given; yet some of them may mix with some rays of the efficacious beam, which are irregularly reflected, and emerge parallel with them, although in a less angle with the incident rays, and thereby form coloured arches within the primary bow, as has been often observed. For Sir Isaac Newton has observed, that there is an irregular reflexion of light from a polished surface, whereby some rays are scattered from the principal beam. Suppose B* were the point of reflexion in a drop of rain, from whence the efficacious rays of any colour were regularly reflected to G, where they emerged, making the largest angle with the incident ray; then, besides these, there are some rays of this beam irregularly reflected to E and D, and to points between these and the point G, and emerging in the lines EK and DI which will make with the incident ray a less angle, and therefore, if sufficiently dense, they may form coloured arches within the primary bow, but they must be fainter than the colours of the bow, because but few rays are thus irregularly reflected in comparison with such as are regularly reflected. These arches will be variously mixed with the colours of the bow, and some reach below it; and thereby alter their colours a little.

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* See Plate 11, fig. 2.

THE REFLEXION OF LIGHT,

BY PLANE SURFACES.

THE surfaces may be either *plane*, *concave*, or *convex*, which reflect the rays of light. The whole doctrine of reflexion depends upon this fundamental principle, which we have already demonstrated, viz. That the angle of incidence is equal to the angle of reflexion; and when any point of an object is seen by reflected light, it is always seen in the direction of the reflected ray, and at its intersection with the perpendicular drawn to the reflecting surface from the object.

In a *plane mirror*, rays diverging from any point of an object are reflected back diverging, as if they had proceeded from an imaginary radiant point in the perpendicular, as far behind the mirror as the object is before it. Let EBC* be a plane mirror, AED, HB, IC, perpendicular to its surface, A, any point of an object, from which rays diverge and fall upon the mirror at B and C; these rays will be reflected back diverging from the point D, in the directions BF and CG, making ED=AE. Because ABH=FBH=BDE, and the angles at E, are right, and EB is common to both triangles AEB, DEB, therefore AE=ED. Hence DBF is the reflected ray. In the same manner DCG will be the reflected ray, when incident on the surface at C from the same point A. And as both the rays BF and CG are reflected from the same point D, this point will be the focus of the rays, which diverge from A: and it is in the perpendicular to the surface of the speculum, as far behind it as A is before it.

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* See Plate 11, fig. 3.

Hence, as this is true of every point of the object, it must necessarily follow, that the image will be similar to, and of equal magnitude with the object. It follows also, from the same proposition, that the distance of the image from the eye will be equal to the sum of the incident and reflected rays.

If the object be parallel to the mirror, it is not necessary that the mirror should be more than half of the length and breadth of the object, to represent the whole of the image, to an eye at the place of the object. Yet when the mirror is so short, that we cannot see the whole of the object, we may nevertheless see it completely, either by bringing the eye nearer to the mirror, or by removing the object farther from it. Yet this change of position will be of no advantage to assist a person in seeing the whole of his own figure in a mirror, which is shorter than one half of his own length; because as the eye approaches to the mirror, the image approaches to it at the same time, whereas it should continue in the same place, to give this advantage. So that at all distances he will see an equal portion of himself.

The whole of the object AC* may be seen by an eye at either extremity of it, in a mirror EF, which is half its length. But if the eye approach the mirror to I, while the object continues stationary, it may be seen, although it extended to G.

In a plane mirror, that, which is on the right hand of the object appears on the left hand of the image, and vice versa, because the face of the image is turned the contrary way from that of the object. While a person moves his right hand, before the glass, his image appears to move its left; and all the letters of a book held before it, will, for the same reason, be reversed, but not inverted, as the image is always erect, behind such a speculum, or in a positive focus.

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* See Plate 11, fig. 4.

But if a plane mirror be held parallel to the horizon, any object that is perpendicular to it, will have its image inverted. This is seen in the inverted images of trees and buildings on the bank of a river, because the roots being nearer to the reflecting surface, must have their images below it nearer to the surface, than those of the tops. In like manner, if any person hold the speculum above his head parallel to the horizon, he will see his image inverted above him. But if he incline it in an angle of 45° to the horizon, his image will be parallel to the horizon, and the images of all objects, that are parallel to the horizon, will appear erect. Hence the image of any object, that is stationary, will appear, by the motion of the speculum, to move over double the space through which the speculum moves, whether the motion be angularly or in a right line.

If an object be placed between two plane mirrors, inclined to each other in any angle, a number of images will be formed by them, which will be placed in the circumference of a circle, whose center is the point of concurrence of the planes, and whose radius is the distance of the object from the said point. One set of these images, formed first by one of the specula, will exhibit one side of the object, and the other set will exhibit the other side of it.* The first image formed by one mirror will become the object, from whence a second image will be formed by the other mirror, and as far behind it as the other image was before it. This second image will also become the object, from whence a third image will be formed by the first mirror, and so on, till this set of images, which began from the first mirror shall end. A second set of images, beginning from the second mirror, and continued alternately between them, will be formed at the same time.

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* See Plate 12, fig. 1. O the object, N, P, the two first images.

The number of images will be determinate; for whenever the place of the last image of either set falls between the planes of both the mirrors, produced beyond their concurrence, no more images can be formed from it considered as an object; because it is then behind the reflecting surfaces of both the mirrors, and no rays from it can fall upon them.

That these images will be all formed in the circumference of a circle may be proved geometrically from this consideration, that each image is as far behind the mirror by which it is formed, as the object is before it, and in a line drawn perpendicular to the plane of the mirror from the object to the image. Now as this line is bisected at right angles by the plane of the mirror, the plane must pass through the center of a circle, whose circumference passes through the places of the object and image.

The angular distanceof the two first images, formed by the two mirrors, is equal to twice the inclination of the mirror. Because each image being as far behind the mirror that formed it, as the object is before it, the sum of the distances of the object from the mirrors, or the inclination of the mirrors, can be but half the distances of the images.

The angular distance of the next two images, formed by two reflexions each, exceeds the angular distance of the first two, by twice the angle of inclination. And each succeeding pair of images, formed by an equal number of reflexions, will exceed the angular distance of the prceding pair by the same quantity, viz. twice the angle of inclination.

If KL, KM,* be two mirrors, and O, an object between them, there will be one set of images formed at A, B, C, D, E, and another set at a, b, c, d, e. Now the angular distance of the two first images, A, a, viz. AKa, is equal to 2 OKL+2 OKM=2 LKM= twice the inclination of the mirrors. And the angular distance BKb, of the two succeeding images, exceeds the angular distance AKa, of the first two images, by the angles BKa+AKb=AKO+aKO=2LKM.

Hence as the distance between each succeeding pair of images increases by double the angle of inclination of the planes, there must be as many images formed by the two mirrors, as there are angles in the circle equal to the angle of inclination. Thus, if the angle of inclination were 10°, 20°, 30°, 40°, or 45°, the number of images would be 36, 18, 12, 9, or 8 respectively. And if the double angle of inclination were commensurate to a whole circle, the two last images would coincide and be formed in the same place, whereby one of them would disappear.

As all the images are formed in the circumference of a circle, if the radius of the circle were increased ad infinitum, or the angle of inclination between the mirrors were diminished ad infinitum, so that the planes would become parallel, the number of images would become infinite, and all be placed in a right line.

The angular distance between the object and either of the second pair of images is double to the inclination of the mirrors: that is, OKB, or $OKb=2LKM$. For $OKB=OKa+aKB=OKa+OKA=aKA=2LKM=OKA+AKb=OKA+OKa=2LKM$.

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* See Plate 12, fig. 2.

Upon this principle is the common reflecting quadrant for measuring angles constructed, so that the angular distance between any object and its image formed by two reflexions, is always double to the inclination of the mirrors. This may also be elegantly demonstrated, in another way, independent of what has been already proved, in the following manner.

Let DCK* represent the quadrant, with a movable index CX, which carries with it the central speculum C, while the small speculum F remains fixed on the radius of the instrument, and parallel to the central speculum, when the index coincides with CD, the other radius; the beginning of the graduations on the arch being at D. Let HQ and FQ be perpendicular to these specula respectively, so that the angle HQF will show their inclination equal to the angle DCX. Now let a ray of light from the object at S fall upon the mirror at C, and be reflected to F, and from thence be again reflected to the eye at I. The image of the object S will therefore be seen in the line IFO, parallel to HCQ. Now SC being produced to I, the angular distance SIO of the object and its image$=SCH=HCF=CFI=2QFI=2FQH=$twice the angle of inclination between the mirrors. So that by the motion of the index over any arch, the image of the object moves at the same time over twice the angular distance. Hence the arch of the instrument is divided into half degrees, which are counted as whole degrees, to give the angle at sight.

In every position of the instrument, in the plane of the objects, whose angular distance is to be measured, the angle at I remains the same, and consequently the reflected image will be stationary, which makes the instrument of such inestimable value at sea. For although the motion of the ship will cause a motion in the image reflected from the central mirror; yet as the second mirror fixed on the radius of the instrument, which reflects the image to the eye, will communicate to it an equal motion in an opposite direction, it will, by this means, destroy the effect of the first motion, and consequently the image will appear to be quiescent: so that the motión of the vessel, so fatal to observations taken with other instruments, will not in the least affect such as are taken with this. There are many advantages attending the use of this instrument above any others, besides that already mentioned; such as, that it will give the altitude of the sun or of a star, or the distance of the moon from them, generally, to less than a single minute; and that, with respect to the sun, when it does not even shine so bright as to project a shadow.

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* See Plate 12, fig. 3.

OF THE REFRACTION OF RAYS
FROM SPHERICAL SURFACES.

The fundamental law, that the angles of incidence and reflexion, are equal, obtains in spherical surfaces, whether concave or convex, as well as in plane mirrors.

Parallel rays, that are incident near the vertex, are reflected by a concave mirror, converging to a focus, in the axis, half way between the vertex and center of concavity; and by a convex mirror, diverging, as if they came from a point in the axis, at the same distance from the center or vertex. Because the angle of incidence is equal to the angle of reflexion, ABC=CBF =BCF.* Therefore CF=FB=FV, when B and V coincide. The point F is therefore called the principal focus, or focus of parallel rays.

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* See Plate 13, Fig. 1.

Hence if rays diverge from the principal focus of a concave mirror, or converge to the principal focus of a convex mirror, they will be reflected parallel to the axis. If they diverge from the center of a concave mirror, or converge to the center of a convex mirror, they will be reflected back in the same line. In a concave mirror, if they diverge from a point between the focus and center, they will be reflected converging to the axis beyond the center, and if they diverge from a point beyond the center, they will be reflected back converging to a point in the axis between the center and principal focus. But if they diverge from a point between the vertex and principal focus, they will be reflected back diverging from the axis, and vice versa. Converging rays are reflected converging to a point in the axis between the vertex and principal focus. What is here said of diverging rays falling upon a concave mirror, is also true of converging rays falling on a convex mirror, and vice versa.—All evident from the equality of the angles of incidence and reflexion.

Diverging rays falling near to the vertex of a spherical mirror, if they diverge from a point more distant from the vertex than the principal focus, will be collected into a focus, or diverge as if they came from a focus, at such a distance from the principal focus, that this distance will be a third proportional to the distance of the radiant point from the principal focus, and the principal focal distance. That is, if RB* be a diverging ray falling on the spherical surface at B near the axis CV, and reflected back to f in the concave mirror, or as if it came from f in the convex mirror, then it will be RF : FC : : FC : Ff.

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* See Plate 13, Fig. 2 and 3.

Let CD, CE, be drawn parallel to the incident and reflected rays, respectively, and cutting them in the points D and E. Then CEBD will be a parallelogram, of which all the sides in its evanescent state, as B approaches to V, will be equal to each other and to half the diagonal or to FC. Then because the triangles REC, RBf are similar, it will be as $RE=RF : EC=FC :: RB=RV : Bf=Vf$.

Cor. As $RF : RF\mp FC=RV :: FC=VF : FC\mp Ff=Vf$. That is, in words at length: If rays diverge from, or converge to, a radiant point, and fall upon a concave mirror, they will be reflected to a focus in the axis, which may be found by the following proportion.

As the difference or sum, of the distance of the radiant point from the vertex, and principal focal distance, is to the distance of the radiant point, so is the focal distance of parallel rays, to the focal distance of the diverging or converging rays from the vertex. If D=the distance of the radiant point, F=the focal distance of parallel rays, and f=the focal distance of the diverging or converging rays, then $\frac{DF}{D\pm F}=f$; or, $D\pm F : D :: F : f$.

In the first term of the above proportion, for diverging rays falling on a concave mirror, or for converging rays falling upon a convex mirror, use the difference between the distance of the radiant point and principal focal distance; and use their sum for converging rays falling upon a concave mirror, and also for diverging rays falling on a convex mirror.

Hence we see the reason why a convex mirror always has a positive focus; because the first term of the above proportion is always a positive quantity, excepting when both the first and second terms, (D=the distance of the radiant point being negative) are negative, in which case, the fourth term must also be positive.

We see also the reason why a concave mirror may sometimes have a negative, and sometimes a positive, focus. When D, or the distance of the radiant point, is positive and less than the principal focal distance, then the focus of diverging rays will be negative.

As any ray passing through the center of a mirror is reflected back in the same line, as being perpendicular to the surface, and any other ray is reflected so as to cross the perpendicular in the focus, whether that ray diverged from or converged to any point in the perpendicular; and as a ray may be drawn through the center from any point of an object, and other rays diverging from the same point or converging to it, will be reflected to some point as their focus in the said perpendicular; the image of any point of an object will be found in the perpendicular from that point. Hence it is easy to understand, that if the object be so small or so far distant from the reflecting surface or center, that all its points are nearly at equal distances from it, all the corresponding points of the image will be nearly at other equal distances from the said surface or center. As the object and image are both terminated by the perpendiculars, which cross each other in the center, they must be both seen under the same angle from the center, and therefore their magnitudes must be in the same ratio to their distances from the center. They are also seen under the same angle from the vertex of the mirror, and consequently their magnitudes are also in the ratio of their distances from the vertex.

In all spherical mirrors, the image is erect, when it is on the same side of the center with the object, but inverted when they are on different sides. Because the image being always in the perpendicular drawn through the correspondent point of the object to the reflecting surface, and the perpendiculars crossing each other only in the center; it must follow, that if both the object and image be on the same side of the center, the image will be erect, but if on different sides, inverted.

Hence we have an easy way of determining the place, position, and magnitude of an image formed by any spherical surface, by drawing lines from the extremities of the object both through the center and the vertex, and producing them both ways, and their intersections will geometrically determine the place and magnitude of the image.

In a concave mirror, as the object and image are viewed under the same angle at the center and vertex, if the eye be close to the mirror, the apparent diameters of the object and image will be the same. If the object touch a concave mirror, the image will also touch it, and both will be equal in magnitude. If the object be nearer to the mirror than its principal focus, but does not touch it, the image is erect, distinct and behind the mirror, greater and more distant than the object: but decreases as the eye departs.

If the object be in the principal focus of a concave mirror, the image is at an infinite distance behind the mirror, magnified, erect, and distinct. As the rays are in this case reflected parallel, the place of the image will be at an infinite distance, and on account of its remoteness it will appear magnified, and as the perpendiculars do not intersect each other, it will be erect, and because of the parallelism of the rays, it will be seen as distinctly as distant objects are usually seen: and in this case, the apparent magnitude of the image is not altered by the motion of the eye.

If the object be farther from the vertex of a concave mirror than the principal focus, and the eye nearer than the place of the distinct image, the image will be behind the mirror, erect, magnified, and confused, but if the eye be more remote than the place of the distinct picture, the image is before the mirror, distinct and inverted; and is increased in magnitude as the eye departs, provided that it does not depart beyond the place of the distinct image.

When the image appears before a concave mirror, if the object be nearer to the mirror than the center, the image will be larger, if more remote, less, and if in the center, equal to the object; and if the eye move one way, the image moves the contrary way; but if the mirror move, the image moves the same way.

If the eye be in the center of a concave mirror, it can see nothing but its own image, because no rays can come to it but what diverged from itself; its image must therefore appear diffused over the whole mirror, and be infinitely magnified and confused.

When the eye is in the principal focus of a concave mirror, the apparent magnitude of an image is not altered by the departure of the object; but when the eye is nearer to the mirror, the image decreases with the removal of the object, but increases, when the eye is more distant, provided it be not removed so far as to bring the image between the eye and the mirror.

The image of an object seen by a *convex* mirror is always behind the mirror, erect, distinct, less than the object, and nearer to the mirror than the object; and when either the eye or object recedes from the mirror, the apparent diameter decreases, and is never equal to the object but when it touches the mirror. So that the object and image approach towards, and recede from, the mirror at the same time. If either the object or mirror move, the image moves the same way: but if the eye move, the image seems to move the contrary way.

From repeated experiments it is found that the best composition for the most copious reflexion of light is made in the following manner. Let the whole mass be divided into fifty equal parts, then 32 must be of copper, 15 of tin, and of silver, brass, and arsenic, each one part. This metal, when broken, should appear of a bright, glassy, and quick-silver complexion. But if it appear of a dead white, more tin must be added: so that 32 ounces of copper will require 16 ounces of tin, if it be very pure. But if the metal when broken appear bluish and rough, more copper or brass must be added.

CAUSTICS BY REFLEXION.

What we have hitherto said concerning the focus of rays reflected from a spherical surface, only respects the rays that are incident near to the axis of the mirror; because those rays that are reflected from more distant parts are collected into different points in the axis, nearer to the vertex than the principal focus. The intersections of these reflected rays, with each other, before they reach the axis, form luminous curves which are denominated *caustics*, which are convex towards the axis, and have their cusps in the principal focus. They are easily seen on the surface of milk or any white fluid in a tea-cup, when the light of the candle is reflected from the opposite side. The more distant that any ray is from the axis, the nearer to the vertex will its reflected part intersect the axis, and consequently as a ray nearer to the axis is reflected to a more distant point, it must intersect the more distant ray, when reflected from the spherical surface. In like manner, the rays that are incident on the mirror, still nearer to the vertex, cross each other, still farther from the mirror and nearer to the axis, in different points, until they finally concur in the principal focus. All these points of intersection of contiguous rays form the caustic curves.

As the reflected parts of any two contiguous rays, on each side of the axis, must intersect each other at different distances both from the axis and vertex of the mirror, there will be two caustics formed at the same time, one on each side of the axis, whose cuspides unite in the principal focus.

REFRACTION OF LIGHT
THROUGH PLANE AND SPHERICAL SURFACES.

When the rays of light fall perpendicularly on any medium, they suffer no change in their direction, but only in their velocity, as we have demonstrated before; but when they fall obliquely on any surface, they will be refracted towards or from the perpendicular, according as the medium, into which they enter, is denser or rarer than that through which they moved before their incidence upon it.

When parallel rays fall obliquely on a plane surface of a medium of different density, they will be parallel after refraction; for, as they have the same inclination to the surface, they will all suffer an equal degree of refraction, being rays of the same kind.

If they diverge before their incidence on the plane surface of a denser medium, they will be made to diverge less; and if they converge, to converge less: but when they are incident upon the plane surface of a rarer medium, diverging rays are made to diverge more, and converging rays to converge more; because, in the first case, they are refracted towards, and in the second case, from the perpendiculars.

Parallel rays, entering a *denser* medium through a *convex surface*, are made to converge; as the perpendiculars, to which they are refracted, meet in the center of convexity. But if they enter through a *concave surface*, they are made to diverge; because the perpendiculars, to which they are refracted, diverge in the denser medium from a point in the rarer medium. But being refracted the contrary way or from the perpendiculars, when they enter a *rarer* medium, either through a convex or concave surface of the rarer, they are made to diverge or converge according as the surface of the rarer medium is convex or concave.

Diverging rays, entering into a denser medium through a convex surface of the denser medium, or a concave surface of the rarer, are made to converge, be parallel, or to diverge less, according to the degree of their divergency. If they diverge from the focal distance of parallel rays when they enter the convex surface of the denser medium, they will proceed parallel after refraction, the medium having the same refractive power, which way soever the rays proceed; and if they diverge from a point nearer to the denser medium, they will proceed diverging but in a less degree; but if they diverge from a more distant point, they will be made to converge. If they diverge from the center of concavity of the rarer medium, they fall in with the perpendiculars and suffer no refraction; if from a point nearer to the rarer medium, being refracted from the perpendicular, they are made to diverge more; but if from a point more distant than the center of concavity, they are made to diverge less, to be parallel, or to converge. If they diverge from a point, which would be the focal distance of parallel rays entering the convex surface of the denser medium, they will be parallel after refraction in the rarer medium, and if from a point more or less distant, than the said focal distance, they will converge or diverge accordingly.

If diverging rays fall upon a concave surface of the denser medium, and diverge from the center of concavity, they fall in with the perpendiculars and suffer no refraction; but if they diverge from a point nearer to the surface, they are made to diverge less, being refracted towards the perpendiculars; and if they diverge from a point more distant than the center of concavity, they are made to diverge more. If diverging rays fall on the convex surface of a rarer medium, they are made to diverge more, being refracted from the perpendiculars.

If converging rays fall upon a convex surface of a denser medium, and if they converge to the center of convexity, they fall in with the perpendiculars and suffer no refraction: if they converge to a point nearer to the surface, they are made less converging, and if they converge to a point more distant than the center, they are made more converging. If converging rays fall upon the convex surface of a rarer medium, and if they converge to the center, they also fall in with the perpendiculars and suffer no refraction. But if they converge to a point nearer to the surface, they are rendered more converging, and if to a point more distant than the center, they are rendered less converging. The reason, in both cases, is, because the refracted rays are bent towards the perpendiculars in the denser medium, and from them in the rarer.

If converging rays fall upon a concave surface of the denser medium, they may be made to be less converging, parallel, or even diverging. But converging rays falling on the concave surface of a rarer medium are made more converging.

From what has been said, it is easy to understand the progress of the rays of light, after passing through lenses of every kind; whose density is always supposed to be different from that of the medium with which they are surrounded. And we may deduce this observation, in general, from what is before laid down; that when diverging rays are made to converge, by passing through a lens, the nearer the radiant point is to the lens, the farther is the focus from it, and vice versa; and when rays diverge from a point as far distant as the focus of parallel rays, they proceed parallel, as the lens has the same refractive power, which way soever the rays proceed. And if rays diverge after refraction, their focus is imaginary, and on the same side with the radiant point.

There is a certain point belonging to every lens, through which if a ray pass, it will proceed parallel to the direction it had before its incidence on the lens. This is called the center of the lens. It is any where in a transparent medium bounded by parallel surfaces; it is in the vertex of the single convex or concave lenses, within the double convex and concave lenses, and removed a little out of the meniscus, and nearer to the surface that has the greater curvity.

That a ray, which passes through the center of a lens, or during its passage through a meniscus tends to its center, is refracted in a direction parallel to that in which it was incident is thus demonstrated.

Let R, r,* be the centers of curvity of the sides of the lens, from which draw two parallel radii to the incident and emergent rays, and join the centers by the line Rr, intersecting the passing ray, produced, if necessary, in the point E. Now E is the center of the lens. And because the triangles REA, rEB are similar, the angle RAE=rBE, RE, rE, will be in the constant ratio of the radii, so that the point E will be constant in the same lens; and as the incident and emergent rays have the same inclination to the perpendiculars, they must be equally refracted in contrary directions at each surface of the lens, and therefore they must be parallel to each other.

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* Plate 13, fig. 4, 5, 6.

THE FOCUS OF RAYS BY REFRACTION.

THE focus of parallel rays, refracted through a plano-convex or plano-concave lens, if they fall near to the vertex, is at the distance of twice the radius of the sphere, to which the lens is ground: which may thus be demonstrated.

It was before observed, that when the refraction is made out of glass into air, the sine of the angle of incidence is to that of refraction as 2 : 3. Now let C* be the center of convexity or concavity of the lens, and CF the axis of the lens, parallel to which let the ray AB be incident on the lens near to the vertex V, and the refracted ray will be in the direction BF, and F will be the focus, either real or imaginary, and VF will be=2CV. For since DBE=ABC=BCF=the angle of incidence, is to DBF, the angle of refraction, nearly as their sines, in such small angles, that is, as 2: 3, their difference EBF=BFC=1, will be to the angle of incidence BCF as 1 : 2, and their opposite sides CB, and BF are in the same proportion, as, 1:2, that is as $CV : VF$, when the point B approaches near to V; therefore $VF=2CV$.

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* Plate 13, fig. 7. and Plate 14, fig. 1.

Hence, if the lens were equally convex or concave on both sides, the focal distance of parallel rays would be only one half of what is above determined, or at the distance of the radius of the sphere to which the lens was ground. Because the rays would now suffer an equal degree of refraction at each side, or twice as much as if it were plane on one side. This is called the principal focus of parallel rays.

Hence, also, if rays diverge from the principal focus of parallel rays, they will be refracted parallel to the axis of the convex lens. The same also will be the direction of the refracted ray, after passing through a concave lens, when the incident ray converges to the focus of parallel rays. Because the lens has the same refractive power, which way soever the incident ray comes.

If rays diverge from a radiant point and be refracted by a convex lens, they will converge to a point in the axis more distant than the focus of parallel rays. The distance of this point from the lens, called the focus of diverging rays, is found by saying, as the difference between the distance of the radiant point and focal distance of parallel rays is to the distance of the radiant point, so is the focal distance of parallel rays to the focal distance of diverging rays. But if the rays converge to a radiant point, the first term of the proportion is the sum of the distance of the radiant point, and focal distance of parallel rays. That is if D=the distance of radiant, F=focal distance of parallel rays, and f=the focal distance of diverging or converging rays. $D \pm F : D :: F : f$.

Let AC* be a lens, whose focal distance for parallel rays is CF, let this distance be set on both sides of the lens, and perpendiculars be erected from it to intersect the incident and emergent rays, and from the points of intersection, let lines be drawn through the center of the lens, which will be respectively parallel to the incident and refracted rays. Because if two rays, GC, GA diverge from the focus of parallel rays, and one of them GC pass through the center of the lens, the other Af will be refracted parallel to it, the first suffering no refraction. Hence the triangles RGC, RAf, will be similar, and therefore RG : GC : : RA : Af, that is, for diverging rays passing through a convex lens, D—F : F : : D : f, and for converging rays, D+F : F : : D : f. The imaginary focal distance of converging rays passing through a concave lens is the same with that of diverging rays passing through a convex lens, viz. D—F : F : : D : f. and of diverging rays passing through a concave lens, the same with that of converging rays passing through a convex lens, viz. D+F : F : : D : f.

Hence, since the focus of parallel rays, refracted through a plano-convex lens or a plano-concave, is at the distance of twice the radius of the sphere=2r; this may be substituted in the above formulas instead of F. And since F=r in double convex or concave lenses, of equal curvity on both sides, the above theorems will become $\frac{2Dr}{D\pm 2r}$=f. and $\frac{Dr}{D\pm r}$=f, for single or double lenses of equal curvity on both sides, respectively; the lower signs belonging to convex lenses, and diverging rays, or converging rays and concave lenses; and the upper signs for converging rays and convex lenses, or diverging rays and concave lenses.

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* Plate 14, fig. 2, 3, 4, 5.

From what has been demonstrated we can easily determine the focal distance of rays, passing through any lens whose radii are different, or any two lenses of different kinds, and different focal distances, by only attending to the inclination of the rays after any refraction. Let it be required to find the focal distance of diverging rays falling upon a lens, whose radii are r and R, retaining the notation above. If the first surface be convex and its radius=r, and the distance of the radiant point D, the focal distance will be as above, $f=\frac{2Dr}{D-2r}$. Now, this focal distance becomes the radiant point to which rays after the first refraction converge, and the theorem for converging rays is $\frac{2fR}{f+2R}=F$=the focal distance of the rays after two refractions. Hence by expunging f, by the common rules of algebra, we have $\frac{2DRr}{DR+Dr-2Rr}=F$=the focal distance of rays passing through a double convex lens whose radii are R and r respectively.

This universal theorem will determine the focal distance of rays, passing through any kind of lenses, by only changing the signs of the expressions, when either of the radii lies on the contrary side, to what is supposed in the theorem; as in the following examples.

If D be infinite, or the rays parallel before refraction, then the expression $-2Rr$ may be left out, as it neither increases nor diminishes the other expressions that involve the infinite quantity D, and then the expression will become $\frac{2DRr}{DR+Dr}=F=\frac{2Rr}{R+r}$; that is, twice the rectangle of the radii divided by their sum will give the focal distance of parallel rays.

If both the radii be equal, then $F=R=r$; that is, parallel rays are converged to a focus by a double and equally convex lens, at the distance of the radius, as we have seen before. If one of the radii as R be infinite, the glass becomes a plano-convex with the convex side turned to the incident ray, and the general expression, $\frac{2DRr}{DR+Dr-2Rr}=F$, becomes, $\frac{2DRr}{DR-2Rr}=F=\frac{2Dr}{D-2r}$: and if the other radius r be supposed infinite, the lens will still be a plano-convex, but having its plane side turned to the incident ray, and then $\frac{2Dr}{D-2R}=F$: so that either side of the lens being turned to the incident ray makes no alteration in the focus. These expressions for the focal length of a plano-convex lens are the same we had before. If D be infinite, then, $\frac{2Dr}{D-2r}=F$, becomes $\frac{2Dr}{D}=2r=F$, or the focal distance of parallel rays passing through a plano-convex lens is equal to twice the radius, as we have seen before.

If both the radii be infinite, that is, when the lens becomes a plane, with parallel surfaces, then the general expression, $\frac{2DRr}{DR+Dr-2Rr}=F$, becomes $\frac{2DRr}{-2Rr}=F$, or $D=-F$; that is, diverging rays proceed diverging after refraction from a negative focus at the same distance with the radiant point, and consequently go parallel to their direction before refraction. And if D were also infinite, or the incident rays parallel, they would be parallel after refraction, as the focus would be at an infinite distance.

If one radius r be infinite, and the other radius R be negative, the lens becomes a plano-concave, with the plane side turned to the incident ray; then by changing the sine of R in the general theorem, we have $\frac{-2DRr}{2Rr+Dr}=F$: and if D be infinite, then $\frac{-DR}{D}=-2R=F$, that is parallel rays diverge from an imaginary focus at twice the distance of the radius of concavity.

If both the radii be negative, the lens becomes a double concave, and the general theorem is $\frac{2DRr}{-DR-Dr-2Rr}=F$, in which, if D become infinite, or the rays parallel, $\frac{2Rr}{-R-r}=F$. That is, the focus will be negative, and parallel rays will diverge, as if they came from a point, whose distance from the lens is found by dividing twice the rectangle of the radii by their sum. And if the radii be equal, R or $r=-F$, that is, parallel rays are refracted through an equally concave lens to a focus, whose distance is equal to the radius of concavity.

If one of the radii, as R, be negative, the lens becomes a meniscus, and when D is infinite or the incident rays are parallel, the general theorem will become $\frac{-2Rr}{r-R}=F$; which shows that the focus will be either negative or positive according as R or r is the lesser or greater: and when $R=r$, the focus is infinite, and the rays proceed parallel after refraction, as is the case of a watch-crystal.

If the radii be equal to each other, the lens becomes a double and equally convex lens, and then the general theorem, $\frac{2\mathrm{DRr}}{\mathrm{DR}+\mathrm{Dr}-2\mathrm{Rr}}=\mathrm{F}$, becomes $\frac{\mathrm{DR}}{\mathrm{D}-\mathrm{R}}=\mathrm{F}$ =the focal distance of diverging rays through a double and equally convex lens. And if D be infinite $\mathrm{R}=\mathrm{F}$, as before.

If both the radii be negative, the lens becomes a double concave, and the general expression is, $\frac{2\mathrm{DRr}}{-\mathrm{DR}-\mathrm{Dr}-2\mathrm{Rr}}=\mathrm{F}$, in which if the radii be equal, $\frac{\mathrm{DR}}{-\mathrm{D}-\mathrm{R}}=\mathrm{F}$, that is, the focus becomes negative, and when D is infinite, $\mathrm{R}=-\mathrm{F}$, as we have seen before.

From the same theorems we can find the focal distance of rays refracted through two or more lenses, whether they be placed close together, or at a distance from each other. For example, let the first lens be convex, and its focal distance be F, determined by any of the preceding theorems; let the second lens also be convex, and its principal focal distance be F; let the distance between them be x, and the focal distance required be f. Now, since the rays converge after passing through the first lens, they will tend to a point in the axis, whose distance from the second lens is $\mathrm{F}-\mathrm{x}$, which is therefore to be considered as the radiant point, to which the converging rays tend when incident on the second lens, and is=D, in the theorem for converging rays, viz. $\frac{\mathrm{D}F}{\mathrm{D}+F}=\mathrm{f}$. Therefore by substituting $\mathrm{F}-\mathrm{x}$ instead of D, we have $\frac{\overline{\mathrm{F}-\mathrm{x}}\times F}{\mathrm{F}-\mathrm{x}+F}=\mathrm{f}$, when both lenses are convex. But since the imaginary focus of converging rays passing through a concave lens is $\frac{\mathrm{D}F}{\mathrm{D}-F}=\mathrm{f}$, when the second lens is concave, we have

$\frac{\overline{F-x}\times \mathit{F}}{F-x-\mathit{F}}$=f. Hence if x be nothing, the theorem becomes $\frac{F\mathit{F}}{F\mp \mathit{F}}$=f, in which the lower sign is to be used, when both lenses are convex, but the upper, when the second lens is concave.

Since the focal distance of a convex lens, whose radii are the same, is $\frac{DF}{D-F}$=f, we may deduce the following observations. If the distance of the radiant point, or object, be greater than the focus of parallel rays, they will be made to converge by a convex lens, but converging rays will be made to diverge by a concave lens, when they converged before; and the focal distance of the convex lens will be affirmative, or on the opposite side of the lens from the object. But if the distance of the radiant point be less than the principal focal distance, the focus of these diverging rays will be on the same side with the object, or negative, and the rays will continue to diverge after passing through the convex lens, but they will in these circumstances converge after passing through a concave lens, though in a less degree than before. If the distance of the radiant be equal to the distance of the principal focus, the rays will proceed parallel, after refraction, and the focus of diverging rays will be at an infinite distance.

Hence we have the contrivance of the magic lantern, in which the rays diverging from the object placed at the focal distance of the lens, proceed parallel after passing through the lens, and by means of a candle, the image is illuminated at a distance. If the distance of the object be equal to twice the focal distance of parallel rays, the focus of rays diverging from it and passing through a double convex lens will be at the same distance as the object, and the image formed there will be of the same magnitude, but inverted. And hence the advantage of such lenses to painters, and such as desire the images they would copy, as large as the objects.

CAUSTICS BY REFRACTION.

In all that we have hitherto said concerning the focus of rays, we have confined ourselves to such as fall near to the vertex of the lens or to its axis. More distant rays are united in different points in the axis, still nearer to the vertex, the more distant that the point of incidence is from it. The more diverging rays will successively intersect the less diverging, in different points before they arrive at the axis; and these points of intersection, gradually diverging from the principal focus, will form luminous curves, denominated *caustics* by refraction. When the light of a candle is refracted through a globe or decanter on the table, and falls upon a piece of white paper laid parallel to the axis, the luminous space which receives the refracted rays will be bounded by two bright curves, one on each side of the axis; which accede towards each other as they approach to the principal focus, where they form a sharp point. As these curves are formed by the successive intersections of every ray with the next to it in order from the axis to the outermost, the brightness of the paper within the curves, and its darkness without them, is caused by the multiplicity of these intersections within the curves. Every ray crosses the next to it, in a point of the caustic, before it meets the axis; those which pass nearest to the axis, proceeding farthest before their intersections, and thereby intersecting nearest to the principal focus; and those that pass farthest from the axis intersect each other soonest and nearest to the globe.

As the candle approaches to the globe, the rays that pass nearest to the axis will soonest become parallel, and afterwards diverge, while more distant rays still converge, until by the continued approach of the candle they also will become parallel and afterwards diverge. Consequently if the emergent rays, when they begin to diverge, were produced backwards, they would intersect the axis beyond the candle, and each contiguous couple of rays would afterwards intersect each other, and these imaginary intersections would form an imaginary caustic, with an acute angle at the radiant point, and diverging from thence. And while some of the rays that pass nearest to the axis are diverging, others are still converging, as they pass at a greater distance from the axis, and the intersections of these will form a caustic beyond the focus.

A lens in the same manner forms a caustic between the principal focus and the lens. Hence the exterior rays, that fall farthest from the axis of the lens are too much refracted to belong to the principal focus, and therefore will cross the axis nearer to the lens. This is called the *aberration* of the rays, by the spherical figure of the lens. A spherical surface therefore having the same degree of curvature every where, cannot refract the rays of different pencils to the same point or focus.

To accomplish this design it must grow flatter towards the edges, that the concurrence of the exterior rays may be prolonged. Nor will a plano-convex lens answer the purpose, for although it occasions less aberration than the double convex lens, yet the lens must be spherical on both sides near the center, to shorten the concurrence of the middle rays, and gradually become concave towards the edges, to lengthen the concurrence of the exterior pencils, with the axis. Nevertheless, the middle pencils are so closely crowded together in the focus, and the exterior rays are scattered so thinly over a plane supposed to pass perpendicularly through the focus, especially towards the verges of the picture formed there, that the confusion thereby occasioned is seldom very sensible, provided the lens have a considerable aperture. It is very little in comparison with the confusion occasioned by the different refrangibility of the rays of different colours.

IMAGES OF OBJECTS BY REFRACTION.

When a pencil of rays diverges from any point of an object, and falls upon a convex lens, the middle ray that passes through the center of the lens suffers no refraction, and all the other rays of that pencil will be refracted to their proper focus in it, where they will consequently form an image of that point; and as this is true of every point of the object, there must be a complete image of the object formed in the focus; the image of each point of the object being formed in the foci of their diverging pencils. As the middle ray of each pencil passes through the center of the lens, they must all cross one another there, so that the image will be inverted, with respect to the object; the rays, that proceed from the points above the axis, having their foci below it.

For the same reason, the image and object will always be viewed under the same angle, from the center of the lens; as the axis of each pencil crosses the other in the center of the lens: and hence the diameters of the object and image will be to each other, as their respective distances from the lens; when the object is parallel to the lens. But as any inclination of the object alters its apparent diameter, by diminishing the angle under which it is seen from the lens, the image will sustain a similar diminution, and still their apparent diameters will be in the same proportion.

If you would receive, on a white screen, in a darkened room, the images of exterior objects, by suffering the rays from them to pass through a convex lens, placed in a window shutter, for the purpose, you must be careful to exclude all other rays of light, which otherwise falling upon the picture would so weaken and dilute the shadings and colourings of it, as to make it obscure or even to disappear entirely. The screen must also be placed where the several pencils unite in their respective foci, and where the distinct images are formed. As the object recedes from the lens, the image approaches to it, but can never come nearer than the principal focus, because that is the place of the image, when the distance of the object becomes infinite. Hence the distinct images of objects placed at different distances from the lens, can never be received at the same time on the screen. Because, while a nearer position of the screen is necessary to render the images of distant objects distinct, the images of nearer objects will thereby become obscure, until the screen be again removed to the place, where the several pencils of rays from them unite. As the images will be more bright and pleasing, the more strongly the objects are illuminated, the lens should be placed in a window opposite to the sun, that the side of the objects next to it may thereby be more copiously enlightened.

As the diameter of an image depends upon the diameter of the object at a given distance, and decreases as the distance of the object increases, the diameter of an image must be directly as the diameter of the object and inversely as its distance. $I=\frac{O}{dist}$. And the area of the image being as the square of its diameter directly, it must be also as the square of the distance of the object inversely, or the square of its own distance from the lens directly; when the diameter of the object is given. $a=I^2=\frac{1}{D^2}=d^2$. a being=the area of the image, I= its diameter, d=its distance, O=the diameter of the object, and D=its distance.

The *distinctness* of a picture depends upon the several pencils of rays being accurately collected into their respective foci upon the screen; but its *brightness*, on the quantity of light transmitted through the lens: which let be=B. Now as the brightness of an image depends upon the quantity of light thrown together in the place of the image, it must be different, according to the distance of the object, the area of the lens, and its focal distance, or the area of the image. The greater that the area of the lens is, the more light it transmits in the same proportion, therefore B=L, L being the area of the lens. The quantity of light that falls upon a given surface, from different distances will be inversely as the squares of these distances; and therefore $B=\frac{1}{D^2}$, when the area of the lens and its focal distance are given. The brightness of an image will also be inversely as the area of the image or the square of its distance from the lens, the other circumstances being equal, and therefore $B=\frac{1}{a}=\frac{1}{d^2}$. Because, when the same quantity of light is diffused over a larger space, it must enlighten it in a less degree, than when it is spread over a less space, and kept closer together. Hence, when neither the distance of the object, the area of the lens, nor its focal distance is given, the brightness of the image will be directly as the area of the lens, and inversely as the squares of the distances of the object and image conjointly. $B=\frac{L}{D^2 d^2}$.

Now as the distance of the object is always inversely as the distance of the image, the one approaching the lens, while the other recedes from it, the brightness of the image will solely depend upon the area of the lens, and will not be altered by the alteration of the distance of the object. For $B=\frac{L}{D^2 d^2}=\frac{LD^2}{D^2}=L$.

The brightness of an image will always be proportional to the brightness of the object, when all other circumstances are equal. Hence the brightness of the solar image, in the focus of a burning glass, is so much greater than that of any other object.

The heat at the focus of a burning glass is always directly as the area of the glass, and inversely as the area of the solar image, or the square of its focal length. For as the area of the glass is larger, so many more rays are transmitted through it, and the smaller the space is, into which they are condensed at the focus, the more intense will be the heat, which will be to the common heat of the sun, as the area of the glass is to the area of the solar image. For the common heat of the sun is the heat of the rays when they are spread over the surface of the glass; and the same rays being collected into the area of the solar image will produce a brightness and heat proportional to the smallness of the space into which they are condensed.

VISION THROUGH LENSES.

When any small object or point of an object is seen by a ray of light refracted through a lens, it appears in the direction in which the refracted ray comes to the eye, and in that place from whence the rays seem to diverge, or their radiant point.

Every pencil of rays, that diverges from any point of an object, will by passing through a lens have the direction of its rays altered, as if they came from some other point in the axis of the pencil, more or less distant than the object; which is called the radiant point of that pencil; and as the eye is not sensible of this change of direction in the rays, we naturally refer the point of the object from whence they proceed, to this radiant point; as the image is painted on the same part of the eye, that it would have been painted on, had there been no lens, and the object was placed in the radiant point. Now, as there are as many imaginary radiants, as there are points in the object, the sum total of these constitutes what is called the *last image*. And as the eye is affected by this last image, or sum total of radiants, in the same manner as if the object had been seen without the lens, in the same place, and of the same magnitude with that image, we say, that it is not the object that we see by refracted vision, but its last image. Now this is universal in refracted vision, and all the various situations, positions, magnitudes, and appearances of objects seen by refracted rays, depend upon the different directions of the rays when they fall upon the eye, and the position of the radiant points, from which they seem to diverge; some of which we shall mention and illustrate.

In vision through any glass, the object will appear erect, if both the object and its last image be on the same side of the lens, but inverted, if they be on different sides. If the object be placed nearer to a convex lens than its principal focus, the rays that diverge from it are rendered less diverging, as if they had come from a radiant point more distant than the object, and consequently the last image will be on the same side of the lens with the object: and as the middle ray, or axis of every pencil that proceeds from any point of an object, passes through the center of the lens without refraction, the other rays of that pencil will proceed diverging after refraction as if they had diverged from some point in the axis of the pencil more distant than the object: so that every point of the object will have its correspondent radiant situate somewhere in the line which passes from that point of the object through the center of the lens, and therefore the points of the image, consisting of all these radiants, must have the same position with respect to each other as the correspondent points of the object have, and the object will appear erect, when they are both on the same side of the lens.

As the rays that pass through a concave lens, converging to a point more distant than the principal focus, are made to diverge, and diverging rays to diverge more, as if they came from a point between the object and lens; the same reasoning proves that the last image, or all the radiant points, will be on the same side with the object, and that it will appear erect. In both these cases, the object and last image being seen from the center of the lens under the same angle, they must be to each other as their respective distances from the lens; the object appearing magnified by the convex lens, and diminished by the concave.

But if the object be placed beyond the principal focus, the rays that proceed from it diverging, will be made to converge by a convex lens to a focus on the opposite side of the lens, where they form a distinct image; and the rays which are converged to the different points of this image will there cross each other and diverge from thence.

Now, if the eye be placed farther off, these intersecting rays will fall upon it diverging from the distinct picture; so that the last image and distinct picture will be in the same place; for every focal point of the one becomes a radiant point of the other. Hence the object will appear inverted, as the distinct image is inverted.

In this case also, the object and its last image will be to each other as their distances from the lens. This we before proved with respect to the object and distinct picture formed in the focus of the lens, and this distinct picture becomes the last image.

As the object and last image appear under the same angle from the center of the lens, if the eye be close to the lens, they will appear under the same angle, and consequently be of the same apparent diameter. This would also be the case if the lens were close to the object. Because the real and imaginary radiants then become the same; the object and last image being in the same place.

If the object be nearer to the convex lens than the principal focus, it will not only appear erect, but also brighter than to the naked eye, and distinct. It will appear brighter, because the rays diverge less after refraction, and more of them will consequently enter the eye; and as they diverge as if they had come from an object at a moderate distance, the object will appear as distinct as objects at a small distance generally appear. It will also appear magnified, unless either the object or eye touch the lens, because the last image is more distant than the object, and both are viewed under the same angle from the center of the lens. But the apparent magnitude will continually decrease as the eye recedes from the lens, because the last image is always seen under a less angle as the eye recedes.

If the object be placed in the principal focus of the convex lens, the rays will proceed parallel after refraction, and therefore more of them will enter the eye than could have entered without the lens, and make the object appear brighter, than to the naked eye. It will appear as distinct as remote objects usually appear, as the rays from them come nearly parallel. It will also appear erect, as the imaginary radiant points are at an infinite distance on the same side with the object. For the same reason, its apparent magnitude will not be altered by the motion of the eye, as no motion of the eye can lessen or increase this infinite distance of the radiant points. It will therefore appear of the same size, whether the eye be close to the lens, or removed from it. But as all objects appear to the naked eye to grow less, when the eye removes from them, and this not taking place when they are viewed through a convex lens, but that they still continue to appear of the same magnitude, upon any removal of the eye; we say, that the object will appear to be magnified, when the eye is removed from the lens, because it could not appear to the naked eye so large, as it appears at the increased distance of the eye, when seen through the lens.

If the object be farther from the lens than the principal focus, and the eye nearer than the place of the distinct picture, the object will appear erect, because the last image is still on the same side of the lens with the object; brighter than to the naked eye, because, as the rays converge after refraction, more of them will enter the eye; and confused, because the eye is nearer to the lens than the place of the distinct picture. It will also appear magnified, in the inverse ratio of the distance of the eye from the place of the distinct picture. So that as the eye removes from the lens to the place of the distinct picture, the apparent magnitude of the object will continually increase, until it become infinite and confused, when the eye arrives at that place; afterwards it will become inverted, confused and diminished, when the eye removes to a greater distance. It will then also become fainter, because the rays now diverging from the place of the distinct picture, fewer of them will enter the eye. If the eye be very close to the place of the distinct picture or focus, it will see the object as confusedly, as if it had been close to the object; since the rays diverge from it, as they do from the object; but when it recedes farther, it will see it more distinctly.

When the eye and object are fixed in their stations, and not farther apart than the place of the distinct picture, wheresoever the convex lens may be between them, as the lens moves from the eye towards the object, its apparent diameter increases until it arrive at the middle point, and then it begins to decrease till the lens arrive at the object, where its apparent diameter is not magnified, any more than when the lens was close to the eye.

When the eye is in the principal focus of a convex lens, the apparent diameter of an object is not altered by the motion of the object from or towards the lens; but the removal of the object increases its apparent diameter, when the eye is more remote, and diminishes it, when the eye is nearer. When the eye is in the focus, as the object departs from the lens, its last image departs with it, and thereby grows less, but at the same time the place of the distinct image is continually approaching to the place of the eye, which makes the apparent diameter of the object to increase, and therefore as it increases as fast on one account as it is diminished on another, it will continue unaltered. That the increase and decrease of the apparent diameter of the object is just equal in this case is plain, because it cannot be infinitely diminished till the object removes to an infinite distance, when the place of the distant picture comes to the eye, and when it sees the object infinitely great. But when the eye is farther from the lens than the focus, the apparent diameter of the object increases faster than it is diminished, and therefore increases upon the whole, and vice versa.

If the object be at a greater distance than the principal focus of a convex lens, and the eye be nearer than the place of the distinct picture, then, if the lens be moved from the right hand to the left, the object appears to move the contrary way, but the same way, if the lens be fixed and the eye be moved. Every thing is reversed, if the eye be farther from the lens, than the place of the distinct picture. For the object will then appear to move the same way with the lens, and contrary to the motion of the eye. By the motion of the lens, the rays fall with different obliquities, and are differently refracted; and by the motion of the eye, different refracted rays come to it, so that the object must appear to change its position continually.

CONCAVE LENSES.

When an object is seen through a *concave* lens, it seems fainter, as the rays diverge and fewer enter the eye; it appears erect, as the rays do not cross each other; but it will appear confused to all such eyes as are not very convex: Yet it may be made more distinct either by removing the eye to a moderate distance, or by making the rays pass through a small hole, so that the most diverging rays may be cut off, and only the middle rays, which diverge least, be suffered to enter the eye.

The apparent magnitude of the object is diminished, as the imaginary radiants are nearer to the lens than the object, and both are seen under the same angle from the lens; and it decreases with the departure of the object; because the last image or imaginary radiant removes with it, and consequently is seen under a less angle.

If the eye and object be fixed in their places, as the concave lens recedes from the eye, the apparent magnitude of the object decreases, until it arrive at the middle station between the eye and object, and then it begins to increase until the lens arrive at the object. For the apparent magnitude of the object is not altered when the lens is at either extreme, and therefore it must be least in the middle.

If the concave lens be moved from the right hand to the left, the object appears to move the same way with the lens; but if the eye be moved, it appears to move in a contrary direction.

Purblind persons, whose eyes are very convex, see distinctly through concave lenses; while old persons, whose eyes are flatter, through the wasting of the humours of the eye, see more distinctly through convex lenses; as we shall see more fully, when we consider the eye.

Objects seen through a plane thick glass appear erect, nearer, brighter and larger, than to the naked eye: and they do not seem to move by the motion of the eye. The imaginary radiant is nearer to the glass than the object, and therefore it will appear magnified as nearer objects appear; more rays will enter the eye to make it appear brighter, and it must appear erect, as the rays do not cross each other: the motion of the eye does not alter the obliquity of the incident rays, nor does the motion of the glass; and therefore the object must appear to be stationary, however the eye or glass may move.

Objects seen through a polygonous glass, with many surfaces inclined to each other, appear to be multiplied, as the rays that fall upon each of the surfaces will be refracted in different directions, and consequently there will be as many images formed, as there are surfaces to refract the rays to the eye.

THE EYE AND VISION.

Having considered how the rays of light passing through a lens are collected into a focus, and there form images of objects; it will be easy to understand, how vision is effected, by the rays that issue from any object, and are converged by the refractive power of the various humours of the eye to innumerable points on the bottom of the eye, where the images of external objects are formed, and by their impressions on the optic nerve make us sensible of the existence of these objects. For the humours are in reality so many lenses, disposed and combined together in such a manner, as to converge the rays of light to a focus on the bottom of the eye, where the image of the external object is completely and accurately formed, when the eye is in a proper state for effecting vision. A complete description of the mechanism of the eye belongs to anatomists to give; but that you may understand how vision is performed by it, we shall give you so much of the general description as will be essential to our present purpose.

The human eye is nearly globular, but not perfectly so, consisting of two segments of different spheres, that which forms the anterior part being the lesser. It consists of three humours of different refractive powers, inclosed in as many coats. The outward coat, which covers the larger segment of the eye, is called the *sclerotica*, being of a hard, white, and opake substance, as is signified by the name. The continuation of this coat, where it covers the fore part of the eye, is denominated the *cornea*, from its resemblance of horn in its consistence and transparency. This being convex and transparent serves to converge the rays of light that fall upon it to a focus, at a small distance beyond the bottom of the eye, could they proceed so far. Next within this lies the *choroides*, of a blackish colour, which serves for a lining to the sclerotica, with which it is connected while it covers the back part of the eye. But in the anterior segment, it is separated from the cornea, and floats in the aqueous humour, where it assumes the name of the *iris* or *uvea*, from the different colours which it has in different eyes. It is composed of circular and radial fibres, by the alternate contraction or dilatation of which, the central hole in the middle of it, called the *pupil*, is lessened or enlarged, to regulate the quantity of light, that is found to be agreeable to the eye. The inner coat is a fine expansion of the optic nerve, which passing through the other coats, spreads like net-work over the inside of the choroides, and is therefore denominated the *retina*. According to some philosophers, the images of external objects are painted upon this coat, and by the impressions of light upon it, the ideas are communicated to the brain, the common origin of all the nerves. Although others are of opinion, that the images are painted on the choroides, and that the sensations are communicated to the brain by means of its own nerves. The principal arguments for the support of these opinions we shall see presently.

The three humours of the eye are the *aqueous*, the *crystalline*, and the *vitreous*. The *aqueous* humour lies immediately behind the cornea, and makes the eye globular before. It is divided into two portions, a part lying before, and a part behind the iris, and communicating with each other by means of the pupil. It derives its name from its resembling water in its limpidity, consistence, specific gravity, and refractive power. As the rays of light enter this denser medium through the convex surface of the cornea, they are refracted toward the axis of the eye, and pass through the pupil.

Behind the aqueous humour lies the *crystalline*, which in shape resembles a double convex lens, being more convex on the back part than before. Being of a lenticular form, it serves to give a greater degree of convergency to the rays, that are refracted through the aqueous humour. It is transparent as crystal, of the consistence of a hard jelly, and exceeds the specific gravity of the aqueous humour in the ratio of 11 to 10. It is inclosed in a fine transparent membrane called its *capsula* and floats in a thin pellucid fluid like water, without any connexion with the capsula; but from the edge of the capsula, all round, proceed radial fibres, denominated the *ligamenta ciliaria*, which connect it with the choroides at the beginning of the iris and cornea, so that it lies between the two segments of the eye. By the ligamenta ciliaria contracting or dilating, the form of the crystalline lens, or at least of the fluid in which it floats, may be altered so as to make it less or more convex; by which means, it may refract the rays that proceed from objects of different distances to the same focus, and thereby afford a distinct vision of objects at different distances.

Behind the crystalline lens, lies the *vitreous* humour, which is also transparent like glass, and nearly of the same refractive power. It is nearly of the consistence of the white of an egg, exceeding the aqueous humour a little in its specific gravity and refractive power, filling the larger segment of the eye, and giving it a globular figure. As it is of less density than the crystalline lens, the rays which enter it through its concave surface are made to converge more towards the axis of the eye; so that all the humours of the eye contribute something to the converging of the rays of light.

The rays that issue from the object falling upon the eye are refracted by its humours, so as to fall upon the retina, where they form an inverted image of the object, by means of which we are made sensible of its existence, position, and motions, although in a manner to us incomprehensible. Yet that vision is performed by means of these images of external objects painted in the eye, is extremely probable from the very existence of them, of which any person may assure himself, by taking away the coats from the back parts of the eye, and presenting it to a candle, whose image will then be painted and inverted on a piece of white paper placed on the back part of the vitreous humour.

The different humours of the eye are of such a determinate degree of convexity and refractive power, as to converge all the rays of light belonging to any pencil accurately into the same focal point on the retina, notwithstanding their different refrangibility; so that external objects may appear without those colours, which arise from the rays of different sorts in the same pencil being refracted to different points, where they form an image of the colour peculiar to each species of them. The different forms and densities of the humours of the eye lead us to believe that the design of Providence, in the construction of it, was to remedy the inconvenience arising from the different refrangibility of the rays of light. The aqueous and vitreous humours have the form of a meniscus, and the crystalline is a double convex lens, and by their composition, being of different densities, the most refrangible as well as the least refrangible rays of any pencil are accurately collected into a point, where the beam must appear white; so that external objects will appear in their own natural colours, and without any thing of that colouring, which arises from the dispersion of the rays of the same pencil.

Vision might be performed without the crystalline lens, but obscurely, since without it the rays would be brought to a focus beyond the retina; as is evident from the case of persons, who have suffered the extraction of it, on account of its having become opake.

As the image of the object is inverted in the eye, many have wondered how the object should appear to us to be erect. But we should remember that *erect* and *inverted* are only relative terms. And when all the parts of a distant prospect are painted on the retina, they all preserve their relative positions with respect to each other; and we have no means of judging an object to be inverted, while it retains its natural position with respect to all the objects around it. There is also a great difference between an object and its image, by means of which we become sensible of its existence and position. There seems to be no necessary connexion between their positions. The perception of things by sight seems to be no other than the remembrance of former perceptions acquired by our other senses, and we constantly conclude that the things which we now perceive will invariably affect us, in the same manner, as they formerly affected us in similar circumstances.

Now if the memory of the same sensations excited in the same part of the retina occasions the same judgment of the position of objects, the inverted images in the eye will answer this purpose as well as if they had been constantly erect. It is only necessary that the object and image should invariably preserve or change their situations and positions at the same time, according to some fixed and established law. And had a person been born with such an eye as would have erected the images of objects on the retina, he would talk and judge of objects just as other persons now judge and talk of them.

It has been also a matter of surprise to some, that an object should appear single to persons, whose eyes are in a sound state, where there is an image of the object formed in each eye. It is also from habit and experience, in a great measure, that we judge an object to be single, provided the images are formed upon correspondent parts of the retina, which is always done, when the optic axes unite at the object. It is principally from experience that we learn to judge of external objects; as appears from persons who have been couched.

From Dr. Cheselden's account of persons couched by him, it appears, that, at first, they had no idea of figure, distance, or colour, by sight, until they had learned to see, as they would express it, or until they had the knowledge of these things by habit, and the assistance of their other senses. They all agreed in this, that they could not, at first, turn both their eyes to the same object, until they had acquired the power by habit; that objects at first seemed to touch their eyes, and that having much to learn, they would forget a thousand things which they had already learned. From experience and the assistance of other senses, we learn to account an object single, when its images in both eyes are formed on the correspondent parts of the retinas, that is, when the axes of the eyes meet at the object.

But when the axes of the eyes meet either between the eye and the object, or beyond it, the images do not then fall upon the correspondent parts, and the object will appear double. Any person may convince himself of the truth of this position, by holding up his finger before his eyes, while he is attentively looking at a more distant object, and his finger will at the same time appear double, as the images of it are not then formed on corresponding parts of the eyes; the optic axes uniting at the object, and not at the finger: but if he should unite them at the finger, by looking attentively at it, the more distant object will then appear double, for the same reason. The mind however seldom attends to the images that are made upon these disagreeing parts of the eyes, because the impression made by the images of the object, where the optic axes meet, is so much more vivid and forcible. Hence, persons that squint seldom attend to the double appearance of a single object; the mind being wholly employed in viewing an object with the eye, that is sound, while it totally neglects the image formed in the converging or diverging eye. Accordingly, they are observed to direct the sound eye invariably to the object, that they would see distinctly. Some philosophers indeed are of opinion, that our seeing objects single, is not acquired by experience and habit, but arises from some natural constitution of the eye; and appeal to the case of a person, couched by Dr. Cheselden, who saw objects single from the first. This however may be accounted for, by his soon acquiring the habit of directing the axes of his eyes to the same object, so that he might not have attended to the instances of double vision which first happened. The doctor mentions another case of a person, who had one of his eyes distorted by a blow on his head, who thereby saw every object double, till by degrees the most familiar objects first, and then others appeared single, without any cure of the distortion of the eye. It is therefore probable that this arose from the acquired habit of neglecting the images formed in the distorted eye: as, I believe, is the case with persons that squint.

When the image of an object falls upon the insertion of the optic nerve, it then becomes invisible. This defect of vision is supposed to arise from the number and largeness of the blood vessels, at the insertion of the optic nerve; or according to some, because the retina is observed in that place to be separated from the choroides, which they suppose to be the principal instrument of vision. Any person may easily assure himself of the fact, by the following experiment, which may be made by a little practice and trial. Let him paste upon an opposite wall, three patches of paper about a foot apart, and shutting one eye, let him direct the other to one of the patches in the mean time receding backwards, until he find that he will lose sight of another of the patches, to which he was not looking directly, because its image then falls upon the insertion of the optic nerve. M. Mariotte, who first proposed the experiment, has contrived the following improvement of it. Place two black patches on a white ground, or two white patches upon a black ground, three feet apart, and recede twelve or thirteen feet back from them: then hold up your finger about eight inches from your eyes, both being open, so that the right-hand patch may thereby be concealed from the left eye, and the left-hand patch from the right eye: then if you look at your finger steadily with both eyes, you will lose sight of both the patches at once. For in this situation, while the object is hidden from one eye by the finger, its image in the other falls on the base of the optic nerve. On this account nature has wisely placed the insertion of it, not in the middle of the bottom of the eye, where we have the most distinct vision of an object, but in the side next to the nose; so that whatever part of the image falls upon this point in one eye, may not also fall upon the same in the other; and therefore to both eyes taken together, no part of the object can be wholly invisible.

This experiment has given rise to a considerable dispute among philosophers, whether the choroides or retina be the principal instrument of vision. Priestley, Mariotte, Mitchell, Rutherforth, and others, allege that the choroides is the principal seat of vision: 1st, Because it appears to be much more sensible of the impressions of light than the retina: as the iris, which is only a continuation of it, naturally contracts the pupil upon the admission of an unusual quantity of light, which it finds to be offensive.

2d, Because it is found to be black in most men, and in such animals as seek their food by day, as in eagles, hawks, &c.; and white in those animals that seek their food by night, such as cats, dogs, owls, &c. And they suppose that this black colour makes the choroides more susceptible of the impressions made by the rays of light.

3d, Because the retina is evidently a transparent pulpy substance, and insensible at the base of the optic nerve, with a very unequal surface, and therefore not proper for the termination of the several pencils of light that fall upon it; whereas the choroides is of a smooth surface, opake, and not lined with a reflecting substance behind it.

4th, Because wherever the retina, which receives the picture of external objects, is close to the choroides, there the choroides can receive the impressions from it, and vision is produced; but where the retina is separated from the choroides, as at the insertion of the optic nerve; there no vision is produced from the picture painted in that place, the choroides not being near enough to receive the impressions made on the retina.

On the other hand, Porterfield, De la Hire, Piquet, and others, are of opinion, that the retina, and not the choroides, is the principal instrument of vision. 1. Dr. Porterfield observes, that the want of vision at the insertion of the optic nerve may be owing to the want of that softness and delicacy, which it has when spread over the choroides; and that in those animals, in which that nerve is inserted exactly in the bottom of the eye, such as the sea-calf, the porcupine, &c. it is observed to be equally delicate, and equally sensible in that part as in any other. For were it not so, it is not probable, that this nerve would be inserted in a place, where the most distinct vision would be naturally expected. 2. Because the nerves in general, when constricted by their coats, have but little sensibility, in comparison with what they have, when they are divested of them, and unfolded in a soft pulpy substance. 3. Haller observes, that the choroides cannot be the principal instrument of vision, because that sometimes in men and birds, but more especially in fishes, it is lined with a black mucus, through which the rays of light cannot penetrate. 4. Because the retina is a large apparatus of nerves, immediately exposed to the action of light; whereas the choroides receives but a slender supply of nerves; and even these are much less exposed to the impressions of light, than the naked fibers of the optic nerve. 5. Another argument for the retina's being the immediate instrument of vision, is derived from the effects of an amaurosis or gutta serena, which totally destroys vision; in which disease, the optic nerve has been found to be lessened, withered, and hardened, when no functions of the eye that depended upon the nerves of the choroides were in the least affected. Now as the optic nerve, which is solely employed in forming the retina, is the only thing that is affected in this disease, which occasions total blind-ness, it is therefore probable, that the retina is the principal instrument of vision.

Upon the whole, it appears to me the most probable conclusion, that although the retina may be considered the principal instrument of sight, by which the sensations excited by external objects may be communicated to the brain, as its naked fibers seem naturally more sensible of these impressions; yet the choroides may also be essentially necessary for the purpose of complete and distinct vision.

The *brightness* and *distinctness* of vision next deserves a particular consideration. Vision is said to be *bright*, when a sufficient number of rays enter the eye at the same time; and *obscure*, when too few are admitted to make a vivid and forcible impression.

That an object may appear with a sufficient degree of brightness, we are furnished with a power of enlarging or contracting the pupil at pleasure, by means of the muscular fibers of the iris, in order to admit a larger or less quantity of light, as occasion may require. But this power has its limits in every animal, although in some it may be more extensive than in others, especially in such as are destined to seek their food in the night; such as cats, foxes, &c. This is evident from what happens to a person coming into a room, after having been out walking in the snow or bright sunshine. At first he scarcely sees any thing, until the pupils of his eyes have had time to open and expand, having been almost shut before, in order to exclude such a degree of light as was found to be offensive. After a few seconds, he will see every object around him with sufficient brightness. For the same reason, the pupil of the eye is observed to be larger in the morning and evening, than at noon; and in a dark day, than in a day when the sun shines bright.

Vision is said to be *distinct*, when the rays of each pencil from an object are accurately collected into one and the same point on the retina; but it is *confused*, when they meet and cross each other in the vitreous humour, or when they tend to a focus beyond the retina. For in both these cases, the rays that come from two or more points of the object would fall upon the same part of the retina, and forming the images of these points of the object on the same place, must render the image confused and indistinct. This may arise either from an undue convexity in the cornea or crystalline lens, or from a disproportionate distance of the object. The last of these we shall consider first. By a fundamental law of refraction, the focal distance of the image depends upon the distance of the object from the lens. As the distance of the one is increased, the distance of the other is lessened. The object therefore at a certain distance from the crystalline lens, will have its image accurately formed upon the retina. But at any other distance, whether greater or less, the image will be formed either nearer to, or farther from, the crystalline lens. So that while the eye continues of the same figure without alteration in any respect, objects at a particular distance only can be seen distinctly, and all others must necessarily appear confused; because the focal distance of the image becoming less, as the distance of the object increases, the image cannot be accurately formed on the retina. But as it is of the last consequence to our happiness in this world, that vision should be distinct, we are accordingly furnished with a certain power of conforming the eye to different distances of objects, within certain limits, whereby they may be seen distinctly, notwithstanding their different distances.

The images of external objects may be formed upon the retina, and the focal distance of the rays be preserved invariable, notwithstanding the different distances of the objects, by increasing or diminishing the convexity of the cornea or crystalline humour, or of both, or by altering the form of the eye, whereby it is made either longer or shorter by means of the external muscles which turn it in different directions. And the eye may be conformed to the different distances of various objects by altering the position of the crystalline lens, bringing it nearer to, or farther from the retina.

That we have some such power over the eye, to conform it to the different distances of objects, is evident from every day's experience, but which of these, or what similar power nature has destined for our relief, is much disputed by philosophers. The coats of the eye, especially in some animals, seem to be too hard to admit of the necessary alteration in the form of the whole eye; and its cavity to be too compactly filled, to allow of the removal of the crystalline humour, from place to place, as occasion may require. It seems therefore most probable to me, that this accommodation of the eye to the different distances of objects, is effected by the alteration of the figure of the crystalline lens, which may easily be accomplished by the contraction or dilatation of the ligamenta ciliaria.

Thus if we would view a distant object distinctly, whose image would be formed in the vitreous humour, before the rays reach the retina, without any alteration in the figure of the crystalline lens; the design may be accomplished by contracting the ligamenta ciliaria which are fastened to the edge of the capsula all round, as this contraction of them would flatten the crystalline lens, and lengthen the focal distance of the rays. Besides, these ligaments being concave on the side next to the vitreous humour, the vitreous humour must be in some measure compressed by their contraction, and thereby made to press upon the crystalline lens, by which it is made still flatter. By these means the rays of light, which would otherwise meet in a focus in the vitreous humour, are now made to converge to a point on the retina. And on the other hand, when we would view a near object, whose focal distance would be beyond the retina, we relax the ligamenta ciliaria, and thereby suffer the crystalline lens to assume a more convex form, and thus converge the rays of light to a nearer focus. Although the crystalline lens is of such a substance as will not be easily compressed, yet it floats in a thin transparent fluid like water, inclosed in the capsula, which can easily yield to the pressure of the vitreous humour, and thereby suffer the necessary alteration in its figure, which will answer the same purpose.

To all this we may add, that although the accommodation of the eye to the different distances of objects, in order to produce distinct vision, may be principally effected by the ligamenta ciliaria; yet it may also be accomplished in some measure, by the contraction and dilatation of the pupil, which power we indisputably possess. When rays proceed from a near object, too much diverging to be collected into a focus on the retina, by contracting the pupil, the most diverging rays are excluded, whereby the others are collected on the retina.

Thus we find, that we can read a very small print distinctly, through a pinhole in a card, held close to the eye, and view an object distinctly at a much smaller distance, than we could otherwise do, without such a contrivance to cut off the rays that diverge too much. And when we would view a distant object distinctly, by enlarging the pupil, we admit more of the diverging rays to enter the eye, which are united at a more distant focus than those that are less diverging, and by their union form a distinct picture on the retina. So that the power of contracting and dilating the pupil of the eye, which was principally designed to regulate the admission of such a quantity of light as is necessary to the brightness of vision, is also subservient to the purposes of distinct vision.

Yet still we must remember, that these powers of the eye are confined within certain limits. An object may be so near or so remote that the best eye cannot see it distinctly. The eye cannot alter its figure, or the figure and position of its parts, so as to answer every distance of the object or to make the images of all objects distinct on the retina. We find by experience, that we cannot see an object distinctly, that is nearer to us than five or six inches, as the rays that diverge from it cannot be refracted by the eye with all its powers to so near a focus as the retina. And if the object be so remote as to subtend a less angle at the pupil than one minute or 45 seconds, we find that vision becomes obscure, and few eyes can clearly see a round object that subtends an angle less than 30 seconds. By various experiments, the smallest angle under which a white spot upon a black ground, or a black spot on a white ground, could be seen, is about half a minute. And yet there are cases, in which an object, whose diameter subtends a much less angle, is, notwithstanding, visible. The fixed stars, the largest of whose diameters do not measure a single second, are visible to the naked eye. A line also is distinctly seen, whose diameter is not above 2 or 3 seconds, at the distance, at which a round spot of 25 or 30 seconds becomes invisible. In order to our perceiving impressions made upon the optic nerve, they must be made with a sufficient degree of force, as from an object whose light is inherent in itself, or from an object of such magnitude in some of its dimensions, so as to make a greater and more forcible impression. Hence we find that a longer line is visible at a greater distance than a shorter. So that the greater visibility of a line than of a spot, and of a longer than a shorter line, seems to depend upon a greater quantity of impression made upon the optic nerve.

But even when the object is of sufficient magnitude, and at the most favourable distance for producing distinct vision, we sometimes find a natural defect in the construction of the eye itself, which produces obscurity in vision, notwithstanding all the powers with which the eye is naturally furnished. This arises from too great, or too small, a convexity in the cornea or crystalline lens, or it may be in both, which cannot be remedied by the natural powers of the eye. When the eye is in a sound state, its different refracting parts are of such a determinate degree of convexity and refractive power, as to converge the rays of every pencil into a focus, precisely upon the retina.

But if the cornea or crystalline lens, or both, should be more convex than just, as is the case of short-sighted persons, such an eye will have too great a refractive power, and will converge the rays to a focus in the vitreous humour, where they form the distinct image of the object, and from whence they will begin to diverge again before they reach the retina: so that when they arrive at it, they will form an ill defined and obscure picture of the object upon it, whereby the vision will become obscure. Nor has the eye sufficient power to remedy this defect, or to converge the rays to a proper focus. Persons who labour under this inconveniency are called *myopes*, from the eye of a mouse, which is observed to be very convex: sometimes they are called short-sighted, because they can see distinctly only at a small distance. This therefore is the natural remedy of this defect: because when the object is brought very near, the rays of light are made to diverge so much that they are collected into a more distant focus, and form the distinct image on the retina. For the same reason, concave lenses are advantageous to them, as the rays are thereby made to converge to a more distant focus on the retina. By either of these means, the distinct picture may be thrown so far back as to fall on the retina. When such persons look at an object, and bring it nearer to their eyes than other persons can do, in order to see it distinctly, their vision must be brighter, as more rays will thereby enter their eyes, than can enter the eyes of others, at the distance of their distinct vision. But when they use concave spectacles to view an object at a given distance, many of those rays, which would enter another eye, are made by the spectacles to diverge so much as not to enter their eyes, and consequently their vision will be less bright. Their eyes will also grow better by age; as the humours naturally waste, and the cornea becomes flatter, and thereby converges the rays less than formerly, provided that there be proper care taken in the use of their eyes. This care is absolutely necessary; because persons, who were not born with this defect, are observed to bring it upon themselves by habit. This is remarkably the case with watch-makers, engravers, and painters in miniature, who are obliged to look very near to their work. For when the eye has been long used to that conformation, which is necessary for seeing near objects distinctly, it will at last settle into that figure, and cannot then be altered at pleasure, to see more distant objects with distinctness. For the same reasons, students will be apt to grow short-sighted, if they habituate themselves to hold their books too near to their eyes. So that this defect is not always remedied by old age. For by looking frequently and long at near objects in youth, whilst the eye is capable of being fixed, by custom, in any shape, a person may keep himself short-sighted, or make himself so, even if he had been naturally otherwise. For this reason, if short-sighted persons, instead of using the artificial helps of concave glasses, would accustom themselves to look frequently at distant objects, and keep the book as far as possible from them when they read, they might, by custom, acquire the power of viewing distant objects with distinctness.

There is another defect of the eye, which arises from a quite contrary cause, namely, too little convexity in the cornea or crystalline lens, or in both. This is the common infelicity of persons, when they arrive at the age of fifty years or thereabouts, and especially of such of them as have been accustomed in their youth to look much at distant objects, and but seldom at those that are near, such as travellers, sailors, and sportsmen. Age wasting the humours of the eye, or custom fixing the conformation of it, that is requisite for viewing distant objects with distinctness, the cornea may become too flat, and possibly the crystalline lens may also feel the effects of old age and thereby become less convex than what is necessary for distinct vision. Hence rays that issue from an object are refracted to a more distant focus beyond the retina; and falling upon it before they are united in their proper foci, they will form a confused image of the object upon it, and thereby render vision obscure and indistinct.

The natural remedy of this defect is to remove the object farther from the eye, which will bring the distinct image nearer to the crystalline lens, so as to fall precisely upon the retina. But this remedy has its limits; for the eye may be so flat, as to require that the object should be removed beyond the bounds of distinct vision, from whence so few rays will enter the eye, that the object cannot be seen with sufficient clearness and precision. And it is sometimes impossible to prevent this by any care whatever; for age will waste the humours notwithstanding all our endeavours. But it may be often prevented for a longer time than to the age of fifty years, by using ourselves to look at near as well as distant objects, and avoiding the use of spectacles as long as possible. For this will in some measure prevent the eye from being fixed to any particular figure. And indeed the muscles, which are designed for conforming the eye to different distances of objects, will best retain their powers by frequent exercise: so that by accustoming ourselves sometimes to view near objects, and then those that are remote, we may commonly preserve our eyes, for a longer time, and neither make ourselves short-sighted when we are young, nor long-sighted when we are old. But when this precaution fails, recourse must be had to the artificial remedy of this defect, which is found in the use of convex spectacles and reading glasses. A convex lens assists the refracting powers of the eye, as the rays that pass through the lens are thereby made to converge more when they fall upon the eye, and consequently to be refracted to a less distant focus; so that the image of the object will be painted on the retina, when the lens is of a proper convexity. By the use of these lenses, more rays are converged into the pupil of the eye, by which the object appears brighter; and it appears more distant, erect, and magnified, as we have seen before.

The more directly any object is placed before the eye, the more distinctly it is seen; and others that are obliquely posited are viewed obscurely: because the rays that fall nearest to the axis of the eye, are more accurately converged to a focus on the retina.

From what we have said of the construction of the eye and the refraction of the rays through its various humours, we may learn, that we are not left entirely to custom and experience in judging of the position and distances of external objects, though we derive a great deal from these sources, but we are also in some cases, especially when the objects are near, greatly assisted by the manner in which the rays are refracted by the eye. As every point of an object is seen in the direction of a ray proceeding from it, when that ray enters the eye, we acquire by experience the habit of referring an object to that place, from which the rays come to the eye. And as the eye is unacquainted with any refractions which the ray may suffer before it enters the eye, by passing through any refractive medium before its incidence on the eye, we entirely neglect the change of direction arising from it, and still refer the object to some point in the line, in which the ray enters the eye. So that we judge of the position of objects by the direction of the rays from them at their incidence on the eye.

We are also enabled in some measure to judge of the distances of objects, by the degree of divergency, with which the rays enter the eye, especially where it is considerable.

And when the object is known and familiar to us, we judge of its distance by its apparent magnitude, or the optic angle under which it is viewed.

The sensible change that is made in the figure of the eye, or of its different parts, by the contraction or dilatation of the muscles, by which it accommodates itself to the different distances of objects, enables us in some measure to judge of these distances.

The brightness and distinctness with which the various parts of an object are seen, contribute something to the accuracy of our judgments concerning its distance, referring those objects that appear brightest to the shortest distance; as we are previously certain that more rays from a near object may enter the eye, than from one that is more distant, to render the image brighter upon the retina.

In all cases, where the distance between the eyes bears any sensible proportion to the distance of the object, we are much assisted in our judgment of such distances by the magnitude of the angle comprehended under the optic axes of the eyes at the object. But in greater distances, we have recourse to the parallax of the object, or its bearing when viewed from different stations. And when all these methods fail, we have recourse to experience and the assistance of other senses, and the recollection of former ideas and judgments.

Upon the whole, we have endeavoured to illustrate these several propositions, which seem worthy to be remembered; viz. That the apparent brightness of an object depends upon the quantity of light received from any point by the eye; that its distinctness arises from the accuracy of the collection of its several pencils of rays into a focus on the retina; that its apparent situation depends upon the last direction of the rays, when they enter the eye; that its apparent magnitude, at a known distance, depends upon the magnitude of the optic angle; and that we judge of the distance of an object, not only by all these, but also by experience and our other senses.

OF OCULAR SPECTRA.

From the stimulus which the retina receives, from the impressions of the rays of light upon it, all vision probably depends: and we are not to suppose, that the retina continues in a passive state, while vision is performed, but that it is stimulated into action, like the other organs of the animal body, by irritation from the impressions of the rays of light, especially as it consists of fibres intermixed with a considerable quantity of nervous medulla; as is evident from its appearance, being easily torn asunder when immersed in warm water. Besides, being excited into action by irritation, in common with the muscles, they are alike fatigued or strengthened by action, are alike pained when stimulated into action in a state of inflammation, and alike subject to torpor and the effects of old age. The impressions, therefore, made on the retina, by the rays of light issuing from external objects, are the immediate causes of those sensations, which give us the knowledge of the existence of such objects. Nor do these impressions wear off instantaneously upon the cessation of their cause, but continue for some time, like the effects of other irritations in the human body. The impressions made by a coal of fire whirled round in a circle, continue until the coal return to the place where its motion commenced, and renews the impression. When any one has attentively viewed a luminous object, for a few minutes, an image of it continues to be visible for some time, after closing his eyes, or removing them from the object; this may be called the *ocular spectrum;* which, in various circumstances, may be attended with different properties and phenomena, and which we shall endeavour to explain.

These ocular spectra may be different, according to the different degrees of irritability of the retina at the time, arising from the force of previous impressions.

1. The retina is not so easily excited into action by less irritation, when it has been lately subjected to greater. Hence, when a person enters a darkened room, after walking in the snow or bright sunshine, although the pupil soon expands, yet the retina will not so quickly become sensible to the weaker impressions of light in the room; and if the room be not too dark, the pupil may be observed to contract again as the sensibility of the retina returns. Look steadily, for the space of a minute, on a square piece of white paper placed upon a black ground, then move the eyes to a sheet of white paper, and a dark square will appear in the sheet of paper. The parts of the retina which received the image of the black ground not having been much irritated thereby, quickly become sensible of the greater impressions of the white sheet of paper, while the parts which have been more irritated by the vi

impressions of light from the white square, are not so quickly excited into action after their fatigue.

In like manner, after looking at the sun, or any other luminous object, for a short time, that part of the retina which received his image becomes less sensible to smaller quantities of light, and therefore when our eyes are turned to less luminous parts of the sky, a dark spot is seen resembling the sun or other luminous object, which we beheld. This also seems to be the source of those muscæ volitantes, which some have mistaken for something floating in the aqueous humour of the eye. We endeavour to bring them into the center of the eye, that we may have a more distinct view of them, by the motion of our eye, but in vain, for they move with the eye, and have no other motion but what they receive from it. Hence, in the extreme debility of fevers, sick persons are observed to be employed in picking something from the bed-clothes, occasioned by their mistaking these muscæ volitantes in their eyes, for realities. Again,

Place a square inch of white paper on a piece of strawcoloured silk, and having looked for some time steadily upon the paper, remove the eye to another part of the silk, and there will appear a yellow spectrum of the same figure with the paper on the silk, and of a deeper colour: for the central part of the retina, having been some time exposed to the stimulus of a greater quantity of white light, is now become less sensible of the lesser stimulus from the silk, and therefore sees only the yellow rays from the silk. Thus we cannot hear lower sounds after we have been accustomed to louder, nor will the stomach feel the stimulus of common food, which has been accustomed to the use of spirituous liquors.

2. But on the other hand, the retina is more easily excited into action by greater irritability, after having been exposed to less.

If the eyes be closed in a bright sunshine, and perfectly covered with a hat, for a few minutes, on the removal of the hat, the eyes being still closed, the retina becomes sensible of the red rays that penetrate the eyelids. A similar light is seen through the fingers exposed to the rays of the sun, or the light of a candle. In a dark day, or the beginning of the evening twilight, if you look steadily upon a window for a few minutes, and then move your eyes a little, so that those parts of the retina, which had before received the images of the wooden frame of the sashes, may now receive the images of the panes of glass, many luminous lines will now appear to lie across the panes. For those parts of the retina, which before were less stimulated by the dark frame work, are now more sensible to the light, than the other parts of it, which were exposed to a greater stimulus, from the lighter parts of the window. The same will happen also when the eyes are closed, by means of the small degree of light, which penetrates the eyelids. And if the frame be painted black, the image of it will be red: through the influence of the red rays, which more copiously enter the eyelids. In the same way, a black spot of any figure made on a white paper, will, upon moving the eye from it a little, appear upon the white paper, something whiter than the other parts of the paper, after the black spot has been steadily viewed for a few minutes. For, the parts of the retina, which received the image of the black spot, are now more sensible to the impressions of light, than the other parts, which received the picture of the white paper.

On closing the eyes, after viewing the black spot on the white paper, the spectrum will appear red, through the influence of the red rays that penetrate the eyelids, if in the day light, but yellow, if viewed by the light of a tallow candle: for tallow candles abound much with yellow light, which passes in greater quantities and force through the eyelids, than blue light: and hence the difficulty of distinguishing blue and green by this kind of light. In the same manner, the flood of light becomes intolerable to the eye that has been accustomed to slight irritations for some time past, upon emerging from darkness into open day, or into a room greatly illuminated. This will occasion an immediate contraction of the pupil, to counteract its force. The impressions made upon the retina continue for a short time, after the irritation ceases, and if continually renewed by the same cause, may be continued at pleasure: as in the case of a coal of fire whirled round in a circle, the luminous circle may be continued as long as the cause of the irritation acts: in the same manner, as the other senses retain the impressions made upon them. When a hard body has been for some time pressed upon the palm of the hand, as in the tricks of legerdemain, it is not easy to distinguish, for a few seconds, whether it remain or be removed. The same may be said of taste.

Hence if you look, for half a minute, through a dark tube of two feet long, upon an area of half an inch of yellow silk, placed upon an inch area of blue silk, upon closing your eyes, the colours of the spectra will appear similar to the areas of the silk, provided the eye be not much fatigued with the quantity of light: but if the eye be kept too long upon them, the colours will be changed. Look steadily, for a short time, upon a window, in the evening or on a dark day, and upon closing your eyes, and covering them with your hands, an exact delineation of the window will appear; but if any light be admitted through the fingers or laterally, the dark parts of the window will appear luminous or white, and vice versa, in a few seconds, when the retina becomes sensible to the lesser impressions of light penetrating the eyelids. This experiment requires some practice and calmness, without any perturbation of the spirits, or fatigue and uneasiness of the organ of vision to make it succeed well.

These appearances may be termed *direct ocular spectra*, which resemble their objects in colour as well as in form, and are produced by a slight irritation. A moderate stimulus excites the retina into action, which quickly ceases, while a greater stimulus excites it into a spasmodic action, which ceases and recurs alternately. And hence the appearance of the window recurs repeatedly before it entirely vanishes. In like manner, upon looking at the setting sun, so as not greatly to fatigue the eye, a yellow spectrum is seen to appear and disappear several times, before it vanishes entirely; but if it be made to fall upon the grass or other coloured objects, it will change its colour by an intermixture with theirs.

OF REVERSE OCULAR SPECTRA.

But besides these ocular spectra, which represent the form and colour of their objects, there are others which represent the form, but a colour the reverse of their objects.

Sir Isaac Newton observed, that the thin plates of air between the convex and plane glasses which transmitted the violet, reflected the yellow rays, those which reflected red transmitted a bluish green, and those which reflected orange transmitted blue. These colours may therefore be denominated the reverse of each other.

Hence a red object produces a green spectrum, an orange produces a blue spectrum, and a yellow object produces a violet spectrum, and vice versa, when the retina having been stimulated by violent impressions of the rays of light falls into spasmodic action of an opposite kind; as when any of our muscles have been long or violently extended, and the action ceases, the limb is stretched the contrary way by the antagonist muscles, or falls down motionless, or in a temporary paralysis. Place a piece of coloured silk upon a sheet of white paper about half a yard from the eye, and look steadily at it for a minute; then, upon moving the eyes to another part of the paper, or closing them and putting the hands before them so as to admit some light, but not too much, to enter the eyes through the eyelids; a spectrum will appear of an opposite colour, namely, such as would result from a combination of all the other prismatic colours. For, as the direct spectrum, without any lateral light, is an evanescent representation of the form and colour of its object, in the unfatigued eye, but partaking of the colour of circumambient objects, when mixed with lateral light from them; so the reverse spectrum, without lateral light, is a representation of the form of the object, in the fatigued eye, with a colour compounded of all the prismatic colours, excepting that of the object; and with lateral light, the colour of it is compounded with the colour of the direct spectrum of the circumjacent objects, in proportion to their respective quantity and brilliancy.

To prove that the green colour of the reversed spectrum, arising from a red object, is a compound of all the other prismatic colours, excepting that of the object; let the prismatic colours be laid upon a circular wheel of pasteboard of four inches diameter in this proportion, red 45°, orange 27°, yellow 48°, green, 60°, blue 60°, indigo 40°, and violet 80°, and the wheel, upon being turned quickly on its axis, will appear white, the compound of all the prismatic colours, as has been formerly observed. But now let the red compartment be entirely left out, and the others be proportionably enlarged to complete the circle: then as the orange is a mixture of red and yellow, the adjacent colours, substitute yellow in its place; and as the violet is a mixture of red and indigo, its adjacent colours, substitute indigo in its place; that all red may be entirely excluded from the painted wheel, as the fatigued eye is supposed to become insensible to the impressions of the red rays, when the green spectrum is excited in it. Now upon turning this wheel quickly upon its axis, a green colour is produced corresponding with great exactness to the green colour of the reversed spectrum obtained in the fatigued eye from a red object.

In contemplating these reverse spectra, they are found to appear and disappear several times successively before they vanish, and when disappearing they may be renewed by withdrawing the hands from before the closed eyes, and admitting a little light through the eyelids. Because the retina, being sensible to all other rays, except those with which it was lately fatigued, is at the same time stimulated into such motions as produce the reverse spectra. On looking on the meridian sun as long as the eyes can bear its brightness, the disk will become pale and then blue.

surrounded with a white halo, and upon closing the eyes, the spectrum will appear yellow at first, and soon change into blue, which may continue for some days; as the retina having been so violently stimulated into action falls into various successive spasmodic actions, which may continue several hours, or even induce a temporary paralysis of the organ of vision. Hence the danger of looking too long at very bright and luminous objects.

The direct and inverse spectra may exist at the same time, excited by different or by the same objects. When six inches square of bright Indian pink paper is viewed for some time, upon a foot square of white paper, the green spectrum, viz. the reverse of the pink paper, appears in the center, while the external spectrum is of a pink colour, which is the direct spectrum of the coloured paper. The same will happen when the internal area is white, and the external area is pink colour. While the rays, from the bright internal object, fall with their full force upon the retina, and by fatiguing it, induce the reverse spectrum; many scattered rays from the internal pink coloured paper, fall upon the external parts of the retina, with such force as is sufficient to produce a direct spectrum of it. In the same manner, six inches square of violet paper, viewed on the same white ground, exhibited a yellow spectrum, the reverse of the violet, while the external spectrum was of the same colour, being its direct spectrum, produced as before explained.

If two colours, mutually the reverse of each other, be viewed, one surrounding the other, the scattered rays from the one, which only tend to excite its direct spectrum, coinciding with the reversed spectrum of the other, will make both the spectra more bright and vivid.

ct-glass, which is the last part of the ratio. And
hatever magnitude the image will appear to have to
e eye placed at the eye-glass, the same it will have
the eye any where in the axis of it, because the
nage is at its focal distance.

In some microscopes, a plano-convex lens is inter-
osed between the object-glass and the distinct image,
that the rays may suffer a second refraction before
ey are converged to a focus. This will contract the
nage and make it less, but the field of view will be
larged, and the vision will be more agreeable.

The area of the object-glass ought to be small, be-
use the object being placed so near to it, the extreme
ys would diverge so much, if the glass were large,
to render the image indistinct. But by contracting
e area of the glass, these very diverging rays are
xcluded from the picture. But hereby also we render
e image obscure, and therefore to increase its bright-
ess, various ways are contrived, such as placing the
bject in the sunshine, or collecting the light by a
onvex lens, or a concave mirror, and throwing it
pon the object.

A microscope of the reflecting kind may also be
ade with a small concave mirror. If the object be
laced nearer to the mirror, than its principal focus,
e image will be behind the mirror, and erect and
agnified in proportion to the distances of the object
nd image from the vertex. But if the object be placed
rther than the focus from the mirror, the image will
e on the same side, inverted, and magnified in the
ame ratio mentioned above.

object, as the lens is placed; but as we have not distinct vision by the eye at much less than sixty times that distance, or six inches, the image will therefore appear to be magnified sixty times. By such a microscope, we shall see the object as large as it would appear to the naked eye, if it were sixty times as near to it as we usually place an object, which we desire to view distinctly with the naked eye.

A double refracting microscope consists of an object-lens of a small focal distance, and one or two eye-glasses. If the object be placed at a little more than the focal distance of the object-glass from it, it will have its image formed inverted on the other side of the lens, and proportionably large, as both will be seen under the same angle from the object-glass. Now, if an eye-glass be applied to view this image at its focal distance from it, the rays will proceed parallel after refraction through it, to the eye placed in the axis of this lens. This compound microscope will magnify an object in the compound ratio of the object's distance from the lens to the limit of distinct vision to the naked eye, which is about six inches, and of the distance of the eye-glass from the image, to the distance of the object-glass from it. The first of these ratios is evident, from what we have said of the single microscope, where the eye being placed at the object-glass, would see the object as much greater, as it is now nearer to it than the limit of distinct vision: but if the eye be placed at the eye-glass, to view the distinct image, it will now see it as much more magnified, as it is now nearer to it, than when it was at the object-glass. Therefore, when the eye is at the eye-glass, it will see the image magnified in the ratio of its distance from the image, to the image's distance from the obdistances of the small mirror and eye-glass, and the quotient will give the magnifying power of the telescope.

Instead of the second concave mirror, if a plane speculum, placed at an angle of 45 degrees with the axis, were interposed to receive the rays from the large mirror before they arrived at their focus, they would thereby be reflected towards the side of the tube, where the image would be formed, which might then be viewed by an eye-glass in the side of it. This is the Newtonian form of the reflecting telescope; and as there is but one image formed in it, its magnifying power will be in the ratio of the focal distance of the eye-glass to the focal distance of the object-mirror.

The refracting telescope consists of one object-lens of considerable focal length, and one eye-glass, placed at the sum of their focal lengths from each other. When rays from a distant object pass through the object-glass, they are collected into its principal focus, where they form an inverted image of the object, which image is viewed by an eye-glass placed at its focal distance from the image, that the rays which diverge from it may fall parallel upon the eye. The magnifying power of this telescope is in the ratio of the focal distance of the eye-glass to that of the object-glass. Because to an eye at the object-glass, or at the same distance from the image as the object-glass is, it would appear under the same angle with the object; and if the eye be placed nearer to it, as for example at the eye-glass, it would appear as much greater as the eye is nearer. Therefore if the eye were placed at the station of the eye-glass, the object would appear as much greater to it, than to the naked eye, as it is nearer to the image at the eye-glass than at the object.

object, as the lens is placed; but as we have not distinct vision by the eye at much less than sixty times that distance, or six inches, the image will therefore appear to be magnified sixty times. By such a microscope, we shall see the object as large as it would appear to the naked eye, if it were sixty times as near to it as we usually place an object, which we desire to view distinctly with the naked eye.

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Although the magnifying power of the telescope is not altered by placing the eye any where in the axis of the rays that pass through the last eye-glass, yet the most advantageous position of the eye is at the focal distance of the eye-glass from it. Because, in that place, no rays from the sides of the tube can enter it, and all the rays that are parallel to the axis are there collected into a focus. Were any rays that do not proceed from the object admitted into the eye, the area of view would thereby be lessened, as some part of it would be occupied by the images of other objects which we do not desire to see. In any other position of the eye, rays from the sides of the tube would enter it, which before refraction converged to the axis, and thereby lessen the field of view.

The Galilean telescope consists of an object-glass of considerable focal length, and a concave eye-glass placed at the distance of the difference of their focal lengths from each other; so that when the rays that pass through the object-glass are converging to their focus, they converge at the same time to the focus of the eye-glass, by the construction of the telescope, and therefore will proceed parallel to the axis after refraction through the concave eye-glass. As the rays do not cross each other in such a telescope, and it is the last image of the object, that we see, the object will appear erect, and it will be magnified in the ratio of the focal length of the eye-glass to the focal length of the object-glass. The area of the field of view in such a telescope is but small, as it depends, not upon the breadth of the eye-glass, but upon the largeness of the pupil. And therefore the most convenient place for the eye is close to the eye-glass. For although the rays of each pencil are parallel after refraction, yet all the pencils are not parallel to each other; and hence the eye will receive more of them, the nearer it is placed to the eye-glass.

The great imperfection of these telescopes arises from the different refrangibility of the rays of light, which prevents their being accurately collected into a focus. The rays of different colours, which proceed from the same point of an object, are, on this account, refracted to different foci, whereby there will be formed several images of the same point of the object, of different colours, and in different places. When Sir Isaac Newton demonstrated that the rays of light were differently refrangible, according to their different colours, the perfection of the refracting telescopes became hopeless, save only by increasing their length, which rendered them cumbersome and unmanageable. But Mr. Dolland has happily discovered a method of remedying this difficulty, and has constructed refracting telescopes of moderate lengths, with great magnifying powers, and free from the inconveniency arising from the different refrangibility of light. These are called *achromatic* telescopes, from their exhibiting the images of distant objects free from colours. The peculiar construction of these telescopes, and the principles from whence it is derived, we shall now endeavour to explain.

From the experiments of Sir Isaac Newton it was generally believed, that there could be no refraction of the rays of light through any given medium, without a dispersion of them into their particular colours, and that every kind of glass would separate them to the same distance, the red from the violet, in all cases, when the beam of light suffered the same degree of refraction. But Mr. Dolland found from his experiments that this was a mistake, and that he could produce a certain degree of refraction without any separation of the beam into the rays of particular colours, by making it pass through prisms of water and glass. From hence he was led to suppose, that the same might be effected by prisms of different kinds of glass, and upon trial found that white flint-glass separated the violet rays to a greater distance from the red, than crown-glass would separate them, when there was an equal refraction of the beam by both; and consequently that when the beam of light had suffered unequal degrees of refraction by both, in a certain proportion, there was an equal dispersion of the rays. Now if these unequal refractions were made in different directions, the dispersion of the rays by one glass, would be corrected by the equal and contrary dispersion by the other; so that there would remain no separation of the rays, while there would still remain a certain refraction equal to the difference of the refractions, which the beam suffered by the two kinds of glass. Hence he composed an object-glass of three lenses, viz. two convex lenses of crown-glass, with a concave lens of white flint interposed, and all ground to the same sphere. The reason of which construction was, that as the flint-glass produced a greater degree of dispersion with a given refraction, there must be a greater refraction by the crown-glass, to produce an equal dispersion with the flint-glass. And as this excess of refraction must serve to converge the rays to a positive focus, the two convex glasses must be made of crown-glass, and the concave of white flint. That the refractions being made in contrary directions, the dispersion of the rays might be thereby rectified fully, while the greater refraction of the crown-glass might serve to converge the rays to a positive focus.

The progress of the rays through such an object-glass may be represented in the following manner. If AB* be a pencil of light falling upon the convex lens, it will be separated into the rays of different colours, and be refracted to different foci, the red at R, and the violet at V, a nearer focus. But being intercepted by the concave lens, they are refracted the contrary way, so that they are made to converge less or even to diverge, but falling with different obliquities on the concave lens, and being refracted differently by the white flint-glass, the diverging red and violet rays are now made to converge, and meet in a point and cross each other in I. But in their passage they meet with the other convex lens, which causes them to converge to a focus, in which they unite in the axis of the lens. Although this lens would of itself converge the violet rays to a nearer focus than the red, if they were incident on it with the same obliquity, yet because the violet rays meet this lens with a greater divergency than the red, they will be refracted to a more distant focus, which will coincide with the focus of the red rays, when the refractive and disposing powers of the lenses are properly adjusted to each other.

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* Plate Z.

We have here supposed that the rays diverged after their passage through the concave lens; but this is not absolutely necessary; they may be made only to converge less, and thereby unite in the axis of the lens, without the third convex lens, provided the necessary degree of convergency be effected by the first glass. But it is better, that it should be effected by two convex lenses than by one, as the aberration of the rays arising from the spherical figure of the glass is thereby corrected in some measure. Nor is it necessary that all the lenses should be ground to the same sphere, or be placed close to each other.

When the error arising from the different refrangibility of the rays is thus corrected; those arising from the eye-glasses, usually called the aberration of the rays, may also be lessened, as was hinted above, by increasing their number. If two eye-glasses of a given focal length were used, instead of one of half that focal length, they would, between them, produce the same degree of refraction that is produced by the one, and the visual angle comprehended between the extreme rays would be the same in both cases: but the refraction being equally divided between them, the error arising from the spherical figure of the lenses would be greatly lessened. This error is found to be proportional to the cube of the visual angle; but as half of the visual angle is produced by each glass, the error of aberration arising from each glass will be as the cube of half the visual angle, and the whole error from both will be as twice the cube of half the angle, which is only one fourth of the cube of the whole angle. For the cube of any number is four times as great as twice the cube of its half. $2\times1^3=\frac{2^3}{4}=2$. For the same reason, if three eye-glasses were used to produce as great a visual angle as is produced by one, the error produced by all would be but a ninth part of the error produced by the one; for one ninth of the cube of any number is equal to three times the cube of its third part. $\frac{1}{9}\times3^3=3\times1^3$. And if four glasses be used instead of one, the error is reduced to a sixteenth part.

Mr. Dolland has not only improved the object-glass of the refracting telescope, but also the eye-tube, by a new disposition of the eye-glasses, using five in number, and disposed in the following order. When the rays are refracted through the object glass, they are thereby made to converge to a distant focus, but meeting with the first eye-glass, they are united in a nearer focus, where they form an inverted image, which might be viewed by a second eye-glass placed at its focal distance from this image, but proceeding parallel through the second eye-glass, they fall upon a third, which would again converge them to a distant focus, but meeting with a fourth lens they are converged to a nearer focus, where a second image is formed, inverted with respect to the first, but erect as to the object. This secondary image is viewed by a fifth eye-glass placed at its focal distance from it. By this construction, it appears, that the telescope is compounded of two, each containing three glasses, counting the compound object-glass as one. They represent objects very distinctly, and are on some accounts preferable to reflectors, as more light is lost by reflexion than by transmission; as they are not so easily tarnished, and as they are as short as the others.

The *solar telescope* is no other than a common refracting telescope, having its end fast in a scioptric ball in a window shutter, that it may receive the direct rays of the sun; and the eye-glass being placed at a little more than its focal distance from the image, the rays from it will be converged to a very distant focus on the opposite wall, in a darkened room, where an image will be formed, that may be viewed by the weakest eyes. Eclipses of the sun, and the transits of Venus and Mercury, may thus be seen with the greatest ease.

The *magic lantern* is only one or more convex lenses, whose focal distance either when single or compounded together is very short, and placed in the side of a box, at a small distance from coloured and transparent pictures on glass, so that a very large and surprising image of the same may be made on the opposite wall in a dark room. It is necessary that they be strongly enlightened by a candle, whose light should also be reflected by a plane speculum, through the convex lens in the side of the box. As this lens is placed at a little more than its focal distance from the object, a large and inverted image of it will be formed at a considerable distance on the opposite wall. No light, but what comes from the candle, should be suffered to fall upon the picture, lest the image be made obscure: and therefore the experiment should be made in a darkened room.

The *solar microscope* is of the same kind with the magic lantern, only here the objects are very small, and are enlightened by the rays of the sun, through a hole in the window-shutter, and reflected parallel to the floor, by a plane speculum on the outer side of the window. The rays thus transmitted pass through a convex lens, whose focal distance is about two inches and a half, fall upon the object, to enlighten it in a higher degree. The rays that come from the small object thus enlightened pass through a convex lens, of one quarter of an inch focal distance, placed at a little more than the focal distance from the object, so that they may be converged to a very remote focus on the opposite wall, where a large and inverted image will be formed.

The *camera obscura*, or darkened room, performs its effects in the same manner, by means of a convex lens of considerable focal distance fixed in the window-shutter, whereby the images of external objects are painted within the room, upon a screen placed so as to receive the distinct images. The distance of the screen from the lens will depend upon the different distances of the external objects, and therefore it must be varied accordingly. No light should be admitted into the room, but what comes through the lens, from the objects; and the sight will be more agreeable, if the objects be enlightened by the sun shining on the side that is turned to the window.

The construction and use of any other optical instruments may very easily be understood from the principles already laid down, so that there is no need of dwelling any longer upon them.

An *equal-altitude instrument* is a telescope with horizontal and perpendicular wires, either three or five, placed at right angles to each other in the focus of the eye-glass; with two cylindrical arms from the middle of the telescope, tapering away to small terminations, which are to be supported in a couple of notches, that when these arms are placed horizontally by a spirit level, the telescope may move in a vertical circle. The supporters of the telescope are also constructed to turn round upon a perpendicular axis, so that it may move in any vertical circle, and be fixed at any altitude at pleasure, while it is turned round on the perpendicular axis into any azimuth. The altitude to which the telescope is set, is noted by a graduated arch.

The principal design of this instrument is to find the time of the day, by observing the time when the sun has equal altitudes both in the forenoon and afternoon. For the middle moment between these observations is the time when the sun was on the meridian, nearly.

ASTRONOMY.

Astronomy is that science which teaches the construction of the solar system, the magnitudes and distances of the bodies of which it consists; together with the fundamental laws, by which the motions of the heavenly bodies are regulated and preserved.

Nothing can exceed the grandeur and sublimity of the idea, which this science gives of the creation around us. By it we learn, that this earth on which we live, although apparently the largest with which we are acquainted, forms but a very small part in that vast assemblage of bodies of which the universe is composed: that it would be absolutely invisible to eyes like ours, at the distance of the sun; nay, that its annual orbit, although near two hundred millions of miles in diameter, dwindles into an invisible point at the distance of the nearest of the fixed stars: That however large this earth appears to us, it would require nearly one million and a half of globes, each as large as it, or of eight thousand miles in diameter, to be moulded together, to form one as large as the sun: That some of the planets, those erratic bodies, which were formerly supposed to wander through the heavens without any fixed paths, are many hundred times larger than the earth; that they are all obedient to the influence of the sun, and circulate round him at different distances and in different periods, by the force of one common law: And that the stars are not equally distant from us, but are scattered at immense distances from each other, each shining with its own inherent lustre, and dispensing light and heat to the revolving worlds, which compose their planetary systems.

Innumerable stars are discovered by the telescope, which are invisible to the naked eye, and their number increases with the perfection of that useful instrument, although it has never been found to magnify the apparent diameter of the nearest of them; so that there may be many of them at such an inconceivable distance from us, that their light, although coming with the amazing velocity of twelve millions of miles in a minute, has not yet reached our globe since the creation of the world. Their distance is so immensely great, that through a telescope that magnifies 6000 times, they appear but as so many shining points. And the more that a telescope magnifies, the more of them are discovered in a given portion of the heavens. Millions of them therefore may not be inferior to our sun in magnitude; and very probably were created for similar purposes of diffusing light and heat to their attendant worlds; which like our earth may be the appointed residence of unnumbered ranks of rational beings, all sharing with us in the unbounded munificence of the Creator. Instead then of one sun and moon, as the unskilful in astronomy are apt to imagine, this science discovers to us innumerable suns and systems, dispersed through the immeasurable regions of space; so that if our whole system, with all its planets and comets, were annihilated, they would no more be missed out of the universe than a single grain of sand from the shore of the ocean; although Herschel, the outermost of our planets, revolves in an orbit of near twelve thousand millions of miles in circumference; and some of the comets make excursions of more than ten thousand millions of miles beyond his spacious round; for still, in this extensive tour, they are nearer to the sun, than to any of the fixed stars, and therefore return to him as the center of their motions.

Now, whoever can imagine, that these countless groups of stars, of which possibly not one in a thousand was ever seen by the naked eye, were only designed as glittering spangles, to adorn the vaulted roof of the heavens, or to answer the purpose of sending a faint glimmering light to our diminutive tenement, must also be weak enough to believe, that infinite wisdom has done much in vain: when a single additional moon would have given us a thousand times more light, than is derived from their united effulgence. The astronomer, who sees that the planets and moons which belong to our system, are opake and solid globes like our earth; some bigger and some less, capable of supporting animals and vegetables, and furnished with the regular returns of day and night, heat and cold, and all the variety of seasons, that we enjoy; must necessarily conclude, that they were made for similar purposes, and that they could not have been designed merely for the accommodation of the human race, the greater number of them being absolutely invisible without a telescope. What an august and enlarged conception does this thought give of the works of the Creator! Thousands and thousands of central suns, multiplied without end, dispensing their light and heat to ten thousand times ten thousand revolving worlds, peopled with myriads of intelligent creatures, all designed for an endless progression in knowledge and happiness! and how great, how wise and good must He be, who made and governs the whole, and who continues, from day to day, to dispense the effects of his benignity and love to the remotest corners of his creation!

Although, to persons unacquainted with astronomy, many of our conclusions concerning the magnitudes, distances and astonishing velocities of the heavenly bodies, appear no better than improbable conjectures; while they are ignorant of the methods, by which we arrive at the knowledge of these matters; yet we are able to give the most satisfactory demonstrations of their truth and certainty.

The great Author of nature has chosen to support and continue the beautiful economy of the universe by the operation of second causes, under the superintendence of his almighty power; and for this purpose he has endowed all matter with an attractive quality, which increases in strength with the quantity of matter in every body, and decreases with the squares of their distances from each other. The sun, the greatest body in our system, is, of consequence, possessed of the largest share of this attractive power, so that all the planets are entirely subject to his superior influence. Had this power been left unbalanced by another, the sun would have soon attracted all the other bodies of the system to himself, but it is wisely counteracted by another of equal efficacy, namely, a projectile force, which every planet received, when it was launched from the almighty hand, through the regions of space, in a direction oblique to the central attraction. So that by the joint influence of these two opposing forces, the planet can neither fall into the central body, nor fly off entirely from it; but, like a stone whirled about in a sling, it must circulate round its center of motion.

In this manner the sun, in the center, gives light and heat and circular motion to his attending planets and comets, which revolve round him at different distances, and in different periods proportioned to the largeness of the orbits which they describe, in the following order: Mercury, Venus, the Earth, Mars, Jupiter, Saturn, and Herschel; which all revolve from west to east. The earth is attended in her annual course round the sun, by her satellite, the moon, which regards her as the center of her motion; while Jupiter is attended by four, Saturn by seven, and Herschel by six satellites already discovered. All these revolve from west to east, in the same direction with their primaries; except the satellites of Herschel, whose orbits are nearly at right angles with that of their primary.

Besides these planets, with their secondaries, there is another class of wandering bodies, denominated comets, which belong to the solar system, and are obedient to the attractive influence of the sun; which revolve round him, in all directions, in very eccentric orbits, and in protracted periods. These were anciently accounted only floating meteors in our atmosphere, and dangerous intruders upon the simplicity of the planetary system; until the more accurate observations of the modern astronomers have assigned them a place in the solar system, and laid a foundation for calculating their periodical returns. Their theory is however but little known, and the periods of only three or four of them have been attempted by calculation; although, from their very different courses among the fixed stars, we have reason to believe, that they are vastly more numerous than the planets. Nor need we wonder at this, when we consider the great number of years requisite for the performance of their extensive circuits, and the immeasurable distances to which some of them fly off from the sun, when they become invisible through the best telescopes; the uncertainty of their proximity to other systems at the extremities of their orbits, and the variety of causes, that may either accelerate their motions, or protract their periodical returns.

This is the true position of the bodies, which compose the solar system; which was first taught by Pythagoras, and after being disallowed or neglected for many centuries, was again revived by Copernicus, an ecclesiastic of Polish Prussia, and finally demonstrated to be perfectly conformable to all the celestial phenomena, on mathematical principles, by the immortal Sir Isaac Newton; and therefore, since his time, adopted by every astronomer in the world, who is even but moderately acquainted with the subject.

Other positions indeed have been assigned to the planets of our system, by Ptolemy, an Egyptian astronomer, who flourished about the 140th year of the christian era; and who supposed the earth to be placed in the center of the universe, and that all the heavenly bodies moved round it from east to west in 24 hours, in the following order:—the moon, Mercury, Venus, the sun, Mars, Jupiter, and Saturn; then the stars, and beyond them, his figment of the primum *mobile*, which he supposed gave motion to all the rest. He knew nothing of Herschel or the comets.

Tycho Brachè, a noble Danish astronomer, supposed, with Ptolemy, that the earth was placed in the center of the universe; and that the moon, and sun, and stars, revolved round it, from east to west in 24 hours; but he differed from Ptolemy in this, that he supposed the sun to be the center of motion to Mercury, Venus, Mars, Jupiter, and Saturn, which he carried with him round the earth. The semi-tychonic system gave a motion to the earth round her axis in 24 hours from west to east, instead of the diurnal motion of the sun moon and stars in the same time. But both these hypotheses were found to be so contrary to the plainest phenomena of the heavenly motions, that they gave way to the force of demonstration, which attended the pythagorean hypothesis, in the hands of the incomparable Newton. If this hypothesis, which commonly passes under the name of the copernican system, be found to be perfectly correspondent to all the phenomena of the heavenly motions, we may safely conclude, that it is the only true hypothesis, especially if no other hypothesis be found sufficient for the same purpose. And this we trust will evidently appear in the course of the following lectures.

Before we enter upon the enumeration of these phenomena, it will be necessary to explain some terms, that frequently occur in this science.

The axis of a globe is any line which passes through its center, and the terminations of this line at the surface of the globe are called its poles; the same points are also called the poles of a great circle surrounding the globe, when every part of the circle is equally distant from the said points; and consequently every great circle divides the globe into two equal parts. The same points are likewise the poles of a lesser circle of the globe, which divides it into two unequal parts, and consequently one pole will be nearer to the circumference of the circle than the other, although no part of the circumference is nearer to the pole than another.

Every circle, whether large or small, is supposed to be divided into 360 equal parts, which are called degrees, each degree into 60 equal parts called minutes, and each minute into 60 seconds, and so on in a sixagesimal division as far as you please.

If the earth be supposed to turn round upon an axis, this axis is called the axis of the world, and its terminations, when produced to the fixed stars, are called the north and south poles of the world. That great circle which is every where 90 degrees from the poles of the earth is called its equator; and if its plane be supposed to be extended to the concave surface of the heavens, it will mark the circle which is called the celestial equator.

The sun, in his apparent annual course in the heavens, describes another circle, denominated the ecliptic, which is not coincident with the celestial equator, and consequently cuts it in two opposite points, called the equinoctial points; and the other two points of this circle, which are each 90 degrees distant from the former, are called the solstitial points of the ecliptic. The planes of these two circles in the heavens are inclined to each other in an angle of about 23° 28′, which is called the obliquity of the ecliptic to the equator. This angle is measured on the surface of the globe by the number of degrees of a great circle intercepted between the pole of the ecliptic and that of the equator, which circle of the globe is known by the name of the solstitial colure: it cuts both the ecliptic and equator at 90 degrees distance from their intersections at the equinoctial points; and is cut at right angles, in the poles, by the equinoctial colure, which is another great circle passing through the poles of the ecliptic and cutting the equator and ecliptic at their intersections.

The ecliptic in the heavens is supposed by astronomers to be divided into twelve equal parts called signs, beginning at the point of its intersection with the equator and proceeding eastward, each sign containing thirty degrees; to which portions they have affixed the following names, in order, viz. Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricornus, Aquarius, Pisces.

If two small circles of the globe were supposed to be drawn parallel to the ecliptic at the distance of eight degrees on each side of the ecliptic, the space comprehended between them is called the zodiac in the heavens; which broad space comprehends the orbits of all the planets, together with all the stars in the twelve signs of the ecliptic above mentioned. The reason of the above denominations, we may comprehend, in the following manner. When the ancient astronomers had marked, on the surface of an artificial globe, the position of the stars, as they appeared in the heavens, with their relative distances from each other; they delineated over them the figures of various animals, persons, and things; and the stars that were covered by these figures, were said to belong to that constellation, which bore the name of the person or animal painted upon them. Thus all the stars, from the intersection of the ecliptic and equator, eastward, for thirty degrees, and lying within eight degrees on each side of it, were said to belong to the constellation Aries; as the figure of the ram, on the globe, was drawn of that size; and so of the rest, quite round the heavens to Aries again. Thus they gave arbitrary names to various assemblages of stars over the whole face of the heavens, according as their fancies had dictated the various figures, which they had painted over the corresponding places of the stars on the surfaces of their celestial globes.

The space of eight degrees on each side of the ecliptic, called the zodiac, from the animals whose names the constellations bear, comprehends the apparent paths or orbits of all the planets, so that none of them were ever observed to be farther north or south of the ecliptic. Their courses through the heavens are all different from one another, each differing more or less from the ecliptic, which is the orbit of the earth round the sun, and which is made the standard, with which the others are compared.

As none of the orbits of the planets are coincident with the ecliptic, they will intersect it in two opposite points which are called the nodes. One half therefore of the orbit of a planet will lie to the north of the ecliptic, and the other half to the south of it.

That point of a planet's orbit, where it begins to appear north of the ecliptic, is called the ascending node, and the opposite point, in which it begins to appear to the south of the ecliptic, is denominated the descending node of the planet.

If two small circles be drawn parallel to the equator at the distance of 23° 28′ from it, on both sides of it, they are called the tropics of Cancer and Capricorn; because they will touch the ecliptic at the beginning of these signs: the space between them is called the torrid zone.

If two small circles be drawn parallel to the equator 23° 28′ from either pole, they are called the arctic, and antarctic circles; and the spaces comprehended between them and the poles are called the frigid zones; while the spaces between these and the tropics are called the temperate zones.

Any great circle of the globe, that passes through the poles of another great circle, is called a secondary to it.

The secondaries of the equator on the surface of the earth serve to determine the latitude of a place, which is counted upon it both ways from the equator to the poles; but in the heavens, they serve to determine the declination of a star, which is counted the same way; and these secondaries are denominated meridians.

The secondaries of the ecliptic, in the heavens, serve to determine the latitude of a star, which is counted both ways from the ecliptic towards its poles.

The longitude of a heavenly body is counted on the ecliptic from the first point of Aries; but the longitude of a place on the earth is counted on the equator from some first meridian. The right ascension of a heavenly body is also counted on the equator from the first point of Aries. So that the right ascension and declination of a celestial body are counted the same way with the longitude and latitude of a place on the earth; while the longitude and latitude of a celestial body are referred to the ecliptic.

That circle which bounds the view of a spectator on the surface of the earth is denominated his sensible horizon, and is a tangent to the earth at the place of the spectator; when the plane of the horizon is extended to the fixed stars it coincides with his rational horizon, which passes through the center of the earth, and is parallel to the former.

The secondaries of the horizon are called azimuth circles, or vertical circles; and that which passes through the east and west points of the horizon is called the prime vertical, while that which passes through the north and south points of it is called the meridian. All these secondaries of the horizon pass through the zenith and nadir, which are two points, directly over the head and under the feet of the spectator.

The foregoing explications being given, we now proceed to consider the orbits, nodes, distances and periods of the planets, which are represented in the following table.

Planets' Names.	Ascending Node.	Inclination of Orbits.	Periodical Times.	Distances a ☉	Rotation on Axis.
	° ′	° ′ ″	D. h. ′ ″	Miles.	D. h. ′ h. ′ ″
Mercury,	♉ 14 43	6 59 20	87 23 16	36,841,468	
Venus,	♊ 14 26	3 23 5	224 16 49	68,891,486	24 8 or 23 20 59
Earth,			365 6 9 14	95,173,117	23 56
Mars,	♉ 18 30	1 52	686 17 18	145,014,148	24 40
Jupiter,	♋ 7 20	1 20	4332 12 20	494,990,976	9 56
Saturn,	♋ 21 50	2 33 30	10759 6 36	907,956,130	10 32
Herschel,	♊ 13 30	0 48 10	83 years.	1898,967,700	} the earth's dist. 100 mill.
				1908,180,000	} by Herschel.

THE SUN.

The sun, being by far the largest body belonging to our system, must necessarily possess the center, and by its superior attraction, regulate all the motions of the planets and comets. It would require 1,383,470 globes, each as large as our earth, to be moulded together, to form one as large as the sun. We are not however to suppose that he invariably occupies the same point of space, so as never to move out of it. While the planets and comets move round him, he moves round the common center of the solar system, which however is not far from his center; because he is so much greater than all the bodies together, which move round the same center of gravity. Hence in common language we generally say that the planets move round the sun, at the center of the system; when, in strictness of speech, both they and the sun revolve round one common center of gravity; which is, however, not so far from his center as to his surface, and which must vary its distance from his center continually, according to the positions of the planets and comets from him.

In estimating the magnitude of the sun, I have considered it as a globe, although to the naked eye, and even through a good telescope, it appears but a circular plane. We arrive at the certainty of its globular figure from the appearance and motion of the spots which are often seen by a telescope upon his surface. They all have a regular motion across his disk from east to west, moving slowly when they first appear on the eastern edge, and gradually quickening their velocity as they approach the center, and then growing slower as they come near to the place of their disappearance on the western limb. Besides this, every spot appears broadest about the middle of its passage over the disk, and narrowest towards the edges, although it still preserves the same length. Now these are the appearances, which should necessarily take place, upon the supposition of the globular figure of the sun, and of there being thin broad substances adhering to his surface, or floating in his atmosphere near to his body; and therefore they warrant our conclusion that the sun is not a luminous plane, but a solid sphere. After these spots have passed over the disk of the sun, they disappear for the same time, which they spent in passing, and many of them reappear in the same place, and pass over him in the same manner as they did before. This has enabled us also, to know, not only that he has a rotation round his own axis, but also the time of this rotation, which is accomplished in about 25^{d}, 15^{h}, 16′, from a fixed star to the same again.

It was found, from the mean of a number of observations, that the solar spots return to the same place again, after 27^{d}, 12^{h}, 20′, but in that time the earth had moved the same way in her orbit 26°, 22′, therefore say as $360°+26°, 22′=386°, 22′ : 360° :: 27^{d}, 12^{h}, 20′, : 25^{d}, 15^{h}, 16′$, the true time of the sun's rotation on his axis, as it would be seen from a fixed star. The observations above referred to, were made in the month of May, when the paths of the spots over the sun's disk were nearly in a right line.

By observations on the apparent paths of the solar spots, astronomers have also found, that the sun's equator is not coincident with the ecliptic, but is inclined to it in an angle of 7°, 30′, and that it cuts the ecliptic in the eighth degree of Gemini and Sagittary, which are therefore called the sun's nodes. When the earth is in either of these nodes, the sun's equator, if visible, would appear to be a straight line. The earth, in the present century is in the 8th degree of Sagittary on the 18th of May, and in the other node on the 19th of November. From the 18th of May, the sun's equator begins to appear as a semi-ellipsis, convex towards the south pole, till the 20th of August, when the ellipsis is widest; from that time it grows narrower, till the ellipsis degenerates into a straight line on the 19th of November. After this time the solar equator becomes convex towards the north pole, with an increasing curvature till the 15th of February, when the curvature is greatest, and then it grows less again till the 18th of May.

As all the solar spots are carried round him in circles parallel to his equator, their apparent paths over his disk will be right lines in May and November, when the earth is in one of the nodes, and in all other months, they will be elliptic curves.

The spots that appear on the face of the sun are by no means permanent; few of them continuing during the time of a complete revolution; some appearing on the eastern edge of the sun and vanishing in a few days, others arising near the middle of the disk, while others split and divide into a number; and on the contrary many contiguous spots have been observed to run together, and form one of larger dimensions. Many have been measured, whose diameters have been found to be three or four times the diameter of the earth. All these spots contain a nucleus of various figures and dimensions, intensely black, and surrounded with an infinite number of small specks, each as black as the neucleus, but very small, separate, and distinct from each other, resembling black sand thinly spread over a sheet of white paper, when viewed through a good telescope, and in a good state of the air. When they are viewed in an unfavourable state of the air, these small specks appear confused, and like a darkish penumbra surrounding the central spot, as they are generally described by the European astronomers. But the purity of the American air has discovered to us, that, what they call a penumbra, is really composed of an infinitude of very small, distinct, specks, each as intensely black as the central spot.

Various hypotheses have been framed by ingenious astronomers to account for the phenomena of the solar spots, but as they are mere hypotheses, unsupported by proofs of any kind, I shall not trouble you with the recital of any of them.

A faint ring of light has been observed to surround the sun, in the time of a total eclipse, and to follow him; which makes it probable, that he has an atmosphere, which is the cause of that appearance.

Although the sun appears to pursue the same course among the fixed stars from year to year, yet he appears to us to have a much greater meridian altitude at one time than at another; but this arises entirely from the motion of the earth and the situation of her axis; as we shall hereafter see.

MERCURY.

MERCURY is the nearest of our planets to the sun which we have yet discovered; but being seldom seen, and no spots appearing upon his face, it is not known whether he has any rotation on his axis, or how he may be situate. But, if we reason from analogy, it is probable that he has the regular returns of day and night, of summer and winter, and all the other seasons of the year. We know that his orbit is inclined to the ecliptic in an angle of 6°, 59′, 20″, which may make an alteration of the sun's meridian altitude if his axis be inclined to the ecliptic. In his course round the sun he moves at the rate of 95,000 miles every hour, which is near 200 times as quick as a cannon ball. As he is but 36,841,468 miles distant from the sun, and we are distant 95,173,117 miles from the source of light and day, he must receive near seven times as much light from the sun as we receive: the quantity of light being as the squares of the distances inversely. And although his heat from the sun should exceed ours in the same proportion, yet this would furnish no argument against his being inhabited, as it is as easy for the Almighty to suit the constitutions of his inhabitants to that degree of heat, as it was to accommodate ours to the temperature of our earth. Mercury appears to us, through a good telescope, when viewed at different times, with all the phases of the moon, save only that he never appears round and full; because when he should appear in this manner, he is so near to the sun as to be lost in his superior splendour. As his enlightened side is always turned towards the sun, this proves that he shines not by any inherent light of his own, but by the light of the sun, reflected from his surface. And that he moves round the sun, in an orbit, included within the orbit of the earth, or at least in an orbit of less radius, is plain, from his never having been seen in opposition to the sun, nor even more than about 23° degrees from him, either to the east or west of him, which is styled his greatest elongation. And besides this, he is sometimes seen on the face of the sun like a round dark spot, where he never could be seen, if he did not revolve in an orbit included by the orbit of the earth, or if he had any inherent light. If he shone with an inherent light he could not be distinguished from the bright disk of the sun, when upon it, and could not be seen like a dark spot upon his face.

These particulars might be easily exemplified in the following manner. Let any person carry a ball suspended by a string round a lighted candle, with a slow and regular motion; and let a spectator, placed at the distance of three or four yards, view the part of the surface that is enlightened by the direct rays of the candle, and he will then observe it, in different positions, to assume the different phases of the moon. When the ball is between the spectator's eye and the candle, it will appear dark, as no part of the enlightened disk is turned towards his eye; but when it is opposite to the candle and beyond it, the enlightened disk or hemisphere is turned both to him and to the candle, and therefore it will in this position appear like the full moon. Nearly in the middle between these positions on each side, only one half of the enlightened surface is then turned to the spectator; and therefore the ball will appear like the moon in her quadratures. But in every other position between these, it will appear more or less horned or gibbous. If this experiment be made with a flat plate, it can never be made to appear horned or gibbous, in any position whatever. If the spectator remove to a considerable distance, he will observe the following appearances. The ball will seem to him to vibrate backwards and forwards on each side of the candle in a right line, moving quickest when it is between his eye and the candle, and slower when beyond it; and at the greatest elongations on either side, it will appear stationary for a small time; spending more time in passing from one of these stations to the other, while it passes beyond the candle, than while it passes between the spectator and the candle; provided still, that the eye, the candle and the ball be kept in the same plane; and that the ball move with a uniform and equable velocity. The ball will appear to move from the right hand to the left in less or more time than in the contrary direction, according as it is between the eye and the candle, or beyond it; although in every revolution it will appear to recede to the same distance, on each side of it.

But if the ball be carried round the candle in an ellipsis, instead of a circle, and the candle be placed in one of the foci, the stationary points, at which the progressive and retrograde motions succeed each other, will be at different distances from the candle, unless the eye and the candle both lie in one of the axes. Now all these appearances take place in the motion of Mercury round the sun, and are precisely such as they should be, upon the supposition of its being an opake globe revolving round the sun in an orbit included within the orbit of the earth. Hence we conclude, that this is the figure and position of his orbit.

When Mercury is between the sun and the earth, he is said to be in his inferior conjunction, and in his superior conjunction, when he is beyond the sun. In each of these positions he must be in every revolution, so that he has two conjunctions with the sun and no opposition in each apparent revolution, and this apparent time would be the same with his true periodical time, if the earth were at rest. But, as the earth is moving the same way, when he has completed his period round the sun as viewed from a fixed star or the center of the sun, counted from the time of his inferior conjunction, the earth will have moved near a quarter of her orbit forward, so that it will require an additional time to overtake it and be again in its inferior conjunction. This may be illustrated by the conjunction of the two hands of a watch or clock, which always requires more time than a complete revolution of the quicker.

If the orbit of Mercury were coincident with the ecliptic, Mercury, at every inferior conjunction, would pass over the face of the sun, and be seen upon it. But his orbit is inclined to it, in an angle of near seven degrees, and cuts it in the 14th degrees of Taurus and Scorpio, in which places the earth is about the 4th of May and the 6th of November. Should an inferior conjunction of Mercury, therefore, happen at either of these times, when he is near to his nodes, and the earth is in or near to the plane of his orbit, Mercury will then appear like a round black spot on the face of the sun, and pass over it from east to west in a few hours. This is denominated a transit of Mercury. As the periodical times of the earth and Mercury are nearly commensurate, and Mercury's motion is so quick, these transits frequently happen. We cannot see Mercury on the sun, at his next transit, on the 4th of May, in the year 1786, as it will be over, before the sun rises. But we may expect to see him transit, on the 5th of November, 1789, at 8^h, 8′, 8″, in the morning.

Mercury is seldom seen by us, because he is generally hid in the rays of the sun, except when near his greatest elongation from the sun, when a line from the earth to him becomes a tangent to his orbit, and for a few evenings or mornings about that time.

VENUS.

VENUS is the next planet in order, which revolves round the sun from west to east, moving at the rate of eighty thousand miles per hour, and performing her periodical revolution in 224^{d}, 16, 49, at the distance of 68,891,486 miles from the sun. She has all the phases and peculiarities of motion which we have observed in Mercury; from which we conclude, in the same manner, that she is a solid globe, shining only with borrowed light from the sun; moving in an orbit included by the orbit of the earth; having an inferior and superior conjunction with the sun, but no opposition; putting on all the phases of the moon; sometimes appearing on the east side of the sun, when she becomes our evening star, and again, on the west side of him, when she is our morning star; being each for 290 days together; appearing stationary between her conjunctions, at her eastern and western elongations, when she is distant from the sun about 48°; and passing over the face of the sun, when her inferior conjunction happens about the time when she is in her nodes, that is about 14° of Gemini and Aquarius.* Her transits do not happen so frequently as those of Mercury. They happen in the months of May and November, when the earth is in her nodes, and generally in eight years after one transit, wanting two days, there will be another, and then not again for near a century: for eight years contain 2922 days; which are nearly equal to thirteen revolutions of Venus, or 2920 days. Three only have yet been observed, viz. in 1639, 1761, 1769, and the next will be seen in Nov. 1874; after which no other will happen till May 1996, and in eight years more there will be another in May 2004.

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* N.B. The place of the descending node of Venus in 1786, August 25, was 8^{s}, 14°, 44′, 38″, by Mr. Bugge, and the annual motion of the nodes is 30″ 37. The inclination of her orbit 3° 23′ 38″ 6.

That her orbit includes the orbit of Mercury is evident from this, that her greatest elongation is about 48°, whereas that of Mercury is but about 23°.

From the spots observed on her surface, astronomers have determined her rotation round her axis to be performed in 24 days and 8 hours; so that she has but 9¼ days in her year; for dividing 224^{d} 16^{h}, by 24^{d} 8^{h}, the quotient is 9¼ nearly. This quarter of a day will make every fourth year a leap year in Venus, as it is with us, which will be more fully explained hereafter.

By the same spots, we have found that her axis, on which her diurnal rotation is performed, is inclined to the axis of her annual orbit, in an angle of 75 degrees, her north pole inclining to the 20th degree of Aquarius, whereas that of our earth inclines to the first degree of Cancer. This will make her summers to be at the same time with our winters, and vice versa. As her axis lies almost in the plane of her orbit, differing from it only 15 degrees, her north pole going foremost in her annual course, her motion resembles that of a ball shot from a rifle, which has a rotatory motion round that axis, which lies in the direction of its motion. Hence those stars, which to us seem to have the slowest motion, appear to her inhabitants to move with the greatest velocity, and vice versa, her north pole being but a few degrees out of the ecliptic.

As the sun alters his declination 75 degrees in Venus north and south of her equator, her tropics must be within 15 degrees of her poles, and her polar circles at the same distance from her equator. This will occasion a remarkable peculiarity in her seasons. At her equator, and polar circles, there will be two springs, two summers, two autumns, and two winters; because, when the sun is at her tropics, his rays will be more oblique at her equator, than they are with us in the middle of winter, which will occasion a winter there when with us it would be the middle of summer. And the case is nearly the same at her polar circles. At her poles, there is one spring and autumn, a summer as long as them both, and a winter as long as the other three seasons. The difference of heat and cold will be much greater in Venus at her poles, than with us, as the sun comes within 15 degrees of the zenith and is again 75 degrees below the horizon. As there are but 9¼ days in her year, in which time the sun changes his declination 150 degrees, if he pass vertically over any place on any day, he will pass many degrees from it on the next, which will give the heated regions time to cool. From this great change of declination in the course of a single day, the longitude may be as easily found in Venus, as the latitude is with us.

That Venus is surrounded with an atmosphere of considerable density, sufficient to refract the sun's rays, was very evident from her appearance, at her internal contact with the limb of the sun, at her last transit; for when she was fully advanced upon the sun's disk, still his limb did not appear quite to surround her, for a few seconds, until her atmosphere also advanced upon him. She appeared to adhere to his limb by a black and dark ligament, which after a few seconds suddenly broke, and left the sun's limb perfectly bright, when that part of her atmosphere passed his limb, which was dense enough to refract his light.

As she is furnished with the regular returns of day and night, with all the variety of seasons, and an atmosphere for the support of animal life, she is probably designed to be the residence of living creatures, whose constitutions are adapted to the temperature of their habitation.

As the orbit of Venus is inclined to the orbit of the earth in an angle of 3°, 23′ 38″, she would appear to a spectator at the center of the sun, to be in the ecliptic, when she was at her node, and to be continually increasing her distance from the ecliptic, until she arrived at her limit, or 90 degrees from her node, where it would amount to 3°, 23′, 5″, which is then greatest; from thence it would decrease again till it vanished at her node. The like appearances would take place in the other half of her orbit. These variable distances from the ecliptic, being measured on a secondary of the ecliptic are called her heliocentric latitudes, when supposed to be viewed from the center of the sun; but the same arches of the secondary, when viewed from the center of the earth, are called her geocentric latitudes, and must appear of different lengths, according to the earth's different distances from Venus. The same difference takes place between the heliocentric and geocentric latitudes of any other planet.

The geocentric latitude of a planet therefore will be continually variable on two accounts, viz. first, it will increase or decrease directly, with the heliocentric latitude; and secondly, it will appear greater as the earth's distance from the planet grows less: so that it will be directly as the heliocentric latitude, and inversely as the earth's distance from the planet. When

Venus is in her inferior conjunction she is nearer to the earth than she is to the sun, and therefore her geocentric latitude will then be greater than her heliocentric latitude. The same will be the case with all the superior planets at their oppositions. But when Mercury is in his inferior conjunction, he is still nearer to the sun than he is to the earth, and therefore his geocentric latitude at that time will be less than his heliocentric latitude; and the same will be the case with all the superior planets at their conjunctions with the sun.

Venus does not appear brightest, when most of her enlightened disk is turned towards us. Her brightness depending on the quantity of light reflected to the eye, and this quantity being directly as the enlightened part of her disk, and inversely as the square of her distance from the earth, she will appear brightest when this quantity becomes a maximum.

THE EARTH.

The earth on which we live is the next planet which revolves round the sun as the center of her motion. She revolves from west to east in $365^d\ 6^h\ 9'$ from any star to the same again, at the distance of 95,173,117 miles from the sun, moving at the rate of 68,217 miles every hour in her orbit, which is nearly 150 times as fast as a cannon ball. The plane of its orbit, called the ecliptic, we make the standard with which the orbits of the other planets are compared. Besides her annual motion in her orbit, she turns round on her axis once in every $23^h\ 56'\ 4'$, which produces an apparent motion of all the celestial bodies from east to west in the same time. Her axis is inclined to the axis of her orbit in an angle of 23° 28′, and points to a star, nearly, which is usually called the north pole, and which is distant from the pole of the world only 1° 45′ in the present century. A secondary of the ecliptic passing through the pole of the world, passes through the first degree of Cancer; so that the axis of the earth declines towards the first degree of that sign.

From the two motions of the earth on her axis and round the sun, we have not only the succession of day and night, but also the different seasons of the year; as may be explained in the following manner.

By the rotation of the earth on her axis from west to east, all the heavenly bodies, of which the sun is one, appear to move from east to west, and every appearance is the same, whether we suppose that the sun and stars really move from east to west, while the earth remains quiescent, or that the sun and stars remain at rest, while the earth revolves from west to east on her axis. If we suppose a broad plane touching the surface of the earth where we are, and extending to the concave heavens, this plane, which is our sensible horizon, will divide the heavens into two parts; one half of the heavenly bodies being above the plane and one half below it. Now if this plane, being fixed to the surface of the earth, move with it while it revolves from west to east, those heavenly bodies, which were below it towards the east, will gradually rise above it, and the sun among the rest; while those that were above it towards the west will gradually sink below it. While the sun is above the horizon it is day, while below it, it is night. In this manner all the stars appear to go round the earth from east to west, in $23^h\ 56'\ 4''$, the time of a complete rotation of the earth round her axis. The sun also would appear to go round in the same time, if he kept his place among the stars without variation. But this is not the case. While the earth is moving in her orbit from west to east, about a degree every day, the sun will appear to move over the same space, among the stars, in the same time. If you now suppose the sun and any particular star to be elevated at the same moment above the horizon, when the earth has completed one rotation on her axis, the same star will rise again above the horizon, but the sun will not rise with it; because in the mean time he has advanced a degree farther to the eastward, and it will require 3′ 56″ of time over and above a complete rotation of the earth to overtake the sun and raise him above the horizon. So that while the stars require $23^{h}\ 56'\ 4''$ to complete their apparent revolution, the sun requires 24 hours, or 3′ 56″ more. Hence the stars rise so much sooner every night, as to rise 366 times in 365 natural days, making 366 sidereal days in the year, as there are so many complete rotations of the earth on her axis in the course of a year consisting of 365 days. This is the difference between sidereal and solar time.

The whole of this may be illustrated by the motion of the hands of a clock. When the minute-hand had completed a revolution, it would overtake the hour-hand, if it had no motion; but this having moved forward, in the time, a small space more time than a complete revolution is necessary to bring them together.

As the motion of the earth on her axis is equable, we derive from it an easy and certain method of regulating our clocks. If a telescope be placed in the meridian—having cross hairs in the focus, then observing when any star passes the middle hair, note the time by the clock, and if it be found to pass the same hair, on the succeeding night 3′ 56″ sooner by the clock, the clock goes regularly and is well adjusted. If the star come sooner than 3′ 56″ to the hair, than what it did the night before, the clock loses time; but if later, she goes too fast, and must be regulated accordingly. And even without a telescope, if through a small hole in a thin plate of tin fixed to a post, you observe the time when a star disappears behind any obstacle at a distance, and find that, on the succeeding evening, it disappears 3′ 56″ sooner than on the night before, the clock goes regularly; but if not, it may be regulated in the manner already mentioned.

SEASONS OF THE YEAR.

The inclination of the axis of the earth to the axis of the ecliptic in an angle of 23° 28′ occasions the variety of our seasons in the course of the year. In order to understand this, we must remember that the sun can enlighten only one half of a planet at a time, and the circle which bounds the enlightened part of the disk is called the circle of illumination. This circle is every where 90 degrees from that point to which the sun is vertical on the surface of a planet. Now when the earth is in Libra as seen from the sun, and the sun appears to enter Aries, as he does on the 22d of March, he will appear to be in the equator, because the ecliptic, which is his annual course, cuts the equator at that point, and his rays will reach from pole to pole, and consequently all the parallels of latitude will be equally divided by the circle of illumination. Hence the days and nights will be equal, all over the earth, which constitutes that season which we call spring. As the sun apparently passes from west to east through Aries, Taurus, and Gemini, to the beginning of Cancer, the circle of illumination gradually extends beyond the north pole, and falls short of the south pole, until it extend at last 23° 28′ beyond the north pole, and fall so much short of the south pole. This will cause all the parallels of latitude to be unequally divided by the circle of illumination, the greater part of them falling in the circle of illumination in the northern hemisphere, and the lesser part in the southern hemisphere. Hence, while the sun is advancing through these three signs, the days will be gradually increasing in length, and the nights growing shorter, in all northern latitudes, and the contrary in all southern latitudes. All this will be greatest, when the sun arrives at the beginning of Cancer, which will constitute the middle of summer on the north side of the equator, and the middle of winter to the south. The north frigid zone, included within the arctic circle, is now wholly within the circle of illumination, and the sun appears to go round without setting; while the south frigid zone is wholly in darkness, so that the sun does not rise to it but passes round below its horizon. Hence the sun never sets for six months at the north pole, nor rises at the south, but appears to go round in the horizon on the day of the vernal equinox, and every day gradually rises higher and higher until he have attained to the altitude of 23° 28′, on the day of the summer solstice, which is accomplished in three months; in the next three months he gradually sinks lower in every revolution until he come to the horizon again. During these three months, while the sun passes through Cancer, Leo, and Virgo, to the beginning of Libra, where he arrives on the 22d of September, the days, in the northern latitudes, gradually grow shorter, and the nights longer, until they become of equal length, at the autumnal equinox, when the sun is again in the equator, and all the parallels of latitude are equally divided by the circle of illumination from pole to pole. This season is called the autumn.

While the sun passes through Libra, Scorpio, and Sagittarius, to the beginning of Capricorn, where he arrives on the 22d of December, the same appearances take place in the southern hemisphere, as were found to take place, from the 22d of March to the 22d of June, in the northern hemisphere. The days grow gradually longer in all south latitudes, and shorter in the northern hemisphere; until it shall be the middle of winter in north latitudes, and the middle of summer in all south latitudes. At this time, the whole antarctic circle is included within the circle of illumination, and the arctic circle is excluded. The sun will not therefore set to the south pole, nor rise to the north, for six months together. In the remaining three months of the year, the sun appears to move through Capricorn, Aquarius, and Pisces, to the beginning of Aries, where he arrives on the 22d of March. During this time, the days in south latitudes decrease in length, and on the contrary increase in all north latitudes, until they shall be each twelve hours long, all over the globe.

PRECESSION OF THE EQUINOXES.

By the diurnal rotation of the earth round its axis, the centrifugal force of every particle on the surface of the earth will be increased, and its gravity lessened, in proportion to its distance from the axis of the earth. So that about the equator, the surface of the earth will rise higher from the center, than towards the poles; and the earth will assume the figure of an oblate spheroid, whose equatorial diameter will be longer than the polar diameter. The proportion between these diameters Sir Isaac Newton fixes to be that of 230 to 229. And the measures that have been made of the length of different degrees of latitude in various parts of the earth, when compared with one another, have induced the astronomers of Europe to acquiesce in his determination, as being nearly a mean of all their measures. As the sun and moon are out of the plane of the equator, for the greatest part of the year, and attract this redundancy of matter, that is accumulated round the equator, more forcibly than the other parts of the earth, they must have a constant tendency to bring the equatorial parts of the earth directly under them. The consequence of this must be, that the equator will be brought under the sun, before an annual revolution is completed, as counted from the time when the sun was at the same equinox before; or in less time than is necessary for a complete revolution of the earth round the sun, by 20′ 17″ of time. So that from the time the sun is in the equator at the vernal equinox, till his return to it again, there will elapse only $365^d\ 5^h\ 48'\ 57''$; whereas the periodical time of the earth is $365^d\ 6^h\ 9'\ 14''$. The first of these intervals of time is called the tropical year, the other is called the sidereal. In the difference of these times, viz. 20′ 17″, the sun comprises an arch of 50″ of a degree: so that after he has left the equator, at an equinox, he crosses it again while he is 50″ of a degree to the westward of the point where he crossed it before. Hence the equinoctial points shift backwards, or to the westward, at the rate of 50″ per annum, or a degree in 72 years. This is called the precession of the equinoxes. This point of intersection, where the ecliptic cuts the equator, however it be continually shifting towards the west, is still called the first point or beginning of Aries, without any regard to the stars of that constellation. This retrocession of the equinoxes towards the west has occasioned a small apparent motion of all the fixed stars towards the east, at the rate of a degree in 72 years. So that those stars, which were in the infancy of astronomy in the sign Aries, are now advanced into the sign Taurus, and those of Taurus are now in the sign Gemini. At the creation of the world, the star that was at the intersection of the equator and ecliptic, in 5790 years will, at the rate of 50″ per annum, have advanced 2° 20′ 25″ towards the east; and after 25920 years the intersection will have receded quite round the equator. The consequence of this motion is, that the pole of the earth's axis would describe a circle round the pole of the ecliptic in 25920 years, denominated the *great year*, keeping at the same distance of 23° 28′ from it, if the obliquity of these circles continued the same. But it will gradually approach towards the pole of the ecliptic in each revolution, at the rate of about half a second per annum, or 3° 36′ in each revolution: and by the same quantity does the obliquity of the ecliptic to the equator decrease. It is now about a third part of a degree less than what it was in the time of Ptolemy, and most of the astronomers since his time have found it to decrease gradually down to the present time. This arises from the same cause with the precession of the equinoxes. Because the sun and moon acting, for the most part, obliquely, on the redundancy of matter about the equator, must necessarily bring these circles to a nearer coincidence with each other.*

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* It has been satisfactorily demonstrated by M. le Grange, that the ecliptic will never coincide with the equator, nor change its inclination above 2 degrees; but that the solar planetary system oscillates, as it were, round a medium state, from which it will never swerve very far. *Ed.*

EQUATION OF TIME.

From the obliquity of the ecliptic to the equator, and the unequal motion of the earth in her elliptical orbit, there arises an inequality in the lengths of days as measured by the returns of a star and of the sun to any given meridian; which is called the equation of time. As the earth's motion on her axis is perfectly equable, never going faster at one time than at another, the sidereal days are all equal, and so would the solar days also be, if the sun always appeared to move in the equator, and the earth's orbit were a perfect circle, so that the sun would appear to advance to the eastward of a given star an equal space daily. But this not being the case, his motion is very unequal, and he sometimes appears to revolve round in less than 24 hours, and at other times he requires more time to reach the same meridian which he left, on the preceding noon. The time shown by the sun is measured by a sun dial, but the time shown by the stars is measured by a clock, which goes regularly. The dial and the clock therefore can never agree together excepting on four days of the year. The clock will be before the dial, from the 24th of December till the 15th of April, and from the 16th of June till the 31st of August; and behind it, at all other times. The difference between the clock and the dial will amount to about a quarter of an hour, when greatest, and will gradually decrease and diminish down to nothing on these four days. As this difference arises from two causes, first, the obliquity of the ecliptic to the equator, and secondly, the unequal apparent motion of the sun in his elliptical orbit, we must consider each of these causes separately, and then determine their joint effect.

As the solar day is longer than the sidereal day, any point on the surface of the earth describes more than a whole circle in the course of a solar day, seeing the sun in the mean time has advanced about a degree to the eastward. This arch which the place describes, more than a complete circle in a solar day, is equal to the sun's daily increment of right ascension. Now these daily increments of A. R. would be different on account of the obliquity of the ecliptic to the equator, even although the sun's motion in the ecliptic were uniform. For if the sun advanced an equal space daily along the ecliptic, yet as this space has very different positions with respect to the equator, being at the tropics parallel to it, and at other places oblique; it will require a greater or less additional turn of the earth on her axis over and above a complete rotation to bring the sun on the meridian; according as these spaces are less or more inclined to the equator. At the equinoxes and solstices, the right ascension and longitude of the sun are equal, but at all other times they are different; and therefore as the motion in right ascension, or on the equator, is measured by a well-regulated clock, while the motion in longitude or on the ecliptic is measured by a dial, the clock and dial cannot agree, except at the equinoxes and solstices, when the sun's longitude and right ascension are equal; provided the whole equation depended upon this cause, viz. the obliquity of the ecliptic to the equator; as would be the case, if the apparent motion of the sun were equable in his apparent orbit.

But this is not the case; for part of the equation of time depends upon the sun's unequable motion in his apparent orbit. He sometimes moves more than a degree in a day, and sometimes less. His motion being quickest in winter, when he is nearest to us, and slowest in summer, when ne is at his greatest distance. His motion, as it is measured by the dial, is slower than his mean motion which is measured by the clock, while the upper half of the earth's orbit is described, and quicker, while the other half is passed over. While his anomaly, or mean motion counted from his apogee, is less than six signs or half a circle, he comes to the meridian before twelve by the clock, because his motion being then slower than the mean motion, his daily increment of right ascension is less, and therefore it requires but a small time for any meridian to come up to him. But while his anomaly is more than six signs, or a semicircle, he comes to the meridian after twelve by the clock, because he has gone farther towards to the east, by his quicker motion, and therefore it requires a longer time for any meridian to come under the sun, after a complete revolution of the earth round its axis. Now the first-mentioned cause of the equation of time making it vanish at the equinoxes and solstices, and the last-mentioned making it vanish at the apogee and perigee; it cannot vanish entirely at any of these times, but at the four intermediate times above-mentioned.

The time measured by the sun-dial is called apparent time, but that measured by the clock is called mean time: and the difference between these is the equation, which being sometimes added to, and sometimes subtracted from apparent time, will give the mean time.

The only difficulty now remaining is, how to find the quantity of this equation of time, and to know whether it be additive to mean time or subtractive from it, to give the true. This may be known in the following manner.

If the sun were supposed to move round the equator in a year, with an equable motion, and a uniform velocity of 59′ 8″ per day, his motion would be measured by a well-regulated clock, and his distance from the first point of Aries on any day of the year would be the same as his mean longitude, which is found by the constant addition of 59′ 8″ for every day after the vernal equinox. He would then come to the meridian every day at noon by the clock; but his real appulse to the meridian is known by the dial, and he will actually come to the meridian, by his proper motion in the ecliptic, with that point of the equator which determines his right ascension. The difference, then, between his mean longitude and right ascension, is that arch of the equator which passes under the meridian between the apparent and mean noons, i. e. between the noons by the clock and dial; and therefore when this arch is converted into time, at the rate of 15 degrees per hour, it will be the equation. Now if the sun's mean longitude be greater on any day, than his right ascension, the difference converted into time must be subtracted from the apparent noon to obtain the mean, because the mean noon will precede the apparent; but if his mean longitude be less than his right ascension, the equation of time must be added to the apparent noon to obtain the mean, because the apparent noon will then precede the mean.

The only thing now remaining is to find the sun's right ascension; which may be done by first finding the latitude of the place, by the altitude of the pole-star on the meridian; and then by finding the declination, from the meridian altitude of the sun; for these, with the known obliquity of the ecliptic, will give the right ascension, by the rules of spheric trigonometry.

The earth, in her annual progress round the sun, is attended by one moon, whose motions and phenomena shall be the subject of some future lecture, when we consider the motions of the satellites that revolve round the planets as the centers of their motions.

MARS.

Mars is the next of the planets, which revolves round the sun, at the distance of 145,014,148 miles from him, and completes his period in 686^d 17^h 18′; and at the same time turns round his axis in 24^h 40′. His orbit is inclined to the orbit of the earth in an angle of 1° 52′, and cuts it in 18° 30′ of ♉ and ♏, which points are called his nodes.* He travels in his orbit, at the rate of 52,500 miles per hour. His diameter is but two thirds of the diameter of the earth, or rather the earth is three times as large as Mars; and on account of his distance from the sun he receives but about one half of the light that we enjoy. His equatorial diameter is to his polar diameter as 1355 : 1272, or as 16 is to 15, nearly. He appears sometimes gibbous, and sometimes shines with a full face in his oppositions and conjunctions, but never appears horned; which shows that his orbit includes the orbit of the earth. To his inhabitants, the earth and moon appear like two moons, changing places with each other, appearing horned, half-illuminated, and gibbous, but never full, and never above one quarter of a degree from each other, although they are 240,000 miles apart. Our earth appears almost as large to Mars as Venus does to us; and sometimes is seen passing over the

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* The ascending node of Mars was in 1^s 17° 54′ 28. 2″ in the year 1783, Dec. 8th. The annual motion of his nodes is 24. 2″.

n's face, attended with our moon, as Venus and
ercury appear to us to pass over him; and the tran-
ts of Venus and Mercury are also observable there as
ell as with us. The axis of Mars is inclined 59° 42′
the ecliptic, and points to 17° 47′ of Pisces. Obliqui-
of ecliptic, 28° 42′. Venus is as seldom seen from
ars, as Mercury is seen by us; and Mercury is ne-
r seen by the inhabitants of Mars, except when
is passing over the sun's disk. Our earth never ap-
ars farther from the sun than about 48 degrees,
hich is the greatest elongation of Venus, and there-
re we shall be the evening and morning star to Mars,
turns, as Venus is to us. As our earth's orbit is in-
uded within the orbit of Mars, it will exhibit all the
fferent phases of illumination, conjunctions, stations,
ogressions, and retrogradations, that Venus does to
s.

As the phenomena of Mars from the earth are the
me, in general, with those of the other superior pla-
ts, we shall consider them together, when we have
entioned the other bodies that compose the solar
stem.

JUPITER.

Jupiter is the next of the superior planets above
ars, in the solar system, and by far the largest, be-
g about one thousand times as large as our earth;
r his diameter is about 81,000 miles, which is more
an ten times the diameter of the earth. He travels at
e rate of 29,000 miles per hour in his orbit, and
nishes his revolution in 11 years, 314 days and
21 hours, turning round upon his axis in 9^h 56′;
that his year contains 10,470 days. By his quick
tation on his axis, his equatorial diameter is large

than his polar diameter in the ratio of 13 : 12, which will make a difference of 6230 miles; so that his poles are nearer to his center than his equator by 3115 miles. This is a necessary consequence of his quick rotation on his axis, because the fluids, and lighter particles washed away by them, will recede from the poles, which are at rest, and retire towards the equator, where the motion is quickest, until there be a sufficient quantity collected, to make up for the deficiency of gravity lost by the centrifugal force; and when this is accomplished by a sufficient accumulation, the equatorial parts can rise no higher. Our earth, being but a small planet when compared with Jupiter, and its rotation on its axis being not half so quick, is of course much less flattened at the poles, its diameters being in the ratio of 230 : 229, which occasions a difference of about 35 miles. Jupiter's orbit is inclined 1°20′ to the ecliptic, his ascending node being in the seventh degree of Cancer. His axis is so nearly perpendicular to his orbit, that he has scarcely any change of seasons. He is surrounded by faint substances, in the direction of his equator, called belts, in which so many changes have been observed, that they are by some supposed to be clouds, or possibly discoloured portions of his surface, less fitted to reflect the light of the sun, and sometimes obscured more than at others, by the clouds that float in his atmosphere. These belts are sometimes observed to be of different breadths, and at others to be nearly equal. Large spots have been seen in them, which have again disappeared; the broken ends of the belts revolving in the same time with the spots, and those near to the equator in about four minutes less time, than those near the poles.

The sun appears to Jupiter but a twenty-eighth part as large as to us, and his light and heat are in the same small proportion; but this inconvenience is compensated by the quick returns of day, and by the revolutions of four moons around him in short periods, some of which are larger and some less than our earth. So that there is scarcely any part of this large planet, whose night is not enlightened by one or more of these moons; excepting near his poles, where the outermost alone is visible. Their motions we shall consider hereafter. His distance from the sun is 494,990,976 miles.

SATURN.

SATURN, the next planet in size and situation to Jupiter, revolves round the sun in 29 years, 167 days and 6⅔ hours of our time, which make but one year to him. His distance from the sun is 907,956,130 miles. He travels at the rate of 21,099 miles per hour in his orbit. He is nearly 600 times as large as this earth, his diameter being 67,000 miles. To such eyes as ours, Jupiter and Herschel are the only planets that can be seen from Saturn, and Saturn and Herschel the only planets that can be seen from Jupiter. The earth is less than some of the moons of Jupiter, and these are all invisible to the naked eye; consequently our earth would be invisible at the distance of Jupiter, and much more so at the distance of Saturn or Herschel. Saturn's orbit intersects the ecliptic in an angle of 2° 33′ 30″, and his ascending node is in 21° 50′ of Cancer, about 14 degrees from the node of Jupiter, towards the east. As visible spots have been discovered on the disk of Saturn, that are distinguishable from the rest of his surface, it is now known, that he has a rotation on his axis, perpendicular to his ring. He has belts similar to those of Jupiter, lying nearly in the direction of his equator, or his ring. The sun appears to be but about one ninetieth part as large to Saturn as to us, and his light and heat are in the same small proportion. However this seems to be wisely compensated by seven moons revolving round him nearly in the plane of his ring, at different distances, and in different periods. Besides these, he is surrounded with a thin broad ring at the distance of about twenty-one thousand miles from him, and nearly of the same breadth, which answers the purpose of a continuation of moons quite round the planet. This appears to his inhabitants like a broad luminous arch in the heavens, as if it did not belong to him, and may be represented by the horizon of an artificial globe, removed to its breadth from the surface of the globe. This ring is inclined to the ecliptic in an angle of 30 degrees, and intersects it in 19° 45′ of Virgo and Pisces, so that when Saturn is in these points, the plane of the ring passes through the earth, the edge of it being turned directly towards us. At this time the ring disappears, and the body of Saturn is seen divested of the ring; as the edge of it is so thin, as to become invisible even with our best telescopes. This will happen twice in every revolution of Saturn: and when Saturn is in the middle between these points, that is about the 20th degrees of Gemini and Sagittarius, it appears most open to us, the eye being elevated about 30 degrees above the plane of the ring, when its apparent diameters are to each other as 9 : 4. As Saturn goes round the sun, his obliquely-posited ring continues parallel to itself, like the axis of our earth, and therefore twice in every Saturnian year, or once in every fifteen years, its edge is turned towards the sun; and being very thin it then becomes invisible to the earth, wheresoever the earth may be. As the sun shines for half a year on the north pole of our earth and then disappears from it for the same time; so the sun, in a like manner, shines for fifteen years on the north side of the ring, and then disappears from it for 15 years more, while he is shining on its south side. In 1789, the ring will disappear and reappear twice, viz. disappear 5th May, and reappear 24th August; disappear 15th October, and reappear 30th January 1790. There is some reason to believe that the ring of Saturn turns round on an axis; because, when it is almost edgewise to us, it appears to be somewhat thicker on one side than on the other, and the thickest side has been seen on different sides of the planet, at different times. Herschel has discovered lately, that it revolves in $10^{h}\ 32'\ 15''$. When the ring is viewed with a very good telescope, it appears double, as if it were composed of two rings, one surrounding the other, and separated by a dark black list or line. The outer ring, as there are really two of them, appears to be but about half as broad as the inner one. Saturn is flattened towards the poles, his diameters being to each other as 11 : 12.

HERSCHEL.

HERSCHEL is the outermost planet of the solar system, which we have yet discovered. It was lately discovered by Mr. Herschel, of Bath, in England, who accidentally met with it, in his telescope, as he was viewing the fixed stars. Upon considering its motion and comparing its present place, with a new star mentioned by Mr. Meyer, of Gottingen, as discovered about thirty years before, in a certain place of the heavens where no star is now to be found, the astronomers of Europe have concluded, that what Meyer had then seen, was the same star that Herschel has lately discovered. Upon this supposition, they have endeavoured to ascertain the elements of its motion to be as follows.

It performs its revolution round the sun, like the other planets, from west to east, in about eighty-three years and four months, at the distance of near nineteen hundred millions of miles from the sun, in an orbit inclined to the ecliptic in an angle of 48′ 10″, cutting the ecliptic in 13° 30′ of Gemini and Sagittarius. Its diameter is about 34217 miles, being to that of our earth as 4.31769 to 1, and its eccentricity 82034. It seems to have passed its perihelion on the 7th September 1779, moving at the rate of 4° 21′ 1″.3 per annum. Its bulk is 80.49256 times that of the earth, though its quantity of matter is but 17.740612 as much. There is but little more known of the motions of this planet; as it has not yet performed half its periodical revolution, since the time it was first discovered. It is attended by two moons already discovered; the first revolves in $8^d\ 17^h\ 1'\ 19''$ and the other in $13^d\ 11^h\ 5'\ 1.5''$.*

ASPECTS OF THE SUPERIOR PLANETS.

Any of these superior planets, Mars, Jupiter, Saturn, or Herschel, may be found in opposition to the sun, as well as in conjunction with him. They appear in opposition to the sun, when they are seen in opposite parts of the heavens; and as they revolve in orbits which include the orbit of the earth, whenever they are on the same side of the heavens with the sun, they are said to be in conjunction with him, and must then appear of a less diameter, because of their greater distance from the earth; but when they are on the opposite side of the heavens, they are consequently nearer to the earth, and must appear of a greater diameter. Thus the diameters of the superior planets, in opposition and conjunction, are to each other, in the following ratios; of Mars, as 25 : 5; of Jupiter, as 62 : 42; of Saturn, as 105 : 85; —of Herschel is not known but by computation.

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* Since the writing of this lecture, four additional moons have been discovered. *Ed.*

As all the superior planets move in the same direction with the earth, and with less velocity, the appearances which they exhibit to us will be the same, if we suppose them to be at rest, and the earth to move with the difference between their respective velocities and that of the earth. Now, while the earth is moving through that part of her orbit, which is nearest to the superior planet, the planet will appear to move from east to west, or retrograde, among the stars; first increasing in its velocity as it approaches to the point of its opposition, where its velocity is greatest, and then gradually decreasing in velocity, until it come to that point where a right line from the planet to the earth will be a tangent to the orbit of the earth, where the superior planet will appear to be stationary for a small time, as the earth is then going directly from it. From thence the direct motion of the planet will commence, and its velocity will gradually increase till the conjunction of the sun and planet, when it will be again greatest, and then it will gradually decrease until it become stationary; after which its retrograde motion will again commence. So that when a superior planet is passing from one station to another through its opposition, its geocentric motion is retrograde; but while it is passing from one station to another through its conjunction, its geocentric motion is direct.

Half the arc of regress of a superior planet is equal to the angle under which the semi-diameter of the earth's orbit would be viewed from the planet; and as this semi-diameter must appear under a less angle, the farther the eye is removed from it, it follows, that Mars's arc of regress is greater than Jupiter's, Jupiter's greater than Saturn's, and Saturn's greater than Herschel's. It follows also, that as the sine of half the arc of regress of a superior planet is to radius, so is the radius of the earth's orbit to the radius of the planet's orbit; so that the proportion of the distance of the earth from the sun, to the distance of any of the superior planets from the sun, may be known by observing the arc of their regress; and if the real distance of the earth be known in any determinate measure, the distance of the rest may be found.

In explaining the progressions, stations, and retrogressions of the superior planets, we have supposed that the planet was at rest, and that the earth moved with the difference of their velocities; and these will be the same that they would be upon the supposition of the proper motion of both; save only, that the stations, and the retrograde and direct motions of the planets, will happen in different portions of the heavens, and not always in the same parts.

The heliocentric latitude of a planet is greatest when it is at 90 degrees distance from either of its nodes: but the geocentric latitude is directly as the heliocentric latitude, and inversely as the distance of the planet from the earth. When a superior planet is in conjunction, its heliocentric is greater than its geocentric latitude, but less, when it is in opposition. The method of determining the mean times of the conjunctions or oppositions of the planets is, by first considering their mean motions as viewed from the sun, and thereby determining how far any one of the planets would appear to separate from the earth in the space of a day; and thus finding how many days will be necessary for their separation to the distance of a whole circle. Thus, if the daily motion of the earth be 59′ 8″=3548″, and that of Herschel, 42″.88, the earth will appear to recede from him with a velocity equal to the difference of their motions, viz. 3505″.12 per day. Then say, as 3505″.12 : 1 *day* :: 1,296,000″=the seconds in a circle : 370 days nearly, which must pass from the time of one conjunction, or opposition to the next. In this manner, the intervals between the conjunctions of any of the other planets may be found from their daily motions, which are as follows; of Mercury, 4° 5′ 32″; of Venus, 1° 36′ 8″; of Mars, 31′ 27″; of Jupiter, 4′ 59″; of Saturn, 2′ 1″; of Herschel, 42.88″. In the same manner the mean conjunctions of the planets are found, which may differ some days from the true times, on account of the unequal motions of the planets in their elliptical orbits. These times are therefore to be corrected from the astronomical tables of their motions, by computing their real distances from each other, as seen from the sun, at the times of their mean distances; and from their known velocities at that time, finding the time of their true conjunctions.

The planets are all opake, globular, and rough bodies; as it is owing to the roughness of their surfaces that they reflect light enough from the sun to make them visible to us. For if their surfaces were smooth and well polished, the image of the sun reflected from them would be no more than a point, and the planet would become invisible; but being rough, they reflect the light in all directions, and from every point, in sufficient quantities, to make them appear in their proper dimensions.

COMETS.

THERE is still remaining the most numerous class of bodies belonging to the solar system, which revolve round the sun in very eccentric orbits, in long periods, and in all different directions; some from west to east, others from east to west; some from north to south, and others in a contrary course; and which are usually denominated comets. They are solid and opake bodies, usually attended with long and shining trains, sometimes extending a hundred degrees from the comet, and projected in a direction opposite to the sun. As they revolve in very eccentric ellipses, they take amazing excursions through the regions of space, and then return, after a long absence, down through the planetary orbits, and many of them approach very near to the center of force, when they move with incredible velocity. They are visible only for a small part of their periods, viz. while they are performing that part of their orbits, which lies within the planetary regions; and hence it is so extremely difficult to ascertain their periodical returns with precision, from the few observations that can be made of their motions, while they are visible. We have therefore no method left for this purpose, but by consulting the histories of comets, and by noting the years in which comets have appeared, that have a similar course through the heavens; and if we find any such appearing after equal periods, and in similar orbits, we have then some probable grounds to conclude that it was the same comet which returned to the sun after these periods. By these means, Dr. Halley first predicted the return of the comet of 1759, whose period appears to be 75.5 years, as he had observed that it had appeared before, in a similar course, in the years 1308, 1456, 1531, 1607, 1682. From the equality of periods and similitude of appearances, he also concluded that the comet of 1532 appeared again in 1661, after a period of 129 years; and if his conjecture be right, we may expect to see it again in January 1790 or 1791. That the comet, which appeared in the 44th year before Christ, appeared (in the consulate of Orestes and Lampadius) in 531, in 1106, and 1680, after a period of 575 years. In the same manner it is concluded, that the comet of 204, 570, 1299 and 1664, was the same, returning after a period of 365 years. It is also probable, that the comets of 1264 and 1556 may be one and the same comet, returning after a period of 292 years; and also that the comets of 1596 and 1699 may be the same comet, returning after a periodof 103 years; and may be expected again in 1802. Although the astronomical elements of between 40 and 50 comets have been calculated in parabolic orbits by Dr. Halley and others, yet we cannot believe, from these elements, that the periodical times of any of them have been ascertained, excepting those above mentioned, and even these are doubtful, until their future returns confirm the predictions. As the comets descend within the planetary regions, they may approach near to the planets in crossing their orbits, and thereby suffer some alterations in their velocities and periods, and also in the inclination of their orbits to the ecliptic, which will ever make it a very difficult matter to predict their returns with precision.

To the naked eye, the head of a comet generally appears like a cloudy star, shining with a dull and obscure light, though some have been observed to shine with a vivid light equal to that of stars of the first magnitude, and some to have even surpassed Jupiter in splendour. But when viewed through a good telescope, the head appears a solid globe, surrounded with a large and gross atmosphere, and most commonly attended with a long and shining train. Various have been the opinions of philosophers concerning the tails of comets; although all acknowledge that they depend, some way or other, upon the action of the sun, for this plain reason, that they are projected from the comet opposite to the sun, and are observed to grow longer upon the comet's nearer approach to him. Sir Isaac Newton, whose opinion, even when delivered as a conjecture, is justly revered by every philosopher, supposed, that the tails of the comets were their atmospheres, greatly rarefied by the heat of the sun, and thereby made specifically lighter than the ether, which he supposes might fill the planetary regions, and regard the sun as its atmosphere, and gravitate towards it. He supposed that this ether, being most rarefied near to the sun, would constantly ascend from it, and thereby carry up with it the reflecting matter of which the tail is composed. This solution, however, is objected to, from the improbability of the ascent of this vapour from the comet; being so much swifter than the motion of the comet, as to cause the tail to ascend before it, when the comet of 1680, at its nearest approach to the sun, moved with the amazing rapidity of 880,000 miles in an hour. Rowning therefore supposes, that the atmospheres of the comets extend as far as their tails, and that the part which forms the tail is distinguished from the rest, by the rays of the sun being most copiously reflected from it. The supposed extent of the atmospheres is an objection to this solution. It is more probable to me, that the atmospheres of the comets are of a moderate size, and are condensed about them in their aphelia; but that when they descend towards the sun, these atmospheres expand, and exhibit the appearance of a coma surrounding the comet when it first begins to be visible; and that when it comes close to the sun, the whole atmosphere becomes so rarefied, as to be carried forward by the action of the sun's rays in their own direction, and by the reflexion of the light, to afford the appearance which is denominated the tail or train of the comet.

The tails of comets appear to be bent a little towards that part which the comet has left, being well defined on the convex side, and always in the plane of their orbits.

When the perihelion distances and periods of comets are known, their aphelion distances are also found from the proportion between the cubes of the mean distances and the squares of the periodical times. Thus the comet of 1759, whose perihelion distance is 58.8 of such parts as the mean distance of the earth contains 100, and period 75.5 years, will be found by that analogy to be at the mean distance of 1786.4; from the double of which, viz. 3572.8, subtract 58.8, and there remains 3514 for the aphelion distance, which is about twice the distance of Herschel from the sun, or 35 times the distance of the earth. By a similar process, the aphelion distance of the comet of 1680, whose period is 575 years, and perihelion distance but 0.612, comes out to be near 138 times the mean distance of the earth, that is, nearly 13,800 millions of miles from the sun; whereas at its nearest distance from the sun, it was not above one third of his semidiameter from his surface; or about half a million of miles from his center. At this distance, the sun appeared to this comet 40 thousand times as large as to us. The astonishing heat that it must have received from the sun in its perihelion, Sir Isaac Newton calculates to be 20,000 times the heat of red hot iron. And the distance to which it goes from the sun suggests to us an idea of the inconceivable distance of the fixed stars; as even at its greatest excursion it must still be nearer to the sun than to the nearest of the fixed stars, of whose attraction it must keep clear, in order to return periodically round the sun.

THE TRUTH OF THE COPERNICAN SYSTEM.

In the foregoing description of the solar system, we have taken notice of the phenomena that have been observed to take place among the heavenly bodies. We now assert farther, that these are just such as they ought to be, upon the supposition that the copernican system is true; and that they are inconsistent with either the tychonic or ptolemean hypotheses. The orbits of the inferior planets encompass the sun, but the earth is placed on the outside of these orbits, and consequently the ptolemean system is false, as it supposes the earth to be the center of their motions, and that their orbits are included by the orbit of the sun. And these planets are found on all sides of the sun, both east and west of him, as well as beyond him and between him and us: therefore they must surround the sun, and the earth be without of their orbits, according to the copernican system. Besides, if the orbits of the inferior planets surrounded the earth, they would sometimes be found in opposition to the sun, though they never are seen in opposition to him, but have two conjunctions with him in every revolution. Mercury is never seen above 23 degrees from the sun, and Venus not more than about 48 degrees. These things are necessary consequences of the copernican system, but impossible upon the ptolemean hypothesis. Besides, their apparent diameters increase and diminish by turns, as they should do upon the copernican hypothesis, whereas they should never vary by the ptolemean. In a word, their stations on each side of the sun, their direct and retrograde motions, their transits over the sun's disk, and their different phases of illumination, are such as they should be in the copernican system, but impossible in the ptolemean.

The earth is not at rest within the orbits of the superior planets, as is supposed both in the tychonic and ptolemean systems; for upon this supposition, the superior planets could never appear stationary or retrograde in their motions; but according to the copernican system, they ought to appear stationary, direct, and retrograde, exactly in the places where they are actually found to be so. As the phenomena of the heavenly motions are inexplicable by, and inconsistent with, the schemes of either Ptolemy or Tycho Brachè, they must be erroneous; and as the copernican system solves every phenomenon without exception, there can be no solid objection brought against it; and it should therefore be accounted true. Especially, when we add, the grand newtonian argument for the truth of the copernican system; that all the various combinations of their motions are the direct result of the single principle of gravitation, which pervades the whole system, and regulates all the complicated motions of both planets and comets, in all their different orbits. This force is directly proportional to the quantity of matter in the revolving bodies, and inversely as the squares of their distances from each other; from whence it follows, that the squares of their periodical times are as the cubes of their mean distances from the common center of gravity: and every one of them describes equal areas in equal times. The moon is retained in her orbit, by the same force, that causes a heavy body here to descend 16 feet in a second of time. The moons of Jupiter and Saturn revolve round their primaries, by this law, and all the planets and comets revolve round the sun by the same. But if the sun and moon be supposed to revolve round the earth by this law, the sun's period would be no less than 475 years!

DISTANCES AND PARALLAXES OF THE PLANETS.

THE distances of the planets from the sun are found by means of their parallaxes.

The parallax of a heavenly body is the difference between the places, among the stars, in which it would be seen by a spectator at the center, and at the surface of the earth. Consequently it is the measure of the angle under which the semidiameter of the earth is seen, from the heavenly body. Therefore, as a spectator at the center of the earth would see an object in the same place in the heavens, as a spectator on its surface would see it, when in his zenith, the parallax of any object in the zenith is nothing, and it gradually increases with its distance from the zenith until it appear in the horizon, when it will be greatest. So that the horizontal parallax is the greatest, and the parallax at any other altitude is as the cosine of the altitude, or sine of the zenith distance. As the true place of a planet is the place in which it would appear from the center of the earth, the apparent place, to which it is referred by a spectator on the surface of the earth, must always be lower than the true. As the parallactic angle of a celestial object is always subtended by the semidiameter of the earth, it must be less as the distance of the object from the earth increases. So that the distance of the heavenly bodies is inversely as their parallaxes. That is, the greater their parallax is, the less is their distance. The parallax of a planet at any altitude is to the semidiameter of the earth, as the sine of its zenith distance is to its distance from the center of the earth.

If we therefore could find the parallax of any planet, we could easily determine its distance from the earth, by the foregoing analogy. Or if we had the horizontal parallax of a planet, we could find its distance by this analogy: as the tangent of the horizontal parallax is to radius, so is the earth's semidiameter to the distance of the planet from the earth. Thus, if the sun's horizontal parallax be 8″.65, as it was found to be at the late transit of Venus, we find his distance from us, by the preceding analogy, to be 95,173,117 miles.

As the periodical times of all the planets are known from observation, if the mean distance of any one of them be found by means of its parallax, the distances of all may be found by the proportion, before mentioned, between the mean distances and periods.

But the parallax of the planets and comets, on account of their great distance, is so small, that it cannot be directly observed with sufficient precision, for so nice an affair as the determination of their distances, as it amounts to a few seconds only; and an error of a single second, in observing the parallax, would produce an error of some millions of miles in the distance.

The most certain way of determining the parallax of the sun, is by observing that of Venus; because she is nearer to us than any of the planets in her inferior conjunction; and consequently she has a larger parallax, and therefore it may be observed with greater precision. But Venus at that time is too near to the sun to be seen by us; as her enlightened disk is then turned from us, unless she should then be in or near to one of her nodes, when she would appear like a black spot on the face of the sun. These transits, from the laws of her motion, happen very seldom. Three only have been observed; and from the two last of them, in the years 1761, and 1769, the parallax of the sun has been calculated to be but 8″.65. Dr. Halley first proposed this use of the transits of Venus, and they have been accordingly observed by the astronomers of the present day, with great diligence and success.

To give a general idea, how this important observation may be made, without entering into the minutiæ of calculation necessary in this nice affair, we shall just mention the outlines of the business.

Let ABD* be the earth, V, Venus in her orbit, S the eastern limb of the sun, and FeE the sphere of the fixed stars. To a spectator at B, Venus appears just entered on the sun at S, and her place among the fixed stars is E; while to a spectator at A, her place among the stars is at F, and the arch FE, or angle FVE=AVB is her horizontal parallax: but the arch Ee, or angle eSE=BSA is the horizontal parallax of the sun. The difference of these parallaxes is FE, or the angle VAv, called the parallax of Venus from the sun; and may be measured by the difference of time between the internal contact of the limbs of Venus and the sun, as observed at the places B and A, when the velocity of Venus in her orbit is previously known from the tables of her motion. Now in the triangle

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* See Plate 16. Fig. 1.

ASV, the proportion between the sides AS, AV, is known by the periodical times of the earth and Venus; as also the ratio between AV and VS. Hence the horizontal parallax of the sun is found by saying, As VS : AV : : the horizontal parallax of Venus from the sun : the horizontal parallax of the sun on the day of the transit; then as the sun's distance from the earth on the day of the transit is to his mean distance, so is his horizontal parallax on the day of the transit, found as above, to his horizontal parallax at his mean distance.

In this method of determining the parallax of the sun, the observations should be made in two places 90 degrees apart; one of the observers having the sun in his meridian or near to it, and the other, in the horizon. But as this may be difficult, the observations may be made in two places at any other considerable distance from each other; where the difference of times between the observations will be great, occasioned by the parallax of Venus from the sun; from which his parallax may be ascertained. And when his parallax is once determined, we have sufficient data to determine the distances of all the planets of the solar system, from the known proportion of their periodical times: and then, from their apparent diameters at these known distances, their real diameters and bulks may be found; their bulks being as the cubes of their diameters; the diameter of the earth being found from different measures of a degree of the meridian, in different latitudes, compared together.

The parallax of the sun being found by observations of this kind, the distances of all the planets are found to be, what we have already given in a preceding lecture. And to this important observation we are indebted for all the precision we have now acquired in the distances and dimensions of the solar system.

The diurnal parallax of a heavenly body always depresses it in a vertical circle; and consequently will produce a parallax of right ascension and of declination, excepting when it is in the meridian, when the whole parallax is in declination: it will also produce a parallax of longitude and latitude; excepting when the vertical circle, in which it appears, is a secondary of the ecliptic, when the whole parallax will be in latitude. The stars, from their immense distance, have no parallax.

ELLIPTIC ORBITS OF THE PLANETS.

THE orbits of the planets differ but little from circles, though all in reality are ellipses, having the sun in the lower focus. If the velocity of a planet in its orbit were such as it would acquire by falling through one fourth of its diameter, that velocity would be just sufficient to retain it in a circle; and it would revolve in it, without approaching towards or receding from the center of force. This balance between the central and projectile forces, must soon however have been destroyed by the mutual attractions of the planets, had it even been so constituted at first. The projectile and centripetal forces conspiring together in some degree during one half of its elliptical revolution, the planet will thereby be brought within the circle, and approach nearer to the central body; whence its velocity will at last become more than sufficient to retain it in the circle, and consequently it will then recede from the center of force: but the projectile and centripetal forces now acting in some measure against each other, the velocity will gradually decrease, until it become less than what would be necessary to retain it in a circle, when it arrives at its aphelion. In that point, having the same velocity, that it had at the beginning, it will describe the same ellipsis over again, in the next revolution.

Hence all the planets are observed to move with different velocities in different parts of their orbits; sometimes slower and sometimes faster, than what would be necessary to describe their whole orbits in their periodical times, which velocity is therefore called their mean motion. And hence our summers are eight days longer than our winters; or, the sun takes eight days more to go from the vernal to the autumnal equinox, than to go from the autumnal to the vernal equinox. Because in the summer season, the earth passes through her aphelion, where her motion is slowest, about the 30th of June; as it is also quickest on the 30th of December, when the earth passes by her perihelion. Since we are nearer to the sun in winter than in summer, his diameter must then, of course, appear larger. Accordingly we find that his diameter in summer is 31′ 33″, whereas in winter it is=32′ 38″. His daily motion in aphelion is 57′ 12″, in perihelion, 61′ 12″, and at the mean distance 59′ 8″.

As the elliptic orbits of the planets are nearly circles, the distance between the centers and foci is but very small in any of them, being largest in Mercury. If the mean distance of any planet from the sun be divided into 1000 parts, the eccentricities of the several planets will be as follows; Mercury's 210, Venus's 7, the Earth's 17, Mars's 93, Jupiter's 48, Saturn's 55, Herschel's 82,034.

All the planets are regulated in their motions by this fundamental law; that however variable their velocities, yet in their orbits they describe areas proportional to the times of description. In order to find their places in their orbits, astronomers have contrived the following method. They have supposed a body, revolving in a circle with an equable motion, to have commenced its motion from the aphelion of the planet at the same time with it, and to complete its revolution in the same time with the planet. The motion of this body is called the mean anomaly of the planet; and as it would be sometimes before and sometimes behind the planet, the difference between them being added to the mean anomaly or subtracted from it, as occasion may require, will give the true place of the planet in its orbit. As the real place of the planet in its orbit is the same with the mean, both at the aphelion and perihelion, there will be no equation to be added or subtracted at these points: but the mean motion will be greater than the true motion from the place of the aphelion to the place of the mean distance, or near to it, when the equation to be subtracted from the mean anomaly will be the greatest possible, in order to give the true place of the planet. From that place to the perihelion the mean place will be still before the true place of the planet in its orbit; but the difference will gradually grow less and less, till it vanish at the perihelion, where the mean and true places will be the same. So that the equation of the center will be subtractive from the mean anomaly to give the true place of the planet, from the aphelion to the perihelion, or while the mean anomaly is less than six signs; and for the same reason it will be additive for the other six signs, or from the perihelion to the aphelion.

When the time of the passage of the planet through its aphelion is previously known by observation, and the mean anomaly is determined from the periodical time, the planet's place is easily known, by applying the equation of the center, according as it is additive or subtractive. This will give the planet's distance from its aphelion. But its distance from the first point of Aries is also found, by first knowing the distance of the aphelion from the first point of Aries, and the motion of the aphelion, if it have any. Astronomers have disposed these motions for all the planets in proper tables, to facilitate the calculation of their places. The table of its mean motion shows its distance from the first point of Aries, and of its mean anomaly from the aphelion; and the equation of the center is the difference between the mean and true anomaly.

It is found that the place of the earth's aphelion is not stationary, but moves forward from west to east at the rate of 16 seconds of a degree in a year, in respect to the fixed stars, or $50''+16''=66''$, with respect to the equinoxes. The place of the sun's apogee on the first of January, in the year 1760, was $3^{s}\ 8^{\circ}\ 47'\ 25''$.

It is found, from observation, that the orbits of the other planets are quiescent, and their aphelion points are found to be in the following places, viz.

The aphelion of Mercury in	♐	12°	44′	0″	
Venus	♒	4	19	54	
Earth	♑	8	47	25	variable.
Mars	♍	0	31	54	
Jupiter	♎	9	9	54	
Saturn. . . .	♐	27	49	54	
Herschel . .	♐	17	13	17	

ELEMENTS AND CONSTRUCTION OF SOLAR TABLES.

THAT you may have some idea of the manner by which astronomers have arrived at the knowledge of the dimensions of the solar system, and the positions and motions of the planets, we shall give you a few examples relative to the earth; from whence you may learn, that by similar observations and methods they have determined the same of the other planets. As the latitude of the place of observation is a principal element in all astronomical observations, we shall begin with the method of finding it.

TO FIND THE LATITUDE OF A PLACE.

Observe the greatest and least meridional altitudes of any star near the pole. If both the altitudes be on the same side of the zenith with the pole, half their sum is the latitude; but if the observations be on different sides of the zenith, half their difference is the co-latitude. There will be a necessity of correcting the observed altitudes for refraction. The pole star itself may do very well for this purpose. There will be about twelve hours between the observations; and therefore they must be made when the night is more than twelve hours long.*

TO FIND THE OBLIQUITY OF THE ECLIPTIC TO THE EQUATOR.

Let the meridian altitude of the sun's center, which is always in the ecliptic, be observed on the days of the summer and winter solstice; the difference of these altitudes is the distance of the tropics from each other, and half this quantity will be the obliquity of the ecliptic. Or the meridian altitude lessened by the co-latitude at the summer solstice, or the co-latitude lessened by the altitude at the winter solstice, will give the obliquity sought. From the mean of a number of good observations made in the year 1772, the obliquity of the ecliptic to the equator was found to be 23°, 28′. And distant observations when compared together prove that it decreases at the rate of about half a second per annum.

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* See Plate 15. Fig. 2.

In this last way, the declinations of the planets or fixed stars are found; observing that their declination is of the same or a contrary name, viz. north or south, with the latitude of the place, according as its complement is less or greater than the altitude.

TO FIND THE TIME OF AN EQUINOX.

In a place whose latitude is known, observe the meridian altitudes of the sun's center on the day of the equinox, and also on the days preceding and following it; then the difference between these altitudes and the co-latitude will be the sun's declinations on these days, and at the times of the observations.

If either of these altitudes should prove to be equal to the co-latitude, that observation was made at the time of the equinox; but if not, the time of it may be thus found.

Let DG* represent the equator, AC the ecliptic, E the equinoctial point, and AD, BF, CG, the declinations of the sun at the times of the observations. Now the angle AED being the known obliquity of the ecliptic, we have it and the declinations in the triangles CEG, BEF, from which we may find the arcs CE, BE, from the two last observations. Then say, as BC : BE : : 24 hours : the time from the second observation to the equinox. Thus may the quantity of the tropical year be determined by comparing distant observations together. But the tropical year may be more accurately determined by a calculation of the moments of the solstices; for the invention of which method we are indebted to Dr. Halley. The observation is exceedingly easy, and the calculation not difficult. It is as follows:

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* See Plate 15. Fig. 3.

TO FIND THE TIME OF THE SOLSTICE FROM OBSERVATION.

LET AVO* represent a small portion of the tropic near the solstice, and KVN a part of the ecliptic. Suppose the sun's places on three several days near the solstice to be K,L,M, then on the meridian he will be in H,F,G, the relation of which may be determined by the shadow of a stile, ab, projected on a board, cd, perpendicular to his rays at noon. From any given point as c, measure ch, cf, and cg, on the noons of the three days, then ch—cf=fh, and cg—cf=fg. By these means we have the proportion of distance of FH, FG; for fh : fg : : FH : FG.

It is known by geometry, that the distances AV, TV, are to each other as the squares of the subtenses AK, TL, so that the curve will have the property of a parabola, of which the solstitial colure VH will be the axis; in which having the three points H,F,G, we can find TV, the time from the second observation to the

* See Plate 16. Fig. 1.

solstice, which let be$=x$, in the following manner. Let the time from the first to the second observation be $=a$, and the time from the second to the third observation be$=na$, then we have $AV=a+x$, and $VE=na-x$. Let $FH=c$, $FG=d$, and let the parameter of the parabola be p. Hence $x^2=VF\times p$, and $VF=\frac{x^2}{p}$. In like manner, $VH=\frac{a^2+2ax+x^2}{p}$, and $VG=\frac{n^2a^2-2nax+x^2}{p}$. Therefore $FH=VH-VF=\frac{a^2+2ax}{p}=c$. $FG=VG-VF=\frac{n^2a^2-2nax}{p}=d$: wherefore $p=\frac{a^2+2ax}{c}=\frac{n^2a^2-2nax}{d}$; and by reducing the equation we have $x=\frac{a}{2}\times\frac{cn^2-d}{cn+d}$. which gives this analogy. As $cn+d : cn^2-d :: \frac{a}{2} : x$. But if the third observation be nearer to the solstice than the second, $x=\frac{a}{2}\times\frac{cn^2+d}{cn-d}$.

To illustrate this by an example. We find that in the year 1500, Bernard Walker, in the city of Nuremberg, observed the chords of the sun's zenith distances as follows:

June 2.	45467
8.	44975
9.	44934
12.	44883
16.	44990

Now if we take the observations of the 2, 9, 16, of June, at 7 days apart, where $a=7$, and $n=1$, we have $c=533$, $d=56$—$c-d=477$, and $c+d=589$; and $589 : 477 :: 3.5 : x=2^d, 20^h, 2'$, which added to the middle observation, will give the time of the solstice June 11^d, 20^h, $2'$, O. S.

In like manner, if we take the observations of the

8^d, 12^d, 16^d, the solstice will be 11^d, 20^h, $23'$, O. S.
8, 9, 16, 11, 20, 20,
9, 12, 16, 11, 20, 38,

And taking the mean of these four determinations, we have the time of the solstice June 11^d, 20^h, $26'$, O. S. or June 21^d, 20^h, $26'$, N. S.

Gassendus also, at Marseilles, in the year 1636, measured, by a stile 55 feet high, the proportion of the gnomon to its solstitial shade, on the days before and after the summer solstice, and the proportions of the measures were as follows.

June 19. 31766
20. 31753
21. 31751
22. 31759

By comparing the measures made on the
19, 20, 21, of June the time of the solstice is, June 20^d, 16^h, $22'$,
20, 21, 22, 20^d, 16^h, $45'$,
19, 20, 22, 20^d, 17^h, $15'$,
19, 21, 22, 20^d, 17^h, $25'$.

By the mean of all, the time of the solstice was June 20^d, 16^h, $56'$, N. S. which is 1^d, 3^h, $30'$ sooner than by the observation of 1500, which was 136 years before it; of this 1^d, 1^h, $3'6.8$ was owing to the precession of the equinox, which makes the tropical year less than the civil year by $11'$, $18''$.

TO FIND THE LENGTH OF THE TROPICAL, PERIODICAL, AND ANOMALISTICAL YEARS.

Let two observations be selected among those of greatest authenticity on record, of the time when the sun had like positions with regard to his longitude, which determine the length of the tropical year; or with regard to some noted star, and from thence determine the length of the sidereal year; or with regard to the line of the apsides, and from thence find the length of the anomalistical year. These observations should be as distant as possible, that the whole time elapsed being divided by the number of revolutions, we may obtain the more accurate result. According to Mayer's tables, these years are as follows:

Tropical	365^d	5^h	$48'$	$42''$
Sidereal	365	6	9	7
Anomalistical	365	6	15	29

The tropical year being $20'\ 25''$ shorter than the sidereal, shows that the sun has returned to the same point of the ecliptic, with respect to the beginning of any of the signs, before he has arrived at the same point with respect to the stars. And consequently any cardinal point of the ecliptic, such as the equinoxes or solstices, has a retrograde motion towards the west, at the rate of about $50''.3$ per annum. For, $365^d, 6^h, 9', 7'' : 360^\circ : : 20', 25'' : 50''.3 =$ the precession of the equinoxes in a year.

A sidereal revolution being performed in $6'\ 22''$ less time than the anomalistical, shows that the sun's apogee, from whence this revolution is computed, advances about $15''.7$ of a degree towards the east per annum. For, $365^d\ 6^h\ 15'\ 29'' : 360^\circ : : 6'\ 22'' : 15''.7 =$ the annual motion of the sun's apogee towards the east.

The length of the year, deduced from a comparison of distant observations made at or near the time of the sun's passing his apogee, is less than that deduced from the like observations when he is about passing his perigee, therefore a mean between them should be chosen; or the length of the year should be determined by observations made when the sun is at his mean distance.

TO FIND THE SUN'S LONGITUDE.

Having a clock regulated to mean time, observe the time when a star, whose right ascension is known, passes the meridian, and also when the sun passes the meridian: the difference of these times gives the right ascension of the sun. From the right ascension of the sun and the known obliquity of the ecliptic, find, by spherical trigonometry, the longitude; from which his place in the ecliptic will then be known.

TO FIND THE GREATEST EQUATION OF THE CENTER.

At the times when the sun is near his mean distances, find his longitude, and the difference between these two longitudes will give his real motion for that interval. From his periodical time find also his mean motion for that interval; then half the difference between his true and mean motions will be the greatest equation of the center.

Thus by observations at Greenwich, 1769, October, -	$0^d\ 23^h\ 49'\ 12''$	M.T. ⊙ Long.	$6^s\ 9^\circ\ 32'\ 0''.6$
and also, in the following year 1770, March, - - -	$29^d\ 0^h\ 4'\ 50''$	M.T. ⊙ Long.	$0^s\ 8^\circ\ 50'\ 27''.6$
The difference of time is - - -	$178^d\ 0^h\ 15'\ 38''$	Diff. Long.	$5^s\ 29^\circ\ 18'\ 27''$

The tropical year is $365^d\ 5^h\ 48'\ 42''=365.2421527$. The observed interval $178^d\ 0^h\ 15'\ 38''$, which is $=$ 178.01085648. Then say as 365.2421527 : 178.01085648 : : 360° : 175°.455948, mean motion, $=5^s\ 25^\circ\ 27\ 21''$, which answers to $5^s\ 29^\circ\ 18'\ 27''$. Half their difference, viz. $1^\circ\ 55'\ 33''$ is the greatest equation of the center, which was required.

TO FIND THE ECCENTRICITY, APHELION AND PERIHELION DISTANCES OF THE EARTH'S ORBIT.

Say, as the diameter of a circle in degrees is to the same in equal parts, so is the greatest equation of the center in degrees, to the eccentricity in equal parts; then the sum and difference of the mean distance and eccentricity will respectively give the aphelion and perihelion distances of the earth. As 3.1415926 : 1 : : 360° : 114.591599 parts equal to the diameter. Then as 114.591599 : 1000 &c. the mean distance : : $1^\circ\,55'\,33''=$ 1.9268166 : 0.0168066=the eccentricity of the earth's orbit. Hence 1.016806=the aphelion distance, and 0.983194=the perihelion distance.

TO FIND THE SUN'S MEAN ANOMALY, FOR ANY GIVEN TIME.

Let the epocha of the sun's passage through his apogee be accurately determined; then say, as a solar year of $365^d\,5^h\,48'\,42''$: 360° : : the time since the sun's passing through his apogee : the degrees of his mean anomaly. Or the sun's mean motion for the given time may be taken from the tables of the sun's mean motion, and this will be his mean anomaly. The tables of his mean motion give his distance from the first point of Aries, and this motion lessened by the motion of the apogee gives his mean anomaly, or mean longitude; which corrected by the equation of the center will give his true longitude.

TO FIND THE SUN'S RIGHT ASCENSION.

The latitude of the place being known, find the sun's meridian altitude, from whence his declination is obtained; with which and the known obliquity of the ecliptic his right ascension is found by trigonometry, thus. As tangent obliquity : radius : : tangent declination : sine right ascension.

TO COMPUTE THE EQUATION OF TIME.

The equation of time is the difference between mean and apparent noon, and consists of two parts, one arising from the obliquity of the ecliptic, and the other from the sun's unequal motion in his elliptic orbit, and may be computed in the following manner.

The difference between the mean and true anomalies, or the equation of the center converted into time, is the first part of it. With the sun's longitude and obliquity of the ecliptic compute the sun's right ascension, and the difference between the longitude and right ascension when turned into time, gives the other part. Then the sum or difference of these will give the whole equation.

A solar day is the interval between two successive transits of the sun over the meridian, which is always more than a revolution of the earth on its axis, and at a mean rate is measured by 360° 59′ 8″; though sometimes more and sometimes less, because of the sun's unequal motion in right ascension. But a sidereal day is the interval between two successive transits of the same star over the meridian, and is always the same, being measured by 360°, and is equal to $23^h\ 56'\ 4''$ of a mean solar day: because the difference between a mean solar day, and a sidereal day, viz. 59′ 8″, when converted into time, gives 3′ 56″ of time; which shows, that if a star come to the meridian at any hour, it will come to the meridian again 3′ 56″ before the same hour on the next day, by a clock which measures a mean solar day by 24 hours. The true and mean solar days are never equal, but when the sun's motion in right ascension is equal to 59′ 8″; which happens about Feb. 11, May 14, July 26, and November 1. The accumulation of these differences produces the equation of time.

TO FIND A TRUE MERIDIAN LINE.

This may be done in various ways, some of which may be accomplished with more accuracy than others. Two of the best I shall now mention. Having a clock well regulated to mean time, and a transit telescope, let the telescope be placed in the meridian, as nearly as can be guessed at, by the pole star; and observe the transits of two stars, one to the north and the other to the south, whose difference of right ascension is accurately known, and does not exceed a quarter of an hour. Now, if the difference of the observed times of passing, be the same with the difference of right ascensions, the telescope is in the meridian. But if the northern star pass the meridian first, as will be the case with β Ursa minoris, and β Libra, and the interval of observed time be greater than the difference of right ascensions, the telescope verges towards the east of the north meridian, and thereby prolongs the observed interval. If therefore in the next revolution of the northern star, it be kept on the middle wire of the telescope for half the number of seconds, by which the observed interval exceeded the true difference of right ascensions, the telescope following the star for that time, and then be suffered to remain in that position, it will now be exactly in the meridian.

But if the observed interval be less than the calculated difference of right ascensions, the telescope verges towards the west of the true meridian; and therefore the southern star, in the next revolution, must be kept on the middle wire of the telescope for half the number of seconds, by which the calculated difference of right ascensions exceeds the observed interval of their passage.

OTHERWISE THUS.

At a season of the year when the nights are more than twelve hours long, choose a northern star that does not set, and observe its passage both above and below the pole, across the middle wire of the telescope, placed nearly in the meridian, noting the times by the clock. Now if the interval of time between the passages of the star above and below the pole be equal to half a revolution of the stars, as measured by the clock, the telescope is precisely in the meridian. But if the star below the pole come later to the telescope than half a revolution of the stars, after it has passed above the pole, the telescope verges to the east of the north meridian: and if the contrary, it verges towards the west. The error of the telescope may then be calculated and corrected by this

RULE.

To the proportional or logistical logarithm of half the difference between half a revolution of the star and the interval of the two observations, add the logarithm cosine of the star's polar distance, the logarithm cosine of the latitude of the place, and the logarithm secant of the star's altitude; and the sum, rejecting 30 from the index, will be the proportional or logistical logarithm of the time between the observation of the star above or below the pole, and the true time of its passage over the meridian.

EXAMPLE.

Suppose β Ursa minoris, whose polar distance is 14° 35′ 30″, and whose altitude above the pole, at Philadelphia,=54° 33′, and altitude when on the meridian below it=25° 22′, was observed, at Philadelphia, whose latitude is 39° 57′ 30″, to pass through the telescope at 9h 0′ 0″ by the clock, which loses 4″ of solar time per day; and again to pass under the pole through the telescope, remaining in the same vertical position, at 9h 3′ 16″; to find the deviation of the telescope from the meridian.

As the clock loses 4″ per day, instead of 12 hours,
it counts for half a revolution only 11h 59′ 58″
To which add the time of the star's passing above the pole, viz. - - 9 0 0
the sum would be the time of its passing under the pole, viz. - - - 8 59 58
had the telescope been right; but it was observed actually to pass at P. at - 9 3 16
The telescope therefore verges to the eastward of the meridian, because the star passes 3′ 18″ later than half a revolution.

To the prop. log. of $\frac{3' \; 18''}{2}$=1′ 39″		2.0378
add cos. pol. dist.	14 35 30	9.9858
cos. lat. Philad.	39 57 30	9.8846
sec. alt. * above the p.	54 33	10.2366

sum of these log. abating 30 from ind. 2.1448=1′ 17″.4 =error in time of telescope above the pole.

For the error of the telescope by the observation below the pole.

To prop. log. of	1 39	2.0378
add cos. pol. dist.	14 35 30	9.9858
cos. lat. Phil.	39 57 30	9.8846
sec. alt. *	25 22	10.0440

1.9522 =2′ 0″.6= the error of the telescope below the pole.

If the star, therefore, in its next revolution above the pole, be followed by the telescope for 1′ 17″.4 of time, and the star be kept on the wire during that time, the telescope will then be in the meridian; in which a mark should be fixed, so as to regain the direction at pleasure. If the telescope had verged too much to the westward, the star must be followed in its passage under the pole, during the time of the error of position under the pole.

DEMONSTRATION.

Let Z* be the zenith of the place of observation, P the pole, and ZAB the position of the telescope near to the meridian ZPC; A, B, the places of the circumpolar star observed above and below the pole. As the telescope is supposed to be placed nearly in the meridian, the angle PZA is but small, as is also the angle ZPA, which measures the difference of time between that of the star's passing through the telescope, and of its passing over the meridian above the pole. For the same reason, AB may be considered as equal to 2AP. The supplement of the angle APB, is the difference between half a revolution of the stars measured by the clock, and the interval of time between the star's passing the telescope above and below the pole, which is to be found.

As S, AB=S, 2AP : S, APB, : : S, AP : (S, PBA or PAB=)S,PAZ$=\frac{\text{S,APB}\times\text{S,AP}}{\text{S, 2AP}}=\frac{\text{S,APB}\times\text{Sec.AP}}{2}$; because half the secant of an arch, is as the sine of the arch divided by the sine of double the arch. In the triangle APZ, As I,PZ : (S,PAZ=)$\frac{\text{S,APB}}{2}\times$Sec.AP

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* See Plate 15. Fig. 4.

$:: \mathrm{S,AP} : \mathrm{S,PZA} = \frac{\mathrm{S,PB}}{2} \times \frac{\mathrm{Sec.\ AP} \times \mathrm{S,AP}}{\mathrm{S,PZ}} = \frac{\mathrm{S,APB}}{2} \times \frac{\mathrm{T,AP}}{\mathrm{S,PZ}} =$ the deviation of the telescope from the meridian.

But in triangle APZ, as $\mathrm{S,AP} : (\mathrm{S,PZA} =) \frac{\mathrm{S,APB}}{2} \times \frac{\mathrm{T,AP}}{\mathrm{S,PZ}} :: \mathrm{S,AZ} : \mathrm{S,APZ} = \frac{\mathrm{S,APB}}{2} \times \frac{\mathrm{S,AZ}}{\mathrm{S,PZ}} \times \frac{\mathrm{T,AP}}{\mathrm{S,AP}} = \frac{\mathrm{S,APB}}{2} \times \frac{\mathrm{S,AZ} \times \mathrm{Sec.\ AP}}{\mathrm{S,PZ}} = \mathrm{S,APZ} =$ the error of the telescope in azimuth.

Now the value of the angle APZ may be calculated from this value of it by a table of common logarithms; but if we desire the value immediately in time, or in parts of a circle, by a table of logistical or proportional logarithms, we must use the reciprocal of each term in the above theorem, according to the rule which we have already given.

TO FIND THE TIME OF THE DAY OR NIGHT, BY THE MERIDIAN TELESCOPE.

When the telescope is fixed in the meridian, and moves only in a vertical circle, observe the appulse of the sun's limbs to the perpendicular wires of the telescope, noting the time by the clock. Then half the time which the body of the sun takes to pass a wire, being added to the time of the western limb's appulse to the middle wire, gives the time of his center passing the meridian, or apparent noon by a dial; which is converted into the mean noon of the clock, by the equation of time being added or subtracted.

At night, observe when a known star passes the meridian, and note the time; then from the right ascension of the star, subtract the right ascension of the sun, computed up to the time of the star's passage, and the remainder, when less than 12 hours; is the time of the star's culmination after noon; but when more, its overplus above 12 is the time after midnight. The difference between this calculated time and the observed time, gives the error of the clock, by which it may be regulated.

OTHER METHODS OF FINDING THE APPARENT TIME.

Observe the altitude of the sun in a place whose latitude is known: and correct the observation for refraction and parallax, so as to obtain the correct altitude. To the logarithm sine of half the sum of the zenith distance, co-latitude and polar distance, add the logarithm sine of the said half sum lessened by the zenith distance, and also the arithmetical complement of sines of the co-latitude and polar distance; then half the sum of these logarithms will be the logarithm co-sine of half the hour angle from noon, which convert into time, at the rate of fifteen degrees for every hour.

The distance of a star from the meridian is found in the same manner as the distance of the sun, from the observed altitude, and consequently the time of its culmination, which is the difference between its right ascension and the right ascension of the sun, by which the time of the night may be found. These altitudes should be taken when the sun or star alters its altitude fastest, that is, when due east or west, or as near to that position as possible, as the result will thereby be the more accurate.

But the method by equal altitudes is much more accurate, and is as follows. Let the altitude of the sun or star be observed by a Hadley's quadrant, or equal altitude instrument, while the object is on the east side of the meridian; and, the instrument remaining set to the same altitude, observe when the object has the same altitude again on the west side of the meridian, and note both the times; then half the sum of these times increased by six hours, will be the time when the star was on the meridian; but there must be an allowance made for the change of the sun's declination in the interval of the observed altitudes. This correction in the latitude of Philadelphia may amount to near a quarter of a minute, and is found ready calculated among astronomical tables.

The method of observing the sun is this. Note the time when the preceding limb touches each of the horizontal wires, when the observation is made by the transit instrument, while he is on the east side of the meridian; and in the afternoon, when the sun descends to the same altitude, having turned the instrument round on its axis, still preserving the altitude of the telescope, observe the appulse of the preceding limb of the sun to the first horizontal wire, and write down the time opposite to the last observed time in the forenoon; also the time of its appulse to the second wire, write opposite to the last observation but one in the forenoon, and so on to the last. Half the sum of any two corresponding times, increased by six hours, gives the time of apparent noon nearly, to be corrected, as above, by the equation of corresponding altitudes; and the mean of the six sets of such observations will give the apparent noon within a second.

When the observation is to be made with the common reflecting quadrant, stand in such a position, that you may see the image of the sun reflected from a bason of oil or mercury, and bring the image, formed by the double reflexion of the instrument, a little below the image seen in the bason, and screw the index fast, without altering until the afternoon observations are over. In a little time the limbs of the two images will be in contact, which time note down as the first observation; when the centers are coincident, note the time also for the second; and finally, when the limbs are in contact before the separation of the images, for the third and last observation. In like manner make the afternoon observations, and write them down in an inverted order, as before directed, and deduce the time of apparent noon from them, correcting the mean by the equation of corresponding altitudes. The elements of solar tables being determined by the preceding methods, the tables themselves may be easily constructed in the following manner.

If 360° be divided by the length of a solar revolution, the quotient will be 0° 59 8″ for the motion of one day, and its multiples show the mean motion for any number of days. The twenty-fourth part of it gives the mean motion for an hour, and in like manner for any other time.

Let the time of his entering the first point of Aries be well ascertained by observation, when his mean motion from Aries is nothing; his mean motion therefore, for as many days, hours, and minutes, as have elapsed from the noon of the first day of January to that time, subtracted from 360°, will be the mean motion for the beginning of that year; for astronomers begin the year at that time. To this add the motion for 365 days, and the sum will give the mean motion for the succeeding year. Thus you proceed for three years; but because the fourth is bissextile, containing 366 days, the mean motion of one day more must be added, to have the motion for the beginning of it. In like manner, the radical mean motion for any year being lessened by the multiples of the yearly motions, observing to allow for leap-years, will give the mean motions of any preceding years.

By the same method, the mean motion of the apogee from Aries being 66 per annum, its multiple will give the mean motion of the apogee for any given time.

Let the place of the sun's apogee, and the time of his passage through it, be accurately determined by observation, when the motion of the apogee is nothing; then the distance of the apogee from Aries is known. Let this distance be lessened by the apogeon motion from the beginning of the year to the time of passing the apogee, and you will have the motion of the apogee at the beginning of the year. Then that radical mean motion being increased or diminished by the multiples of the yearly motions, will give its mean motions for any succeeding or preceding years. And the sun's mean motion for any time being lessened by the motion of his apogee, gives his mean anomaly for that time, to which applying the equation of his center, his longitude will be obtained.

The table of equation of the center is thus constructed. To every degree of mean anomaly assumed, find the true anomaly, and the difference between these will be the equation of the center. If these equations be computed for six signs, the equations for the other six signs are the same, only placed in an inverted order; because equal anomalies are at equal distances, on each side of either apsis.

THE CIVIL YEAR.

In order to record past transactions, mankind soon found it necessary to have some large measure of time, such as a year, which they naturally deduced from the return of the same seasons, for procuring or gathering the fruits of the earth. This they have subdivided into months, and these again into weeks of seven days each. Observing that the moon appeared to pass through all her phases about 12 times in the return of the same seasons, they divided the year into twelve months, of about 30 days each; so that the year of the ancients consisted of 360 days. But in process of time, they found that this method of estimating the length of the year did not agree with the revolution of the sun; which put them upon various ways of correcting it, either by adding a few days every year, or a whole month after a certain number of years.

At length, Julius Cæsar, about forty years before the commencement of the christian æra, took this troublesome affair in hand. Having obtained from Sysogenes, an eminent mathematician of Alexandria, information that the length of the solar year was about 365 days and a quarter; hence he ordered that the year should consist of 365 days for three years together, and that every fourth year should contain 366. This intercalary day, every fourth year, was inserted after the 23d of February; which, in their way of reckoning the days of the month, was called the sixth of the calends of March, sextilis calendarum. This sixth of the calends was reckoned twice that year, or the 23d and 24th of February were both called the sixth of the calends of March, and hence the year obtained the name of bissextile. In our almanacks, this day is added after the 28th of February.

The above is called the Julian account, or old style, from this rectification of Julius Cæsar, who thought he had thereby made the civil year to keep pace with the solar year; so that the equinoxes and solstices would constantly return on the same days of the month, in all succeeding years. But this mean length of the civil year, viz. 365 days and six hours, exceeds the tropical year of $365^d\ 5^h\ 48'\ 57''$, by $11'\ 3''$, which amounts to a whole day in 130 years; so that in that space of time, the equinoxes will anticipate a whole day, or happen one day of the month sooner. In 400 years by the Julian account, there would be contained 400 years, three days, one hour, fifty-three minutes and twenty seconds; hence if three days were left out of the civil account in every 400 years, the civil and solar years would pretty nearly agree together; as the $1^h\ 53'\ 20''$ would not amount to a whole day in less than 5082 years, and therefore might be neglected as inconsiderable.

In the year 325 of the christian era, when the council of Nice settled the canon for the celebration of Easter, the equinox fell on the 21st of March; but in the year 1582 it fell on the 11th of March, 10 days sooner. Pope Gregory therefore determined to restore the vernal equinox again to the 21st of March, and accordingly ordered the 5th of October 1582 to be called the 15th, and thereby struck out 10 days from the calendar in that year. And to prevent the like anticipation of the equinoxes for the future, by the use of the Julian account, he ordered three days to be left out of it, in every 400 years; by making the centurial years, not divisible by 4, such as 1700, 1800, 1900, 2100, &c. to be common years of 365 days, whereas they would have been bissextile, by the Julian account. Thus while the years 1600, 2000, 2400, are allowed to continue bissextile, the three days of anticipation in 400 years are left out of the calendar. This rectification is called the new stile, or Gregorian account. It was immediately adopted in the countries where the Pope's authority was acknowledged, but was not admitted in the British dominions, till the year 1752, when they were obliged to strike out 11 days, which they did by calling the 3d of September the 14th, to bring the vernal equinox back to the 21st of March, as it was at the time of the council of Nice.

ERAS AND CYCLES.

In computations of time, we find it necessary to fix upon some remarkable transaction, as the beginning of our reckoning, which is called an era or epocha. Thus some compute from the creation of the world, 4004 years before Christ; the ancient Greeks from the institution of the Olympiads of 4 years each, 776 years before Christ; the Romans from the building of Rome, 753 years before Christ; the Chaldeans and Egyptians from Nabonassar, king of Babylon, 747 years before Christ; the Mahometans from the flight of Mahomet, called the Hegira, 622 years after Christ; and Christians from the vulgar account of his birth, which is now generally accounted to be 4 years erroneous, or too late.

Astronomers have found that their computations are much facilitated by the invention of certain cycles, of which the following are the most remarkable.

The Cycle of the Sun, is a period of 28 years, after which the same day of the month returns on the same day of the week, and the dominical letters return in the same order. If 365 be divided by 7, it will quote 52, and leave a remainder of one, which shows that there are 52 weeks and one day in a common year; that the first and last days of the year are on the same day of the week, and consequently that the next day of the week will be the first day of the succeeding year. Now it has been customary to place the first seven letters of the alphabet opposite to the days of the month, and as there are but seven days in the week, the same letter that stands opposite to the Sunday of any week, will stand opposite to all the Sundays of that year, and is therefore called the dominical letter. Now the first of January having the letter A placed opposite to it, in every year, should this day be Monday, the letter G will be the dominical letter for that year, and Monday will be the last day of the year; so that the next year beginning on Tuesday, and having the letter A placed opposite to it, F will be opposite to all the Sundays of that year. For the same reason, E will be the dominical letter for the third year. Thus in seven years all the seven letters would be dominical letters in their turn, beginning at the last and going on in a retrograde order to the first, if they were all common years of 365 days. But as this order is interrupted every fourth year, which contains 366 days, or 52 weeks and two days, the next year must begin two days further in the week than the last, thereby leaping over one, for which reason the fourth is called leap-year. This series cannot return until it be as often interrupted as there are days in the week, that is, seven times. Now as this interruption happens every fourth year, and thereby one day in the week is passed over, it will require 28 years to pass over all the days of the week, after which time the same days of the month will happen on the same days of the week, and the dominical letter be the same that it was 28 years before. As the 28th and 29th of February are supposed to have the same letter affixed to each of them, as was formerly the case with the 23d and 24th, when the intercalary day was inserted after the 23d of the month, it is evident that the letter which pointed out Sunday before, will not stand opposite to the Sundays after that day. So that in leap-years there will be two dominical letters, the one serving for January and February, and the other, being the letter preceding it in the alphabet, for the remainder of the year. At the birth of Christ, 9 years of this cycle of the sun were elapsed: and therefore if 9 be added to any given year of the christian era, and the sum be divided by 28, the quotient will show how many complete cycles have elapsed, and the remainder will show what year of the cycle, the given year is.

To find the dominical letter; to the given year add its 4th part, rejecting fractions, divide the sum by 7, and the remainder taken from 7, leaves the number of the letter from the beginning of the alphabet:* in leap-years, it and the foregoing are the dominical letters.

The Metonic Cycle, so called from Meton an Athenian, who invented it about 432 years before Christ, is a period of 19 years, containing all the variations of the days in which the new and full moons, with the eclipses of the sun and moon can happen; after which they occur again on the same days of the month, on which they occured 19 years before; only with a variation of $1^h\ 28'\ 15''$ sooner. So that we may expect a new or full moon on the same day of the month, on which it happened 19 years before; only that it will be nearly an hour and a half sooner in the day. This anticipation will in a little more than 16 lunar cycles, or 310.7 years, amount to a whole day, and it has now anticipated five whole days since the council of Nice.

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* In the 19th century, 1800 being a common instead of a leap-year, 1 must be added to this number. *Ed.*

If you place a unit opposite to the days of the months, in which the new moon happens, through the whole of the first year of the cycle, the number 2 opposite to the like days in the second year of the cycle; and so on, through the whole 19 years, you will have by these numbers, properly disposed in a table, the days of the month opposite to them, in which the new moons happen in each year of the cycle. These 19 numbers for their use and excellence, were denominated the primes or golden numbers. But on account of the beforementioned defect of $1^h\ 28'\ 15''$, this table can answer the purpose intended only for 310.7 years, and then it will require that these numbers be set one day higher in the table, by which a pretty regular correspondence may be preserved between the lunar and solar years. These golden numbers thus disposed, must indicate the number of the current year of the cycle at any time. Now as one year of it was elapsed at the birth of Christ, and the prime was 2 in the year in which he was born; if we add 1 to any year of the christian era, and the sum be divided by 19, the remainder will show the year of the current cycle.

At the time of the Nicene council, and for a long time after, it was thought that 19 years, or 228 solar months, were exactly equal to 235 lunar or synodical months, and therefore that the golden numbers, set in their calendars opposite to the days on which the new moons happened through one lunar cycle, would invariably serve for the new moons of every successive lunar cycle. Therefore they ordained that Easter should be kept on the first Sunday after the full moon, which in common years happened after the 21st of March, or after the 20th of March in bissextile years. Hence it can never happen after the 25th of April.

The epact, which depends upon this cycle of 19 years, indicates the age of the moon at the end of the year. As the time of a lunation is $29\frac{1}{2}$ days in the nearest whole numbers, there are 354 days$=12\times29.5$ in the lunar year; but this being 11 days less than the solar year, it follows, that if these years should begin together at a new moon, the moon would at the end of it be 11 days old. For the same reason, it would be 22 days old at the end of the second, and 33 days old at the end of the third; but because she will have changed 3 days before the expiration of the year, she will only be 3 days old, and at the end of the next, 14, &c. to the end of the cycle of 19 years; always rejecting 30 for an intercalary month added to the lunar year, when the epact would exceed 30. At the end of the cycle, the epacts would run over again, if the cycle were perfect, and they would always be 11 times the primes.

To find the epact; multiply the golden number for the year by 11, divide the product by 30, and from the remainder take 11, if it be greater than 11, or add 19 to it if it be less, and the difference or sum will give the epact, till the year 1900.

By the intercalations of seven months in 19 solar years, to the lunar year, each month consisting of 30 days, except the last of 29 days, the lunar and solar years would keep pace with each other, if the Metonic cycle had been perfect.

The Dionysian Cycle or period, is the product of the cycles of the sun and moon $=28\times19=532$ years; after which time, not only the new and full moons return on the same days of the month, but also the days of the month return on the same days of the week. Being a compound of the former two cycles, it must have the properties of both. So that after the expiration of this period, the dominical letters, days of the week, and of the month, with the new and full moons, return again in the same order. For this reason it is denominated the great Paschal Cycle, and the Dionysian Period.

The Cycle of Indiction is a period of 15 years, used by the Romans for indicating the times of taxation in that empire; but as it has no connexion with the motion of the heavenly bodies, we would have nothing to do with it in these lectures, had it not been that it made an element in another cycle which is of much importance in chronology, and which is called the Julian Period, from Julius Scaliger, who has eternized his name by the invention. Three years of this cycle were elapsed at the birth of Christ.

The Julian Period is a cycle of 7980 years, arising from the continued multiplication of the cycles of the sun, moon, and indiction. As it consists of the other three cycles, it must commence when they all began together, being carried backwards. Therefore its commencement is placed in that imaginary point of time, which was 710 years before the creation of the world; so that 4714 years of it were elapsed at the birth of our Saviour. Being of such extent, it comprehends all other periods, epochas and cycles, with all their memorable transactions, and is not yet completed; so that it is well adapted to the purposes of chronology.

SECONDARY PLANETS.

Four of the primary planets belonging to the solar system, viz. the Earth, Jupiter, Saturn and Herschel, are attended in their annual course round the sun by secondary planets, usually denominated moons. The Earth has one, Jupiter four, Saturn five, and Herschel two already discovered. All these revolve from west to east, in the same manner as their primaries, in similar orbits, and observing the same law of gravitation which regulates the planets in their elliptic orbits round the sun, with respect to their periodical times and distances from their primaries, and the description of areas proportional to the times of description. The moons of Herschel, however, revolve from north to south nearly.

The periodical times of these moons are as follows.

OF JUPITER.

					Distances in semid's. of Jupiter.	In miles.
1.	1d	18h	28′	36″	5⅔	229,473
2.	3	13	18	50	9	364,500
3.	7	3	59	40	14 23/60	582,535
4.	16	18	5	6	25 18/60	1,024,658

*OF HERSCHEL.

1.	8	17	1	19
2.	13	11	5	1.5

OF SATURN.

					Distances in semid's. of Saturn.	In miles.
1.	1	21	18	30	8⅞	140,000
2.	2	17	41	22	11¼	187,000
3.	3	12	25	12	15	263,000
4.	15	22	41	15	36	600,000
5.	79	7	48		108	1,800,000

OUR MOON.

			Distances in semid's. of Earth.	In miles.
27	7	43	60.25	240,000

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* The orbits of the two discovered satellites of Herschel lie nearly in the same plane, and nearly at right angles with the plane of the orbit of their primary: being inclined to it in an angle of $\left\{\begin{array}{l}99^\circ\ 43'\ 53''.3\\ 81\quad 6\quad 4\ .4\end{array}\right\}$ respectively, as determined by Mr. Herschel the discoverer of this planet. Their ascending nodes are in $\left\{\begin{array}{l}18^\circ \text{ of Virgo}\\ 6 \text{ of Sagittar.}\end{array}\right\}$ We shall have eclipses of these satel-

These satellites sometimes appear on the east side of their primaries and sometimes on the west, and are at their greatest elongations, when a line drawn from the earth to them is a tangent to their orbits. They appear to us in conjunction with their primaries, either when they are between the earth and their primary, or beyond it; the first is called their inferior conjunction, and the other their superior conjunction. From their greatest western elongation to their greatest eastern elongation they are direct in motion; and retrograde in the other half of their orbits,

lites about the years 1799 and 1818, when they will appear to ascend through the shadow of the planet, almost perpendicularly to the ecliptic. They are probably not less than the satellites of Jupiter. Four additional satellites have lately been discovered.

As the radius of our moon's orbit, which is nearly 240,000 miles, would appear under an angle of no more than 27″.1866, if seen from the mean distance of Herschel, and his outermost satellite appears to us under an angle of 44″.23 at its mean distance from its primary; and the first or innermost satellite under an angle of 33″, they must be more distant from their primary, than our moon is from us; the first being nearly $\frac{1}{5}$ farther, and the other $\frac{3}{5}$.

Herschel has lately discovered two new satellites revolving round Saturn within the five before discovered. The periodical time of the nearest is less than 24^h, and it is but 26″ distant from the center of Saturn, the exterior boundary of the ring being 22″ distant from the same. The periodical time of the second is 32^h 53′ 9″, and the periodical time of the nearest is 22^h 40′ 46″, it is much the smallest of the whole system. The distance of the nearest is from the center of Saturn but 27″.366 and of the second 35″.058. The revolution of the nearest is not yet so accurately determined as that of the others, arising from its proximity to the ring. The ring of Saturn revolves in 10^h 32′ 15″, and Saturn himself in something less than half a day.

namely, while they are passing between the earth and their primaries.

The orbits of the satellites of *Saturn* lie nearly in the plane of his ring produced, which intersects the ecliptic in an angle of 31°, and therefore they will appear to us to move in ellipses similar to his ring, and never in right lines, but when the ring becomes invisible; the orbit of the fifth satellite only excepted, which makes an angle with the orbit of Saturn of 13° 8′. Their ascending nodes, by the observations of Cassini, 1714, were in ♍ 20° 26′, excepting that of the fifth which was in 3° 20′ of the same sign. From the inclinations of the orbits of Saturn's satellites, they can seldom pass behind the body or ring of Saturn, or through his shadow, excepting when they are near to their nodes. So that their eclipses are but seldom, and being so distant from the earth, and never seen but with the best of telescopes, we can derive but little advantage from observing them. They were discovered gradually; the largest first, and afterwards the rest, as telescopes were brought to greater perfection. Hugens first discovered the fourth, which is the largest, in 1655; the others were discovered by Cassini, two in 1671, and the other two in 1686. They are not to be seen but in a serene night, and with a very good telescope.

The revolution of the nearest sat. of Saturn is	0d 22h 37′ 22″
———— of the next - - -	1d 8h 53′ 10″
———— of the outer ring - -	0d 10h 31′ 6″

The orbits of the satellites of Jupiter lie nearly in the plane of the ecliptic, for which reason they generally appear to us almost in a straight line on both sides of Jupiter, passing over his disk in their inferior conjunctions and lying hid behind him in their superior conjunctions. This is the case with all the satellites in every revolution, excepting the fourth, which for four years passes above and below the body of Jupiter. When the satellite is concealed from us by the interposition of Jupiter, it is called an occultation, and when it passes through the shadow of Jupiter, it is called an eclipse, at which time it is invisible even by our best telescopes. The three interior satellites are eclipsed in every revolution round Jupiter; but the orbit of the fourth being a little inclined to the ecliptic, and its distance from Jupiter so great, it escapes being eclipsed, when it is above 52 degrees from its nodes. The eccentricities of their orbits is but small, the micrometer not showing any difference in their greatest elongations at different times, excepting the fourth, whose orbit is found to be an ellipsis, having its eccentricity about $\frac{7}{1000}$ parts of its mean distance. No eccentricity has yet been discovered in the first satellite. The inclination of its orbit to that of Jupiter was settled by Cassini at 2° 55′ and its ascending node in ♒ 15°. Mr. Dunthorne has determined the ascending node of the second to be in ♒ 5° and to be at rest; that of the third to be in ♒ 16° 30′ in the year 1727, moving forward at the rate of 8° in 60 years; and that in 1730 the ascending node of the fourth was in ♒ 13° 30′, where Cassini found it about 60 years before.

The inclination of the orbit of the second satellite is variable from 2° 50′ to 3° 52′. It was least in 1668 and greatest in 1752, and seems to have made one and a half revolution in the intermediate time. The second is supposed to have some eccentricity, but the quantity is unknown. The orbit of the third has an eccentricity about a third part of that of Venus; its inclination to the orbit of Jupiter is also variable, it was 3° in 1695, and about 3° 24′ in 1765, when it was near its maximum. Its apojovium in 1728 was in ♈ 10°, and seems to move forward in 50 years about 90°. The apojovium of the fourth, in 1728 was in ♓ 12° 30′, and seems to move forward 60° in a century. And the inclination of its orbit is 2° 40′, and does not seem to vary above one or two minutes any way.

The periodical times of these satellites as given above, are subject to some variations, both on account of their mutual gravitations to each other, and the progressive motion of light. The irregularity arising from their mutual gravitation seems to have its period in 437 days, in which time the three interior satellites return nearly to the same position. This irregularity is small in the first, smaller in the third, and not perceptible in the fourth, but very considerable in the second, amounting sometimes to near 40 minutes.

The other irregularity is only apparent, as it arises from the motion of light, which requires about eight minutes to pass from the sun to us. The distance of Jupiter from the earth in opposition, is less than when in conjunction, by the whole diameter of the earth's orbit; and we accordingly find that the eclipses of the satellites, which happen in the former case, are 16 minutes of time sooner, than those which happen in the latter. From this retardation of the time of the eclipses, as Jupiter is removing from the earth, the celebrated Reaumur determined the velocity of light in the above ratio. From this progress of light we know, that we can see a satellite 8 minutes after it is involved in the shadow of Jupiter, and cannot see it for 8 minutes after its emersion.

The rays of light emitted from the sun while he is in any place do not arrive at our eyes, for 8 minutes after he has left that place.

The beginnings and endings of the eclipses of Jupiter's satellites are almost instantaneous, and therefore are very proper for determining the longitudes of places on the earth. The eclipses of the first are the most proper for this purpose, because of their frequency, and its greater velocity. As they are the same to every spectator on that side of the earth that is turned to Jupiter, if two persons observe the beginning or end of the same eclipse, called the immersion or emersion, and note the precise times of their observations, the difference of these times converted into degrees, at the rate of one hour for every fifteen degrees, will give the difference of longitude between the places of observation.

When Jupiter is near to his conjunction with the sun, he and his satellites are concealed in the rays of the sun. From the time of his appearing after his conjunction till near his opposition, his shadow being projected to the right hand of his body, the immersions of the satellites are visible on the west of the planet, when they fall into his shadow. From the opposition to the conjunction, his shadow being projected to the left hand, the emersions are visible on the east side of Jupiter, when they emerge out of his shadow on that side.

The beginning and end of the same eclipse of the first satellite can never be seen by us, because one side of the shadow of Jupiter is always hid behind his body, at the distance where the first satellite passes through it. Nor can they be both seen in the second, unless the satellite be near to its limits, or 90° from its node, and Jupiter at the same time be near his quadrature with the sun, and near his perihelion. As to the third, when Jupiter is more than 46° from his conjunction with, or opposition to, the sun, both the beginnings and ends of the eclipse may be seen on the same side of Jupiter; as they are also of the fourth, when Jupiter is 24° from his opposition or conjunction. When Jupiter is in quadrature with the sun, his shadow is then most exposed to our view, and the exterior satellites pass through so small a section of it, that both the immersions and emersions are seen either before or after the occultation behind the body of Jupiter. When the fourth satellite is about 52° from its node, the inclination of its orbit is such, as to make it pass either above or below the shadow, so that it will escape being eclipsed for two years together, which happens twice in every revolution of Jupiter, and therefore for two years in every six there are no eclipses of this satellite.

THE MOON.

THE moon accompanies the earth in its revolution round the sun, at the same time revolving round the earth as the center of her motion, and finishing her period in $27^d\ 7^h\ 43'$, moving at the rate of 2290 miles every hour in her orbit. She turns round her axis in the same time that she spends in going round the earth, which is the reason that she constantly turns the same face towards the earth, so that her day and night is as long as her revolution. As her axis is almost perpendicular to her orbit, she has scarcely any change of seasons. Yet the lunarians may determine the length of their year and of ours, which is the same, by observing when either of our poles begins to be illuminated.

As she has no light inherent, and only shines by light from the sun reflected to us, she exhibits all her different phases, according as more or less of her enlightened side is turned towards us. When she is between us and the sun, her enlightened side is turned away from us, and therefore she is invisible. When she has advanced in her orbit towards the east, a little segment of her enlightened surface is turned towards us, and therefore she exhibits the appearance of a section of two circles, less than the area of a semi-circle, until she has arrived at 90° from the sun, when the segment of her enlightened surface appears to be a complete semi-circle. From thence more and more of her enlightened face is gradually turned towards us, until she come to her opposition, when she shines with a full face. From thence to her change again she gradually assumes the same appearances, which she exhibited in the first half of her revolution, constantly turning her illuminated disk more and more from the earth, until she become invisible again in her change. All these phases of the moon may be exemplified by the sun shining on a round stone on the top of a gate. Place yourself in such a position that the lower limb of the moon may appear to touch the upper side of the stone, while the sun shines upon it, and the enlightened part of the stone, that is then turned to the eye, will appear precisely similar to that of the moon.

One half of the inhabitants of the moon have no absolute darkness at all; for during the absence of the sun, our earth appears to them under all the different phases of the moon to us, waxing and weaning, being full to them when she is new to us, and vice versa: and affording them thirteen times as much light as the moon affords to us. It is owing to the light reflected from the earth, that the unilluminated part of the moon is visible to us for a few days after her change. The inhabitants of the other half of the moon can never see our earth, unless they travel round to the side that is exposed to our view, and therefore they have no moon-light in the absence of the sun, which continues for a fortnight together. From the center of that hemisphere that is next to us, the earth appears in their zenith continually, neither rising nor setting, but turning round on her axis, once in 24 hours, and thereby affording to them a complete and regular dial for the measure of their time, by means of the spots on her surface. From the parts which appear to us as the verge of her disk, the earth appears in their opposite horizon, without rising or setting, except as they travel towards the center of the disk, by which means the earth will appear higher and higher until she come to their zenith. Thus the lunarians can, from the altitude of the earth to them, easily determine both their longitude and latitude, with sufficient precision.

From the light observed to surround the moon in a total eclipse of the sun, some think that she is furnished with a rare and transparent atmosphere. But others are of opinion, from the sudden occultation of the fixed stars by the moon, without any previous diminution of their light, that she is destitute of an atmosphere, unless it be exceedingly rare: so that the stars will appear at noon day to her inhabitants, when they turn their backs to the sun. It is owing to our atmosphere, that the heavens appear bright to us, so as to obscure the stars: as we can see them through a telescope or from the bottom of a deep pit, in the day time, when the glare arising from the refracted light of the sun is thereby removed. If she have no atmosphere she can have no twilight, but must suffer an immediate transition from noonday light to midnight darkness. Nor can she have storms and rains as we have.

Her face, when viewed through a good telescope, appears to be exceedingly rough and uneven, diversified with high mountains and deep cavities, which are easily distinguished from each other, by the different projections of their shadows, from the different positions of the sun, and from the irregular and indented boundary of light and darkness; which if her surface were smooth and even should appear to be a straight line at her quadratures, and a regular elliptic curve at every other time. But it is quite otherwise, and although her limb appears to be a well-defined circle, this is owing to the mountains rising behind the cavities, and by their oblique positions to us, preventing us from discovering the irregularity of the boundary of light and darkness, near the limb; while we can easily see it, in any other position. She revolves round the earth in an orbit inclined to the plane of the ecliptic in a variable angle, which is about 5° 18′ at a mean, being sometimes more and sometimes less. This causes her to appear to be not quite round when she is full, near her greatest distance from her node; a small deficiency of light being discovered about her south pole, when she is full in the upper part of her orbit, and a similar deficiency about her north pole, when she is full in the southern part of her orbit; because we can at these times see farther than what is enlightened by the sun on one side, and not so far on the other. Hence it is that she will appear to nod backwards and forwards, alternately withdrawing her poles from our sight.

As she revolves round the earth in an elliptic orbit, she moves slower in her apogee, and faster in her perigee than at her mean distance, and therefore she will pass over the lower half of her orbit in less time than half her revolution, and consequently before she has revolved half round her axis, the motion on which is equable and exactly commensurate to her periodical time. The consequence of this is, that she will alternately exhibit to us a small segment of her eastern and western limbs, which causes her to appear to librate from east to west on her axis.

Although the moon revolves round the earth as her center of attraction, and is sometimes between the sun and earth, yet her path is always concave towards the sun. For she only approaches nearer to the sun than the earth is, by 240,000 miles, when at her change; but the versed sine of half the arc of the earth's motion for a month is at least 400,000 miles.

The moon when new is more attracted by the sun than by the earth; because the point in which any body situate between the sun and earth would be equally attracted by them, is at least 70,000 miles nearer to the earth than the moon is at her change, and consequently she must be more attracted by the sun than by the earth at that time. It may then seem strange, that she has not long ago deserted the earth, and revolved round the sun. But we must remember, that the distance between the earth and moon is so small, that if the sun could carry off the moon, the earth must also follow, which is prevented by the projectile force of the earth. One body would move round another in the cabin of a ship under sail in the ame manner, as if it were at rest. For an impulse on a system of bodies affects them all equally, and they would circulate round one another, by any other orces, in the same manner as if no such impulse had existed.

TIME OF THE MOON'S RISING, AND HORIZONTAL MOON.

As the moon performs her revolution in about 27½ days, she will advance to the eastward about 13° daily, and the sun advancing the same way only at the rate of one degree daily, she will gain on him about 12° every day; which to the inhabitants about our equator will retard the time of her rising about 48′ daily. But in different latitudes, this retardation will be different, so that in high latitudes she will rise for some evenings together with much less difference of time. To illustrate this, we may consider her path as nearly in the ecliptic, as it differs from it but about 5° 18′. Now in high north latitudes, the ecliptic is very much inclined to the horizon about the beginning of Aries; and therefore a short space of time will be sufficient, to bring 30 or 40′ of the ecliptic above the horizon, in this part of it, by the rotation of the earth on her axis. At the poles, one half of the ecliptic never sets, but is constantly above the horizon, and therefore while the moon is in this half, it can never set, nor does it rise while in the other half, for a fortnight together. Now the moon is twelve times in the year about the beginning of Aries, and must at these times rise for a few days, with very little difference of time. But her rising nearly at the same hour for a few days together is never so particularly noticed, as when she is full about the beginning of Aries, and this sign rises in the evening, the sun being in the opposite sign Libra, which is in the autumn. The full moons at this season of the year rising for a few evenings together nearly at the same time, have occasioned them to be particularly remarked, and to obtain the name of the *harvest moons*. In the winter, this part of the ecliptic rises at noon, and if the moon be in it, she must then be about her first quarter, when the time of her rising is not observed. In the spring, it rises with the sun, when the moon, if in it, must be new, and therefore invisible. In summer, it rises about midnight, and the moon being in it must be in her third quarter, when few are awake to observe the time of her rising. But in the autumn, it rises in the evening, when the moon, in it, must be full; and therefore it is more particularly observed.

The moon and sun, when seen in or near the horizon, appear larger, than at any other altitude, although they subtend nearly the same angle. This is owing to an optical illusion, and the method we have of judging of distances by sight. We judge the concave heavens to be a part of a greater sphere, whereof the horizontal base is nearly four times as long as the height above our heads: hence we naturally refer the sun and moon to a greater distance when seen in the horizon, than when near the zenith, and therefore we judge them to be larger to cause them to appear under the same angle at the eye. The indistinctness with which they are viewed in the horizon, on account of the greater refraction of the rays of light through a denser part of the atmosphere, may also contribute to this optical deception. For observing that an object gradually appears more obscure, as its distance from the eye increases, we naturally judge it to be more distant, the more obscurely it appears: and if more distant, consequently larger. And we are also compelled to judge the horizon to be more distant than the zenith, by the number of intermediate objects along the horizon, whereas we have none between the eye and the zenith. This will contribute to the same effect.

LUNAR INEQUALITIES.

ASTRONOMERS have found that the moon goes through her orbit from any given star to the same again, at a mean rate, in $27^d\ 7^h\ 43'\ 4''$; from her apogee to her apogee, in $27^d\ 13^h\ 18'\ 43''$; and from the sun to the sun again, in $29^d\ 12^h\ 44'\ 3''\frac{1}{20}$. Now since the moon is $5^h\ 35'\ 39''$ longer in going from her apogee to her apogee again than from a fixed star to the same, this shows that her apogee moves from west to east, as she moves in her orbit. And because she requires less time by $2^h\ 38'\ 27''$ to revolve from her node to her node again, than from a star to the same again; this shows that her nodes revolve from east to west. So that her apogee makes a complete revolution in eight common years, 311^d and 8^h; its annual motion being $40^\circ\ 39'\ 52''$, and daily motion $6'\ 44''\ 1'''$. And her node makes a complete revolution in eighteen common years, $254^d\ 7^h\ 33'$, its annual motion being $19^\circ\ 15'$, and daily motion $3'\ 10''\ 38'''$.

A mean lunation is $2^d\ 5^h\ 1'$ longer than a periodical revolution; because the earth has advanced so far in her orbit during a revolution of the moon, that it requires this time for the moon to advance to the same position again, with respect to the sun and earth, that she had at the beginning of her revolution.

If the moon in her motion round the earth was only affected by her gravitation towards it, she would follow the same laws which regulate the motions of the primary planets, describing areas proportional to the times of description. But as her distance from the sun is nearly the same with that of the earth, she must also feel the force of his attraction, sometimes conspiring with the attractive force of the earth, and sometimes acting against it, and thereby producing great irregularities in her motions, according to her positions with respect to the sun and the earth.

When the moon is in quadrature, the attraction of the sun increases her gravitation or tendency to the earth. The sun's attraction affects both the earth and the moon, and did it attract them in parallel lines, so that if their projectile force were destroyed, and they were to descend to the sun in parallel lines, such an attraction could not bring them nearer together in their descent; but the sun attracts them in lines which meet in the sun; and therefore the effect of this attraction of the sun must be to bring them nearer together, or to increase the tendency of the moon to the earth in her quadratures.

When the moon is in the syzygies, her tendency to the earth is diminished by the attraction of the sun. The sun attracts both the earth and the moon, but with different forces, as one is nearer to him than the other; and as the attraction of two bodies is mutual, whatever diminishes the attraction of the earth towards the moon, diminishes the tendency of the moon to the earth, and therefore in the conjunction and opposition of the sun and moon, their tendencies will, by the action of the sun, be diminished. But the diminution in the syzygies is double to the augmentation in the quadratures. Had this diminution and augmentation been equal, at the octants the disturbing force of the sun would make no alteration in the moon's tendency to the earth, but as one is double to the other, the four points at which they balance each other, will be about 54° 44′ from the syzygies on each side.

The diminution of the moon's gravitation to the earth in the syzygies, and its augmentation in the quadratures, tends to flatten her orbit in the syzygies, and to lengthen it in the quadratures; so that her orbit, even suppose it had been circular at first, is thereby made an oval or ellipsis, having the shortest diameter in the line of the syzygies, the longest, in the line of the quadratures, and the earth in the center. Hence the curvature of the moon's orbit will be greatest in quadrature, and least in syzygy. For when the moon's tendency to the earth is lessened in the syzygies, she is at liberty to proceed nearer to the direction of the tangent, and therefore to run out farther from the earth by the time that she arrives at her quadratures, where her tendency becoming greater towards the earth, she is again more deflected from her tangent, and brought nearer to the earth again in the succeeding syzygy. And as the moon is going farther from the earth as she advances towards her quadrature, her velocity will be continually decreasing till she arrives at that point, from whence it will again increase to her other syzygy. The small quantities by which the moon's velocity is diminished in going from the syzygy to the quadrature, and is again augmented from the quadrature to the syzygy, grow greater and greater from the syzygy to the octant, and then lessen in the same proportion from the octant to the quadrature. It is always in the ratio of the sine of twice the moon's distance from the syzygy. This inequality is called the moon's variation.

The moon describes areas proportional to the times, in the syzygies and quadratures, but in no other part of her revolution. In all others, the areas are less than in proportion to the times; because the sun's disturbing force is in the same direction with the earth's, at the syzygies and quadratures, but in other places oblique to it, and does not tend to the earth.

This disturbing force of the sun produces five remarkable inequalities in the moon's motion.

1. In the time of several successive periodical revolutions.

2. In the progressive motion of the line of the apsides.

3. In the variable eccentricity of the moon's orbit.

4. In the continual change of the inclination of the lunar orbit, to the plane of the ecliptic.

5. In the retrograde motion of her nodes.

I. The moon's orbit is dilated when the earth is in perihelion in winter, and contracted, when the earth is in aphelion in summer, and consequently the moon's *periodical time* is longer in winter than in summer. As the moon's tendency to the earth is more diminished than increased, upon the whole, by the sun's force, and as this force is greatest when the sun is in perihelion, the orbit of the moon must be more dilated, in this situation, than when the earth is in aphelion; and therefore the moon will require a longer time to perform her revolution, when her orbit is more dilated and larger. The revolutions of the moon, whether from apogee to apogee, or from node to node, are longer in winter than in summer.

II. When the moon is in the syzygies, the line of the *apsides* moves from west to east; but when she is in quadrature, it moves the contrary way; having the greatest progressive and retrograde velocity in these points. But as it moves more forward than backward, upon the whole, it will perform a complete revolution in the order of the signs, or forward, in 8^y 309^d 8^h 20′. Since the alternate increase and decrease of the moon's gravity towards the earth, is the cause why her central force is sometimes too great and sometimes too small, to be inversely as the squares of the distances: consequently the moon cannot describe an immovable ellipsis, nor indeed move in an ellipsis; unless the line of the apsides be supposed to be movable. Now the central force being least in the syzygies, she would recede farther from the earth than the elliptic curve, and would not be found in it, unless the apsis moved forward the same way. So that the apsides move from west to east in the syzygies. But the moon's central force being increased by the action of the sun in the quadratures, the contrary happens, and she would fall within the ellipsis, if the apsis did not move backwards. Therefore the apsis will move backwards in the quadratures. But as the force of the sun which diminishes the moon's tendency to the earth, is double to the force which increases it, and also extends to 54° 44′ on either side of the syzygies, while the augmentation of it extends only to 35° 16′ on either side of the quadratures, it is evident that the direct motion must exceed the retrograde; and therefore the apogee advances from west to east, upon the whole, and completes its entire revolution in the aforementioned time.

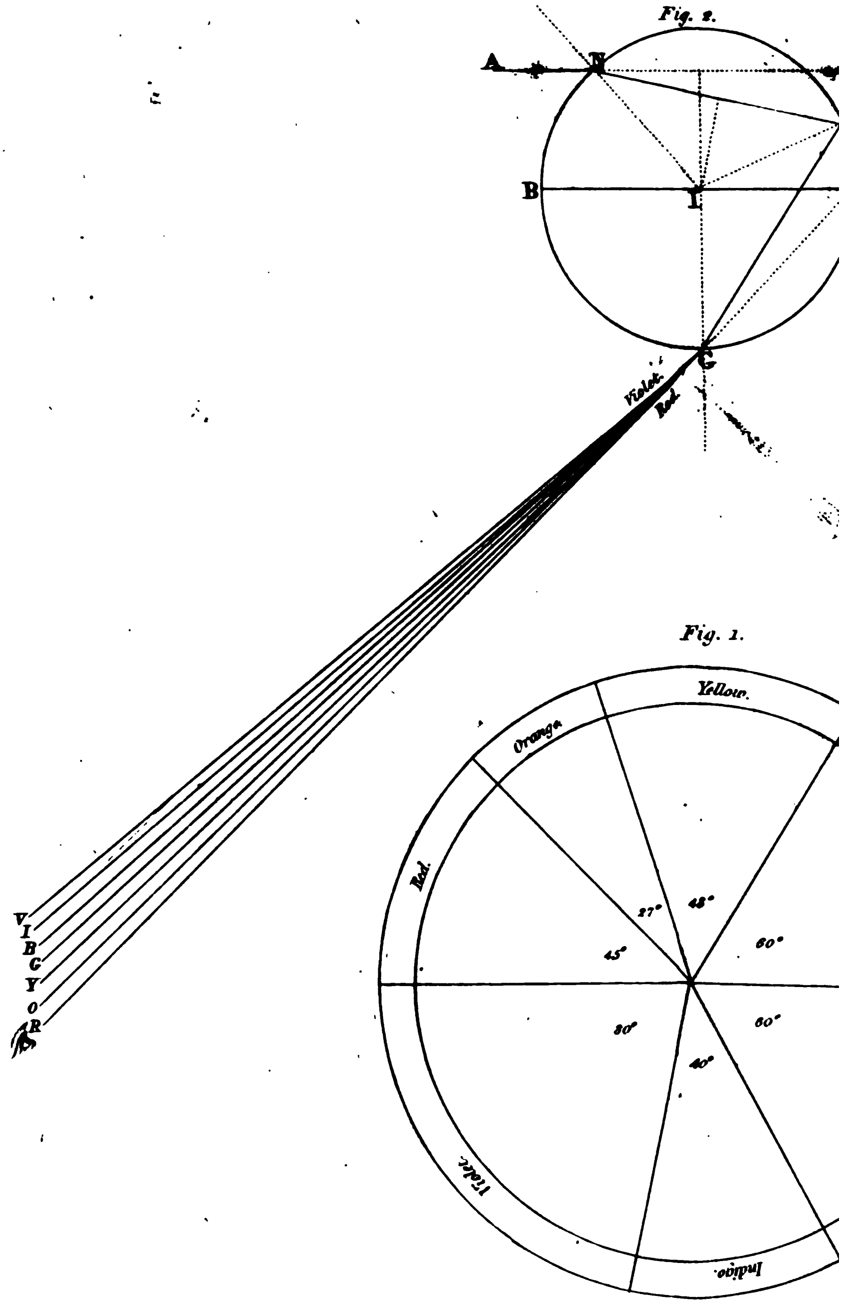
III. The *eccentricity* of the moon's orbit, varying contricity is greatest when the line of the apsides concurs with the line of the syzygies, and least when it concurs with the line of the quadratures. If the moon's central force were not diminished by the sun in the syzygies, it would increase from the apogee to perigee just as it would decrease in the other half of her orbit, and the eccentricity would not be altered. Or if the increase or diminution of force were always the same at the same distances in her ascent and descent, the case would be the same. But if her central force be more diminished in her ascent, than it is increased in her descent, the moon will depart more in her ascent, and run out to a greater distance from the earth in her apogee, and approach nearer to the earth in her perigee, than she would otherwise do; so that her apogeon distance will be proportionably greater than her perigeon distance, and her eccentricity will be increased. Now as the inverse ratio of the squares of the distances, in which the moon's central force should vary, is most disturbed by the sun when the line of the apsides concurs with the line of the syzygies, and least when it concurs with the line of the quadratures, the eccentricity will be greatest in the former case, and least in the latter.

In explaining the foregoing inequalities, we have taken no notice of the inclination of the lunar orbit, because they would all take place if her orbit coincided with the ecliptic. But there are inequalities in her latitude arising from the alteration of the plane of her orbit, which are now to be explained.

IV. In every revolution of the moon, the angle of *inclination* of the moon's orbit to the plane of the ecliptic, increases in the interval from the syzygy to the next quadrature, and decreases from the quadrature

Fig. 2.

Fig. 1.


A
N
B
I
G
Violet.
Red.
V
I
B
G
Y
O
R
S
Yellow.
Orange.
Red.
27°
48°
45°
60°
80°
60°
40°
Violet.
Indigo.

upon the whole, and is largest of all, when it arrives at that position.

V. When the moon is in quadrature, the nodes of its orbit are always stationary; but commonly retrograde when the moon is in the syzygies. From one quadrature to another, they are most retrograde when the line of the nodes concurs with the line of the quadratures; and least, when it concurs with the line of the syzygies.

When the nodes are in quadrature, while the moon is passing from quadrature to quadrature again, she is constantly solicited towards the ecliptic by the ablatitious force of the sun, and therefore she will arrive at it before she has performed half a revolution, and consequently will meet her node, which has retrograded in the mean time so as to meet her before she has gone 180° from her other node. As she moves from west to east, her nodes move from east to west, and perform their revolution in the time before mentioned. But they are quiescent, when the line of the nodes is in the syzygies, because the ablatitious force of the sun then acts in the plane of the lunar orbit.

When the line of the nodes is in the octants, from the quadrature to the first octant, while the moon is passing by her node, the ablatitious force of the sun is directed to a point on the opposite side of the line of the node from the moon, when it increases the obliquity of the lunar orbit, and causes the node to move forward during her passage through two octants in the revolution or 90°, but they move retrograde for 270°, so that upon the whole the nodes move more backward than forward in any revolution, and therefore move round in something less than nineteen years.

What has been said of the irregularities of the moon's motions is also true of the satellites of Jupiter and Saturn; but because of their great distances from the sun, these irregularities are generally insensible, excepting in those that are nearest to the planet, on account of their nearness to it.

The same method of reasoning shows, that the precession of the equinoxes is variable from 43″ per annum to 57″, being at a mean rate 50″, depending on the position of the moon's nodes; being greatest when they are in Aries, least when in Libra, and at a mean, when in Cancer and Capricorn.

ECLIPSES OF THE SUN AND MOON.

An eclipse of the sun is occasioned by the interposition of the moon between the sun and the earth, at the change of the moon; but an eclipse of the moon is occasioned by her passing through the earth's shadow, when she is full, or in opposition; so that there can be no eclipse of the sun, but at the change of the moon, and no eclipse of the moon, but when she is full. We shall first consider the

ECLIPSES OF THE MOON.

If the earth were as large as the sun, its shadow would be of equal breadth at all distances. But being much less, its shadow must terminate in a cone, whose vertex cannot reach to any of the primary planets, and therefore none of the celestial bodies can pass through it, or be eclipsed by the earth, but the moon. The semi-angle of the conical shadow of the earth, is equal to the sun's apparent semi-diameter lessened by his horizontal parallax, which is but 8″.65. For TES,* the measure of the semi-diameter of the sun seen from the earth, is equal to EAC, the semi-angle of the cone, +ESC, the horizontal parallax of the sun; S, being the sun, and C the earth. Hence, from the known semi-diameters of the sun and earth, and the distance between them, the length of the shadow may be easily determined, which is found to be about 864,000 miles. And from the known distance of the moon, the diameter of the earth's shadow, where the moon passes through it, is also easily found, by plane trigonometry. The difference too, between the moon's horizontal parallax, and the semi-angle of the cone of the shadow, is equal to the semi-diameter of the shadow at the distance of the moon. For $EBC - EAB = BED$. Or it is equal to the difference between the apparent semi-diameter of the sun, and the sum of the horizontal parallaxes of the sun and moon. Hence the diameter of the section of the earth's shadow, where the moon passes through it, is nearly three times the breadth of the moon. For $62' - 16' = 46'$ and $2 \times 46 = 92'$, the diameter of the section, almost three times 31′, the moon's diameter.

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* See Plate Y

But notwithstanding the breadth of the earth's shadow at the distance of the moon, she does not always pass through it at every opposition. Her orbit is inclined to the ecliptic in an angle of 5° 18′, at a mean rate, and therefore these planes will be distant from each other about a degree, at the distance of 12 degrees from her node, where the semi-diameters of the moon and the earth's shadow added together, being about one degree, would just touch each other, so that she will not enter into any part of the shadow when full, at more than 12° from her node. This therefore is called the ecliptic limit for eclipses of the moon. If the moon be nearer to her node than 12°, she will pass through a part of the earth's shadow, her northern or southern limb entering into it, according as she has south or north latitude; and if she be in her node, at the time of opposition, she is then in the ecliptic, where the center of the earth's shadow always is, and therefore will pass through its diameter, entering it with her eastern limb, and making the duration the longest possible, viz. 3^h 57′ 6″, if she be then at her greatest distance from the earth, where she moves slowest; or only 3^h 37′ 26″, if she be then at her nearest distance, where she moves quickest. In this case, she is centrally eclipsed. Hence if the latitude of the moon, at her opposition, be less than the sum, but greater than the difference of her own semi-diameter and the semi-diameter of the earth's shadow, she will be partially eclipsed.

The moon even in total eclipses receives light enough to become visible; occasioned by the refraction of the sun's rays through the earth's atmosphere; by which means, many rays are refracted into the shadow, especially those of a red colour, which have the greatest momentum, and make their way through it, while others are turned off in other directions. Now these rays cross each other in a point between the earth and moon, and thence diverging into the shadow, greatly diluted, enlighten it, at the distance of the moon; from whence she not only becomes visible, but also appears of a reddish brown and dusky colour. She never enters into the dark shadow of the earth, for then she would be absolutely invisible.

Although the shadow of the earth is diametrically opposite to the sun, yet the moon has been seen totally eclipsed before the sun was set; which was occasioned by the refraction of the atmosphere, raising both the sun and moon above their true positions, near 34 minutes.

ECLIPSES OF THE SUN.

Eclipses of the sun are occasioned by the interposition of the moon between him and the earth, and therefore he can never be eclipsed but at the change of the moon, when both luminaries are in the same part of the heavens. Hence the preternatural darkness at the crucifixion of our Saviour could not be effected by an eclipse of the sun, because he suffered at the Jewish passover, which was to be celebrated by their law at the full moon, or on the 14th day of their month. Unless the sun be near to the node of the moon at the change, there can be no eclipse of the sun, for otherwise she will pass by him either to the north or the south of him in the heavens. If she be then in her node her center will pass over the center of the sun, and if her diameter be then large enough, she will cover his whole disk and cause a central and total eclipse of the sun; but if she be at any distance from her node less than 17° she will only obscure a part of the sun, according as her latitude is smaller or greater.

As the moon is less than the sun, her shadow will terminate in a cone, the semi-angle of which is equal to the apparent semi-diameter of the sun viewed from the moon, when lessened by the sun's horizontal parallax seen from the moon. But as the earth and moon are nearly at the same distance from the sun, we may say, that the semi-angle of the cone of the moon's shadow is nearly equal to the apparent semi-diameter of the sun without any sensible error. As the moon is much less than the earth, her shadow must be much shorter, and generally does not reach much farther than the earth, and sometimes not quite so far. Its diameter where the earth passes through it is not large enough to cover the whole surface of the earth, but only a small spot on it, not above 180 miles in diameter, when the sun's diameter appears least, and the moon's greatest; so that the total darkness in an eclipse of the sun can extend no farther.

The semi-diameter of the section of the moon's shadow, where the earth passes through it, is equal to the difference between the apparent semi-diameters of the sun and moon. To those places only on the surface of the earth, which are involved in the dark shadow of the moon, while the earth is passing through it, the sun will appear totally eclipsed. But he will appear to be partially eclipsed to a large space beyond the shadow, and all round it, as far as a right line would reach, passing from one side of the sun by the opposite side of the moon. Beyond this line, the sun will not appear to be eclipsed at all. If this line be supposed to revolve round the sun and moon, always passing by the opposite points in their disks, it will describe a conical figure, whose vertex will be between the sun and moon; enlarging as it extends beyond the moon, and surrounding its shadow. The space comprehended between this increasing cone and the dark conical shadow of the moon, receives but a part of the sun's light, the rest being intercepted by the body of the moon; and the places have gradually less and less of the sun's light as you approach towards the dark shadow, where the sun is entirely concealed by the moon. This increasing cone, which surrounds the dark shadow, is called the moon's penumbra. To all the places, on the earth's surface, over which this penumbra passes, the sun will appear to be partially eclipsed, and that in a greater or less degree, as they are nearer to or farther from the dark shadow of the moon.

The semi-angle of the moon's penumbra is equal to the sun's apparent semi-diameter, added to the apparent semi-diameter of the moon; as seen from the sun. For $DCE = ADB + DBE$,* but this angle DBE may be neglected as insensible. And the halves of these are equal.

The semi-diameter of the moon's penumbra, where the earth passes through it, is equal to the sum of the semi-diameters of the moon and sun, or of the moon and semi-angle of the cone. $FMH = MHD + HCM$. The semi-diameter of the earth's disk, as seen from the moon, is equal to the moon's horizontal parallax. Therefore if the moon's latitude at her change be equal to or greater than the sum of the semi-diameters of her penumbra and horizontal parallax, there will be no eclipse of the sun, but if less there will be an eclipse; and if less than the sum of her horizontal parallax and the semi-diameter of her dark shadow at the earth, the eclipse of the sun will be total to some part of the earth. And if the moon change in her node, the eclipse of the sun will be central.

As there can be no eclipse when the latitude of the moon is greater than her horizontal parallax and the sum of the semi-diameters of the sun and moon, which altogether make about a degree and a half or 92′ at a mean; the limit for eclipses of the sun is found to be about 17°, from this subtense of 92′ and the angle of obliquity of the moon's orbit to the ecliptic.

The penumbra of the moon cannot cover the whole disk of the earth, which is double to the moon's horizontal parallax, or about 120′, while the whole diameter of the penumbra is not more than about 64′.

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* See Plate 16. Fig. 2.

As the semi-angle of the cones, both of the earth and moon's shadows, is equal to the semi-diameter of the sun; these cones must be similar, and their altitudes proportional to their bases. Now the diameter of the earth is to that of the moon as 100 : 28, and the altitude of the earth's shadow about 217 semi-diameters of the earth. Therefore as 100 : 28 : : 217 : 60¾ the height of the moon's shadow, which is about the distance of the moon from the earth. So that her shadow does not reach far beyond the earth, and sometimes it even falls short of it, its vertex passing through the air over the earth; when she changes at her greatest distance from the earth. Should she then be in her node, her diameter would not appear large enough to cover the whole face of the sun, and the eclipse would be annular, a small ring of light appearing round her in the middle of the eclipse, her shadow terminating in a point before it reaches the earth. To all the places over which this point passes, the eclipse will appear annular. When she changes at her mean distance, the vertex of her shadow just touches the earth, and she eclipses the sun totally to all places, over which it passes, but the total darkness is only of 3′ 13″ continuance. The moon's apparent diameter when greatest, at her nearest distance, exceeds the sun's when least, only 1′ 38″ of a degree, and the moon moves over this distance in 3′ 13″ of time, and no longer can the total darkness continue at any given place.

When the moon's diameter appears largest and the sun's least, her dark shadow covers a spot on the earth's surface of 180 miles in diameter, and the total darkness can extend no farther. At the distance therefore of 90 miles from the center of this spot, there is no total darkness, yet the moon's penumbra may cover a circular spot of 4900 miles in diameter, within which space, the sun will appear to be more or less eclipsed to places more or less distant from the center of this penumbra. When the moon changes in her node, this penumbra appears circular at the middle of the general eclipse, but at all other times it is elliptical, and passing over the earth's diameter takes the longest course possible; continuing on it 5^h 50′ of time at a mean rate, but something longer, if the moon be at her greatest distance from the earth, where she moves slowest, and shortest when she moves with her greatest velocity, being at her least distance. The earth's motion on her axis being from west to east, the same way that the shadow moves, will cause the duration of the general eclipse to be a little longer, than if there were no such rotation. The motion of the moon's shadow is equal to the motion of the moon from the sun, which is about 30′ 30″ per hour, or 1830 miles on the earth, that is, 30½ miles per minute; almost four times as swift as the motion of a cannon ball.

As the limits for eclipses of the sun, viz. 17°, are greater than for those of the moon, viz. 12°, there must be more eclipses of the sun than of the moon in any given year; yet more eclipses of the moon will be visible in any particular place, than of the sun: because the eclipses of the moon appear the same to all the inhabitants of that hemisphere of the earth which is turned towards the moon at that time. But an eclipse of the sun is visible only to a few inhabitants of the earth at the same time.

There cannot be less than two eclipses in one year, and these will be both of the sun, nor can there be more than seven, and four at least of these must be of the sun: because the sun passes by the nodes of the moon but twice in the year generally, or at most but three times. When he passes them twice in a year; if the moon should change in the node, she will then eclipse the sun, but before the opposition, 15 days after, she will be 14 or 15° past her opposite node, and will not therefore be eclipsed at that time; so that there will be but one eclipse during the time of the sun's passing that node; the same may happen at the next node; so that there may be but these two eclipses of the sun in that year.

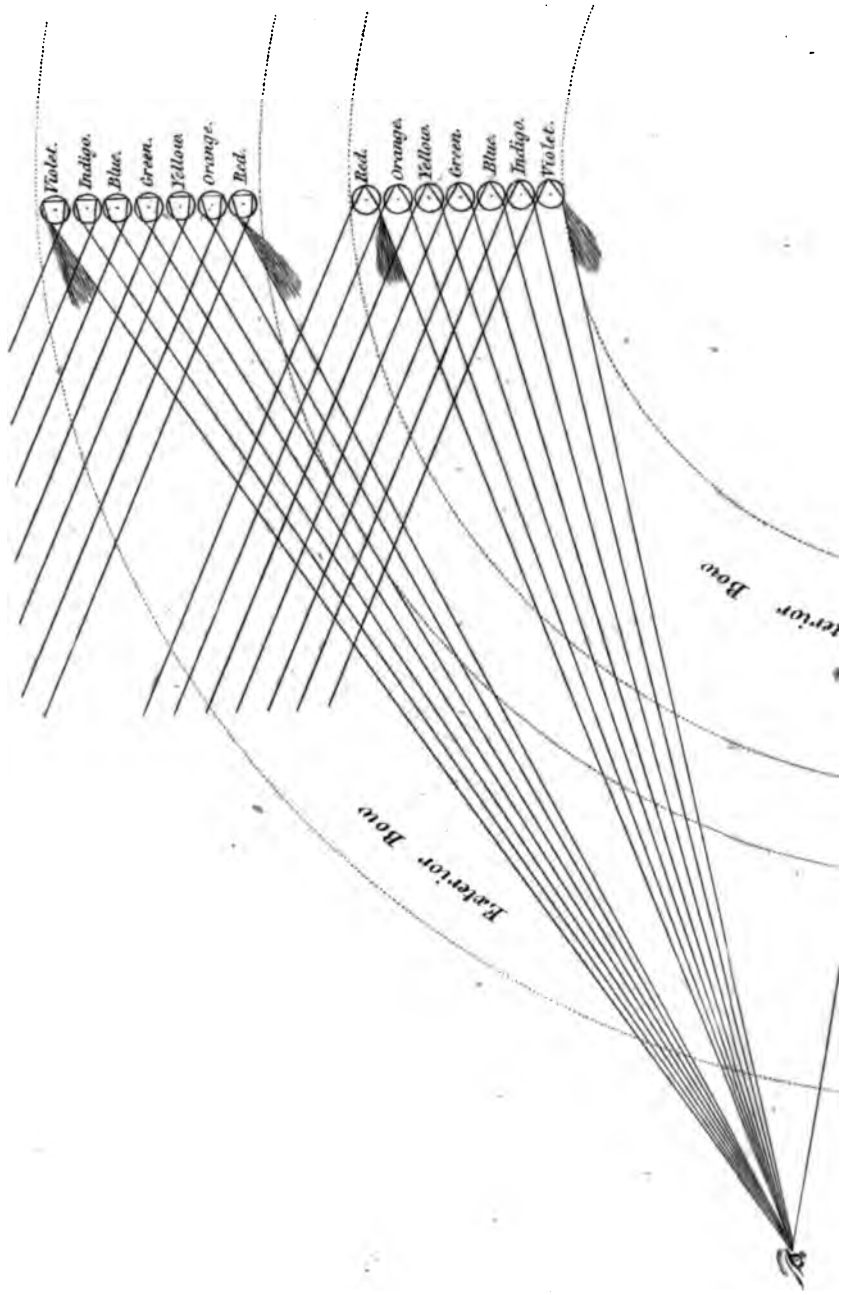
But if the sun come within 17° of the moon's node on the beginning of the year, the new moon at that time will eclipse him; the full moon, 15 days after, will be eclipsed near the opposite node, and the moon will come round again to the first node and eclipse him once more before he gets out of the ecliptic limits, which extend to 34°, and are more than the sun can pass over in a month. Three more eclipses may happen near the other node, about half a year afterwards. But as the nodes of the moon move backwards 19° 20′ per annum, the sun may come from one node to the other in 173 days, or after 6 lunations, which contain but four days more. So that in 346 days, or 19 days before the year is ended, the sun will come again to the same node which he passed before at the beginning of the year, and there will be one other eclipse either of the sun or moon in that year; the other eclipses, that happen about this node, will however fall in the subsequent year.

The sun passes by either of the moon's nodes 19 days sooner than it passed it on the preceding year, on account of the retrograde motion of the lunar nodes.

And as they go round in 18 years, 225 days, and 8 hours; if there had been a complete number of lunations in that time, there would then be a complete and regular period of eclipses in the same time. But this never happens. However in 18 years 11^{d} 7^{h} 43′ 20″, including four leap-years, or one day less, if five be included, the sun, moon, and nodes return to the same state, in which they were at the beginning of the period; save only, that the line of conjunction of the sun, moon, and nodes, will fall backwards, or to westward, 28′ 12″ of a degree in every period. Add therefore this quantity to the time of any eclipse, and you will have the time of an eclipse of the same kind, in the next period, very nearly; or subtract it, in order to have the eclipse of the preceding period; without any trouble of calculation. This is called the Plinian Period or *Chaldean Saros*, as Pliny records it, to be known by the Chaldeans. But as the line of conjunction falls westward 28′ 12″, in every period, it would pass through the whole ecliptic limits in about 1388 years, as it would shift about 36 degrees in that time; and near 12,500 years would be required to describe the remaining 324 degrees; after which time, the same period of eclipses would begin and run over again. A longer and more exact period for calculating eclipses is 557^{y} 21^{d} 18^{h} 30′ 11″; for then, after that time, the sun, moon, and node, meet so nearly again, as to be but 11″ distant.

It is however generally much safer to depend upon a calculation from astronomical tables, or, if great accuracy be not required, upon graphical constructions and delineations.

PLATE VIII.


Violet.
Indigo.
Blue.
Green.
Yellow
Orange.
Red.
Red.
Orange.
Yellow
Green.
Blue.
Indigo.
Violet.
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Exterior Bow

hand, while in the other six signs. Open the sector till the distance of the two 90°, on the line of sines, is equal to V h, and take the sine of 77° 49′ 48″, the sun's distance from the nearest solstice, which set from V, the middle of h g, to P, to the right hand in this case, and draw the earth's axis CP, cutting the disk in h, from which towards VI and VI set off the chord of the co-latitude 38° 30′, making CA rad. and draw the line VI K VI; then from the points where this line cuts the disk, set off the chord of the sun's declination both ways towards E and G, and D and F, and draw the lines DE, and FG, cutting the earth's axis in L and XII, which bisect in K; then VI K, and K XII, will be the two semi-axes of the ellipsis, which will represent the path of the place of observation, London, which describe and divide into hours and minutes. Making CB radius on the line of sines, take the co-latitude 38° 30′, from the line of sines, and set it both ways from K to the hours VI and VI in the line VI K VI, which will be just in the disk at the equinoxes, but in no other season of the year. N. B. When the sun's declination is south, the elliptic path is convex toward the pole, and vice versa.

Take the chord of 5° 35′, the angle of the moon's visible path, from the line of chords, making AC radius, and set it off from H, the axis of the ecliptic, to the left hand, to M, when the moon's latitude is N. ascending, but to the right hand, when it is N. descending; draw CM for the axis of the moon's orbit, and bisect the angle HCM by the line Cxz. Take the moon's latitude from the same scale with CA, and mark the point x in the bisecting line Cxz, whose nearest distance from the line CA shall be equal to the moon's latitude, and through x draw a line at right angles to CM, the axis of the moon's orbit, which line shall be the path of the moon's penumbra, over the disk of the earth. The point x marks the conjunction according to equal time by the tables, the point y, where this line cuts the axis of the ecliptic, is the point of the ecliptical conjunction, and the point w is that of the middle of the eclipse.

Take the moon's horary motion from the sun, 27′ 54″, from the same scale with AC, and with that extent make marks along the path of the moon's penumbra, each being one hour; subdivide them into 60 equal parts for minutes, and number the hours so that the instant of new moon by the tables shall fall upon the point x, half way between the axis of the ecliptic and the moon's orbit. Then apply one side of a square to the path of the penumbra, and thus move it backwards and forwards, until the other side cuts the same hour and minute, both in the path of the penumbra, and the path of the place, London, which will be, in the present example, at 10^h 47′. With the semi-diameter of the sun, taken from the same scale CA, viz. 16′ 16″, describe a circle round 10^h 47′ in the path of London, representing the disk of the sun; and with the semi-diameter of the moon 14′ 57″ taken from the same scale, describe a circle round 10^h 47′ in the path of the penumbra, to represent the disk of the moon: and the portion of the sun's disk hid by the moon's disk, will show the phases and quantity of the eclipse at that time.

Lastly, take from the same scale with AC, the semi-diameter of the penumbra, and carry that extent, viz. 31′ 3″, backwards and forwards, along the paths of the penumbra and the place of observation, London, on both sides of the axis [illegible] - until both points of the dividers fall into the same points of time in both the lines, and these times will represent the time of beginning and ending of the eclipse, at that place, according to apparent time; which, by applying the equation of time, is converted into mean time.

This construction supposes that the moon's motion is uniform and rectilineal during the whole time of the eclipse, and that the angle under which the moon's disk is seen, continues the same, which is not strictly true; however, it will give the times of the beginning and end within a few minutes, when executed with sufficient care.

PROJECTION OF LUNAR ECLIPSES.

THE requisites for the projection of a lunar eclipse, are these eight following; which we shall exemplify in the eclipse of May 1762, at London.

1. The true time of full moon, May, $8^d\ 3^h\ 50'\ 50''$: 2. The moon's horizontal parallax at that time $57'\ 23''$: 3. The sun's semi-diameter $15'\ 56''$: 4. The moon's semi-diameter $15'\ 38''$: 5. The semi-diameter of the earth's shadow at the moon, $41'\ 37''$: 6. The moon's latitude south descending, $32'\ 21''$: 7. The angle of her visible path with the ecliptic, $5^\circ\ 35'$: 8. Her true horary motion from the sun, $30'\ 52''$; all which may be collected from the tables.

Make a scale of any convenient length, and divide it into 60 equal parts, each equal one minute. With the sum of the semi-diameters of the moon and earth's shadow, taken from this scale, $57'\ 15''$, describe a semicircle ADB:* and also another with a rad. = the semi-diameter of the earth's shadow, $41'\ 37''$, taken from the same scale, to represent the earth's shadow. Draw two diameters at right angles to each other; one representing the ecliptic and the other its axis. Open the sector to the radius of the first circle on the line of chords, and take off the chord of 5° 35′, from D to E, or from the vertex of the axis of the ecliptic, to the right hand, because the moon's latitude is S. descending, but to the left hand if the latitude had been N. or S. ascending, and draw the line CE for the southern half of the axis of the moon's orbit. Bisect DE by the line CGg, on which set off the moon's latitude 32′ 21″, taken from the same scale, from C to G, and through G draw the line NGP, for the path of the moon's center, perpendicular to CFE: then F will be the point, where the moon's center is at the middle of the eclipse, G the point where her center is at the tabular time of her being full, and H the point, where her center is at the time of the ecliptical opposition.

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* Plate 18.

Take from the same scale the moon's horary motion from the sun, and with extent=to an hour, graduate the moon's path into hours and minutes, marking the graduations so that the time of full moon, viz. 3^h 50′ 50″ may fall in the point G, where the bisecting line cuts the moon's path.

With the moon's semi-diameter 15′ 38″ as a radius, from center F, describe a circle for the position of the moon at the middle of the eclipse, and also two circles from the centers N and P, in the line of the moon's path, for the positions of the moon at the beginning and end of the eclipse, when it is in contact with the earth's shadow, represented by the interior circle before described. The point N represents the instant when the eclipse begins, and the point P, when it ends; the first being at 2^h 15′ 10″ A. M., and the other at 5^h 18′ 0″.5; the moon being eclipsed 10 digits.

The disk of the sun and moon is supposed to be divided into twelve equal parts, and the quantity of the eclipse is estimated in these parts.

THE FIXED STARS.

The fixed stars are generally supposed to be the central suns of so many systems of planets, which revolve round them at different distances and in various orbits, deriving from them the benefits of both light and heat. They are said to be fixed, because they generally preserve the same relative distances from each other.

The number of those that are visible to the naked eye is less than one would imagine, from a cursory view of the heavens. The British catalogue of them which is the largest, contains but 3001, of which many are invisible to the naked eye, being only discoverable by the telescope, and thence denominated telescopic stars. They shine with native and inherent light, for otherwise they would be invisible; as their distances from us are inconceivably great. Dr. Bradly says, from his observations, that if they had an annual parallax of a single second he must have discovered it. How immensely great then, must their distances be, when the diameter of the earth's orbit, which is near two hundred millions of miles, does not subtend an angle of a second at the distance of the nearest of them; hence not less than 40 millions of millions of miles! Their twinkling appearance is occasioned by their momentary occultation, by every particle of dust that floats in our atmosphere. To the naked eye they appear of a sensible magnitude, which is destroyed by a good telescope, as no micrometer can measure their diameters. The rays which the eye receives from them are not all collected accurately into a point on the retina, but are diffused over a circular space around the spot where the image is formed, and by their vibrations or impressions on that space, cause us to imagine them to be larger than they really are.

From their apparent magnitude, astronomers have distinguished them into six or seven different magnitudes. And for the convenience of distinguishing them from one another, they have ranged them into different combinations, under various particular names, which combinations are called asterisms or constellations. Twelve fill that space which is called the zodiac, and are denominated Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricornus, Aquarius, Pisces; lying eastward of each other in this order, and each extending over a space of thirty degrees, and of the breadth of about 16 degrees, comprehending all the stars in that space. The number of the ancient constellations was 48, but on our present globes, they are near 70, each star having, by the contrivance of Boyer, a letter annexed to it, whereby it is easily distinguished; the first letters of the Greek alphabet being applied to the brightest, and so on through the whole; and then the letters of the Roman alphabet.

The Milky Way is a luminous tract surrounding the heavens, arising probably from the confused refulgence of innumerable stars disseminated through that part, as they are visible in great numbers through a good telescope; although some astronomers are of a different opinion.

Cloudy stars are so called from their misty appearance to the naked eye, by which a star or two are dimly seen; but through a good telescope they seem to be broad illuminated parts of the heavens, in which one or more stars are seen. There is one in the right hand of Perseus,—one in the middle of the Crab,—one near the sting of Scorpio,—three in Sagittarius, in his eye, bow, and right shoulder,—two in Orion, in his head, and in his sword; and the Magellanic clouds near the south pole, where the stars shine as through very thin clouds; also one between the great and little Dog, very full of little stars. In the first of these, there appear more stars, than in any others, although 21 have been counted in the head of Orion, and above forty in the Crab. The most remarkable of them is that in the sword of Orion, which seems like a gap in the heavens, through which we look, as it were, into a much brighter region. Seven stars are seen in this luminous space. Although these spaces appear to us to be very small, yet they must, in fact, be very large,—probably more extensive than our whole solar system.

Several stars have been observed to appear and disappear periodically, others to suffer a periodical diminution and increase of splendour, and others to have disappeared entirely. Gemma observed one in the chair of Cassiopea, in November, 1572, surpassing Sirius in splendour and magnitude, but gradually decaying till the month of March following, when it became invisible.

Kepler, in the year 1604, discovered a star near the right heel of Serpentarius, surpassing Jupiter in magnitude and lustre, which disappeared between October, 1605, and February following, and has not been seen since. Jansenius, in the year 1600, discovered one in the breast of the Swan, which in time became so small as to disappear entirely. In the years 1657, 1658, 1659, it recovered its former splendour, but soon decayed; its period is about 18 years. The Stella mira in the neck of the whale, first observed by Fabricius in August, 1596, has been observed to appear and disappear periodically, seven times in 6 years, its period being 333 days; continuing for 15 days in its greatest lustre, and never quite extinguished. In 1686, Kirch discovered a new star in the Swan's neck, which returns periodically in 404 days. There is also a star in the Swan, whose lustre is alternately increased and diminished in every twelve days.

Some have supposed, that the appearing and disappearing of these stars may be accounted for, upon the supposition of their having large spots upon their surfaces, which render them invisible when these spots are turned towards the earth: and when their rotations on their axes turn these spots from us, they begin again to appear. Others, from their quick rotations on their axes, may acquire such a centrifugal force, as will occasion them to assume the form of millstones, or of Saturn's ring, which will make them visible only when their sides are turned towards our earth. Some have thought also, that the sudden blaze observed in some of them might be accounted for, by supposing some of their comets to have dropped into them, to supply their exhausted fuel.

Some of the stars, such as Capella, Sirius, Procyon, and particularly Arcturus, are observed to change their places, above a minute of a degree, with respect to others. But whether this arises from any real motion in the stars themselves, or to our whole solar system's changing its place in universal space, must require the observations of many ages to determine. If our whole system be changing its place, as some late observations seem to indicate, at least in the opinion of Herschel, who supposes that it is moving northward, with considerable velocity, this must, in process of time, occasion an apparent change in the distances of the stars from each other. On this supposition, the stars which are nearest to us, must be most affected by such a motion, and therefore their relative distances would suffer the greatest change.

From the precession of the equinoxes, already explained, the stars appear to alter their longitude annually 50″ of a degree, as it is counted from the intersection of the ecliptic and equator. But this does not alter their latitude, as the pole of the earth moves round the pole of the ecliptic, without sensibly approaching towards it.

Catalogue of variable Stars reduced to the beginning of 1786.

Names of the Stars.	Right ascension in time.	Declination	Greatest and least mag.	Periods.
	h ′ ″	° ′ ″		
Nova of 1752 in Cassiopeia	0 13 0—	62 58 + N.	1—0	150 years, but possibly les
ο Ceti	2 8 33	3 57 25 S.	2—0	334 days Cassini.
Algol	2 54 19	40 6 55 N.	2—4	2ᵈ 20ʰ 49′ 3″.
Meyer's 240 in Leo	9 36 5	12 25 0 N.	6—0	
Hevelius's 30th of Hydra	13 18 4+	22 9 38 S.	4—0	494 days, invisible about 6 months.
Nova 1604 in Serpentarius	17 18 0—	21 10 ½ S.	1—0	
β Lyræ	18 42 11	33 7 46 N.	3—4.5	6 days and 9 hours.
Near the Head of Cygnus	19 38 58	26 48 ½ N.	3—0	10 or more years.
η Antinoi	19 41 34	0 28 14 N.	3.4—5	7 days 4 hours 15 min
χ In the Swan's Neck	19 42 21—	32 22 58 N.	5—0	405 3 days, or rather 2 days 21 hours, invisible for 6 months.
In the Swan's Breast	20 9 54	37 22 37 N.	3—0	about 18 years, invisible years.
δ Cephei	22 21 0	57 20 0 N.	3.4—4.5	5 days 8 hours 37 min.
α Draconis	13 58 36	65 24 8 N.	2—4	
δ Ursæ major.	12 4 45	58 13 24 N.	2—4	
σ Sagittarii	18 42 0	26 32 34 S.	2—4	

ABERRATION OF THE STARS.

THERE is also another apparent motion of the fixed stars, arising from the motion of the earth, and the progressive motion of light combined together, called their aberration; which causes their longitudes and latitudes, their right ascensions and declinations, to be different from what they are in reality; and as it depends upon the earth's motion, all the variations of it must be completed in the course of a year.*

It is known from the eclipses of Jupiter's satellites, that light takes 8′ 7″ of time to pass from the sun to the earth, in which time the earth moves in her orbit over an arc of 20″.25; so that the velocity of light is to that of the earth, as radius to the tangent of 20″.25. It is also known that the visibility of objects depends upon the impression made on the eye by the rays of light, which they transmit, and that we judge of the position of objects by the direction in which the ray enters the eye, without any regard to any motion of the eye, or any antecedent alteration in the direction of the ray. These things being premised, we now say, that the impression of a ray of light from a star upon the eye, is neither in the direction of the ray when first emitted from the star, nor in the direction in which the earth is moving carrying with it the eye of the spectator; but like a body urged by two forces in two directions; so light is felt in the direction of a diagonal of a parallelogram, whose two sides are in the directions of the ray of light and of the earth's motion, and also proportional to the velocities of light and of the earth. So that the star's apparent place will be at that end of the diagonal of the parallelogram to which the earth is moving. Now as one side of this parallelogram is the line which joins the earth and star, and the other is the tangent to the earth's orbit, where she may be, the plane of the parallelogram will always be in this tangent; and as the direction of the tangent is continually changing, the plane of the parallelogram must, in the course of a year, revolve quite round the line that joins the earth and star. Hence the star's apparent place must always be in the periphery of an orbit, similar to that of the earth's, described round the star's true place as the center, and in a plane parallel to the earth's orbit, which may be considered as a circle. But if this circle of the star's aberration, were projected on any other plane oblique to the former, it would then become an ellipsis. Hence if the star were in the pole of the ecliptic, the heavens being there parallel to the plane of the ecliptic, the star's apparent path would be in the arc of a circle; and if the star were in the ecliptic, its course would be a right line, because its circle of aberration then passes through the eye. But in any intermediate place, between the pole and the plane of the ecliptic, the circle of aberration, which is parallel to the ecliptic, must be projected into an ellipsis in the heavens, whose transverse axis is parallel to the ecliptic, and whose conjugate is perpendicular to that plane. The transverse axis of the ellipsis of aberration of all the stars is the same, and found to be 40″.5 of a degree, but the conjugate axis increases with the star's distance from the ecliptic; and therefore they are to each other as radius to the sine of the star's latitude: that is, as radius : S, star's latitude : : 40″.5 : the lesser axis of the ellipsis.

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* If the earth be at A* and moving from A to B in the time that a ray of light comes from the star in the direction CB; so that AB is to AC, as the velocity of the earth is to the velocity of light; then the star will appear at D, in the direction BD parallel to AC. For if a telescope AC reaching to the star were carried parallel to itself, the ray of light emitted in the direction CB, would be still in the axis of the telescope until it should arrive at the eye at B, where it would appear to have come in the direction DB, of the telescope, and thereby represent the star at D, advanced before its true place C.

* See Plate 16, Fig. 5

As the distance between the sun and the earth is so small in comparison with the distance of the stars, we may consider it as nothing: then the parallelogram above-mentioned, in the diagonal of which the star's apparent place is always found, may be considered as turning round on the line that joins the sun and star, as the earth moves round the sun, keeping its plane always parallel to the direction of the earth in her orbit. The star therefore revolves in the periphery of the ellipsis from west to east, being at the eastern extremity of the ellipsis when it is found in opposition with the sun, and at the western end when in conjunction. When the sun and star have the same longitude, the plane of the parallelogram of aberration is perpendicular to the plane of a circle of latitude, and the star is in the western extremity of its transverse axis; consequently its aberration is wholly in longitude, and then greatest, being, as far as possible, 20″.25, removed from its true circle of latitude; and its aberration in latitude is nothing, as it is at the same distance with the center of its ellipsis from the ecliptic. But when the star is in quadrature with the sun, the plane of the parallelogram of aberration is parallel to the plane of the circle of latitude passing through the pole of the ecliptic and the star, and the star appears in the extremity of the lesser axis, so first emitted from the star, nor in the direction in which the earth is moving carrying with it the eye of the spectator; but like a body urged by two forces in two directions; so light is felt in the direction of a diagonal of a parallelogram, whose two sides are in the directions of the ray of light and of the earth's motion, and also proportional to the velocities of light and of the earth. So that the star's apparent place will be at that end of the diagonal of the parallelogram to which the earth is moving. Now as one side of this parallelogram is the line which joins the earth and star, and the other is the tangent to the earth's orbit, where she may be, the plane of the parallelogram will always be in this tangent; and as the direction of the tangent is continually changing, the plane of the parallelogram must, in the course of a year, revolve quite round the line that joins the earth and star. Hence the star's apparent place must always be in the periphery of an orbit, similar to that of the earth's, described round the star's true place as the center, and in a plane parallel to the earth's orbit, which may be considered as a circle. But if this circle of the star's aberration, were projected on any other plane oblique to the former, it would then become an ellipsis. Hence if the star were in the pole of the ecliptic, the heavens being there parallel to the plane of the ecliptic, the star's apparent path would be in the arc of a circle; and if the star were in the ecliptic, its course would be a right line, because its circle of aberration then passes through the eye. But in any intermediate place, between the pole and the plane of the ecliptic, the circle of aberration, which is parallel to the ecliptic, must be projected into an ellipsis in the heavens, whose transverse axis is parallel to the ecliptic, and whose conjugate

PLATE XIV.

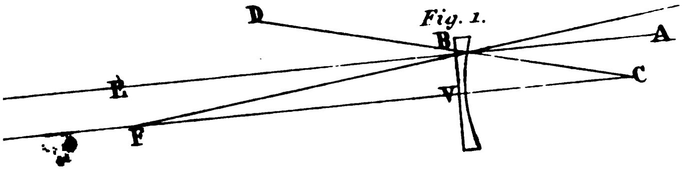
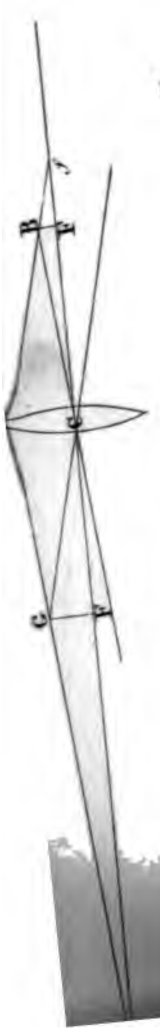


Fig. 1.



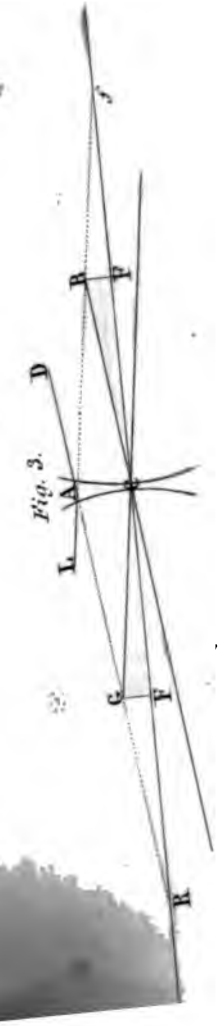


Fig. 3.

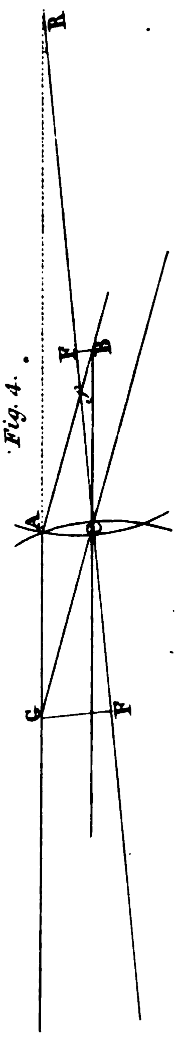


Fig. 4.

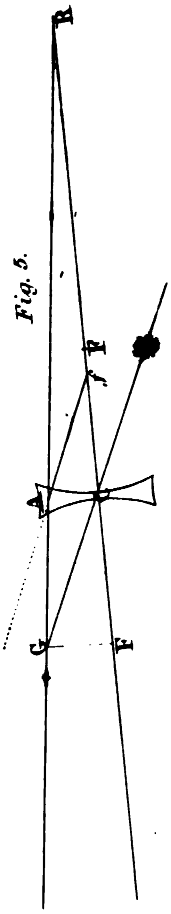


Fig. 5.

that its aberration is entirely in latitude, and nothing in longitude.

The same may be said with respect to the aberration in declination and right ascension. When the right ascension of the sun and star is the same, the plane of the parallelogram of aberration is parallel to, or coincident with, the plane of the circle of declination, and the aberration in right ascension is nothing; for the star appears in the plane of the true circle of declination. But when the plane of the parallelogram of aberration is at right angles with the plane of the circle of declination, the star appears in a parallel to the equator, and consequently has no aberration in declination. At any intermediate position, the aberration will be both in right ascension and declination. Yet the aberration in right ascension is not greatest when the aberration in declination vanishes, excepting in the solstitial colure, when a star is found in it, and vice versa. This is the case however with the aberration in longitude and latitude. The aberration in longitude is greatest, when the aberration in latitude is nothing, and vice versa.

From what has been said, we may easily compute the aberrations of a star for any particular time. As for example, let the aberrations of a star be required, whose latitude is 36° north, and longitude 12° in Taurus, at the time when the sun is in 26° of Leo.

With a radius taken from any scale of equal parts, $=20''.25$, describe a circle to represent the ecliptic, and divide it into signs and degrees.* Through the star's place in the ecliptic draw a diameter, which shall represent a portion of a small circle parallel to the ecliptic, and the greater axis of the star's ellipsis of aberration; and beginning from the center, divide this diameter each way to the circumference into as many parts as shall be found by this analogy. As cosine star's latitude : radius : : 20″.25 : the number sought, which in this case will be 25″.02. Because any arc of a small circle is to its corresponding arc of a great circle, as the cosine is to radius.

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* See Plate 19.

At right angles to this diameter, draw another to represent the star's circle of latitude, whose radius divide into 20″.25 equal parts, because it is part of a great circle, and in this diameter take CE : CF : : radius : sine of the star's latitude, and CF will be the lesser axis of the star's aberration. Upon the axes CA and CF describe the ellipsis CFBT, which shall represent the apparent path of the star through the year; its true place being at C the center of the ellipsis. From S the place of the sun in the ecliptic let fall the perpendicular SG on the diameter CA, and cutting the ellipsis in L the place of the star in its ellipsis, and LG will be its aberration in latitude. Draw LN perpendicular to the lesser axis of the ellipsis, from the place of the star, and it will be the aberration in longitude. These may be measured on their respective scales, or computed as follows.

As EC : FC : : radius : S, star's latitude : : 20″.25 : 11″.9=the maximum of aberration in latitude=FC. Hence as EC : FC : : SG : LG : : S, of the sun's distance from his opposition to the star : aberration in latitude at the time, that is as radius : maximum=11″.9 =FC : : S, SCB=76 : 11″.55=the aberration in latitude at that time.

Or as radius : S, star's latitude×S, sun's distance from opposition to the star : : 20″.25 : the aberration in latitude. Or, lastly, as cosine star's latitude : S sun's distance from point opposite the star : : $20''.25$: the aberration in latitude at the time.

Let ⊙ represent the sun's place in the ecliptic, when the star's apparent longitude or latitude is the same with the true, and tends to excess. For the aberration in latitude, ⊙ is the point opposite to the star.

TO FIND THE ABERRATION IN LONGITUDE, FOR WHICH PURPOSE ⊙ IS 3 SIGNS AFTER THE STAR'S PLACE.

As cosine star's latitude : radius : : $20''.25$: maximum of aberration$=BC=25''.02=M$.

As radius : S, sun's elongation from ⊙ : : maximum : aberration in longitude at the time$=A=6''.21$. Or, without the maximum. As cosine star's latitude : S, sun's elongation from ⊙$=14°$: : $20''.25$: $A=6''.21$.

TO FIND THE ABERRATION IN RIGHT ASCENSION AND DECLINATION.

We must determine the angle made by the star's circle of latitude and its circle of declination previous to any construction or computation of the aberration in declination or right ascension, which is easily done by spherical trigonometry; by saying, as cosine latitude of the star, is to the sine of its distance from the solstitial colure; so is the sine of the obliquity of the equator and ecliptic, to the sine of the angle between these circles, commonly called the star's angle of position; which in this case is 26° 50′.

As the north part of the star's circle of declination is so much removed to the east of its circle of latitude, make the angle $ECP=26°\ 50'$ to the eastward of EC, and draw the diameter PCQ, to represent the star's circle of declination, and divide its radii into $20''.25$

ach. At right angles to PCQ, draw the diameter RCT, to represent a portion of a small circle parallel o the equator passing through the star; and divide ts radii into as many parts as are found by this analogy; as cosine star's declination : radius : : 20″.25 : the number sought, which in this case is 30″.5. Now draw LM, LN, and these lines being perpendicular to PC, and CT, respectively, will be the respective aberrations in right ascension and declination; and may be measured on their respective scales.

As there is frequently occasion, in practical astronomy, for computing the aberrations in right ascension and declination, we shall deliver these practical rules following, without their demonstrations, referring you, for this purpose, to the writers on this subject.

FOR THE ABERRATION IN RIGHT ASCENSION.

In the astronomical tables, find that point in the ecliptic, which answers to the right ascension of the star, which call N. This point is that, wherein the sun being found, the aberration makes the right ascension the least. Take, in the same tables, the angle made by the ecliptic, and a meridian passing through the point N; and then say, as radius×cosine star's declination : cosine sun's distance from N×S, angle between the ecliptic and meridian of N : : 20″.25 : aberration in A.R. which is additive or subtractive, according to the sun's situation with respect to the point N.

Otherwise thus; as S, star's latitude : radius : : cosine P=the star's angle of position : tangent of an angle Z.

Then if the star be, (with respect to that pole which is of the same name as the star's latitude) in a sign ascending or descending, P being acute, Z added to the star's true place, or taken from that opposite to its true place; but P being obtuse, if the star be in an ascending or descending sign, Z taken from the star's true place, or added to that opposite to its true place, will give ☉. N. B. ♑, ♒, ♓, ♈, ♉, ♊, are ascending signs with respect to the north pole, and descending signs with respect to the south pole, the rest are the contrary.

As cosine star's declination×SZ : cosine P×radius : : 20″.25 : maximum of aberration in A.R.

Radius : S, sun's distance from ☉ : : M : aberration in A.R.

Or, cosine star's declination×S,Z : : S, sun's distance from ☉×cosine P : : 20″.25 : aberration in A.R.

N. B. ☉ is the sun's place in the ecliptic when the star's apparent longitude, latitude, declination, or A.R. being the same as the true tends to excess. And the angle Z is the sun's distance from the nearest syzygy with the star, at the time of ☉.

FOR THE ABERRATION IN DECLINATION.

As S, star's latitude : radius : : tangent P=the star's angle of position : tangent of an angle, which is Z. Then if P be acute, and the star, (in respect of that pole of the equator which is of the same name with the star's latitude,) be in a sign ascending or descending, Z taken from or added to the point opposite to its true place gives ☉; but if P be obtuse, and the star be in an ascending, or descending sign, Z added to or taken from its true place gives ☉; provided always, that the star's declination and latitude be both north or both south, but if one be north and the other south, then *for the star's true place*, read *point opposite to its true place*, and vice versa.

As S,Z : S,P : : 20″.25 : maximum of aberration in declination.

As radius : S, sun's elongation from ⊙ : : maximum : aberration in latitude at the time proposed.

Or, as radius × S,Z : S,P × S, of sun's elongation from ⊙ : : 20″.25 : A=the aberration in declination at the time.

Note. These rules give the value of A and M, in seconds of a degree; and if the sun's place be in that semi-circle of the ecliptic which precedes ⊙, the aberration A must be taken from the star's true longitude, latitude, declination, or A.R. to give the apparent; but if in that semi-circle which follows ⊙, A must be added.

The libratory variation of the equator to the ecliptic, which arises from the action of the moon and sun upon the redundant matter of the earth about the equator, is termed the nutation of the poles, which occasions another small apparent motion of the fixed stars, amounting to about 9″.3 of a degree when greatest. This principally depends upon the position of the moon's nodes. The following method will serve to determine it with sufficient precision.

FOR THE NUTATION OR DEVIATION OF A STAR IN A.R.

With a radius equal to 9″.3 describe a circle,* which graduate into signs and degrees. At the points 3^s and 9^s draw a diameter, on which as a transverse axis describe an ellipsis, whose axes are as 9.3 : 7.1. Graduate the same into signs and degrees, from the right hand to the left, making 0^s, of the ellipsis to coincide with 3^s in the circle. Graduate the ellipsis by drawing lines from every degree in the circle perpendicular to the

* See Plate 29.

transverse diameter of the ellipsis, intersecting the ellipsis in the corresponding points. Through the degree and minute in the circle answering to the star's A.R. draw a diameter, and from the place of the moon's node in the ellipsis measure the shortest distance to this diameter passing through the star's A.R. and apply it to the scale of the radius; then this length multiplied by the natural tangent of the star's declination, will give the nutation in A.R. in seconds of a degree.

Hold the place of the star from you, and if the place of the moon's node be on the right hand of the diameter passing through the A.R. of the star, the nutation is additive, but if on the left, subtractive. The equation of the equinoxes, taken from the tables, is to be applied to the nutation in A.R. according to its sign in the tables, and the sum or difference will be the correction in A.R. of the star.

FOR THE NUTATION OR DEVIATION OF A STAR IN DECLINATION.

Make a similar projection with the former, only let the graduations of both ellipsis and circle begin at the right-hand extremity of the transverse axis of the ellipsis; and proceed towards the left, as before. The nearest distance from the place of the moon's ascending node in the ellipsis, to the diameter passing through the place of the A.R. of the star, measured as before, is the nutation in declination in seconds.

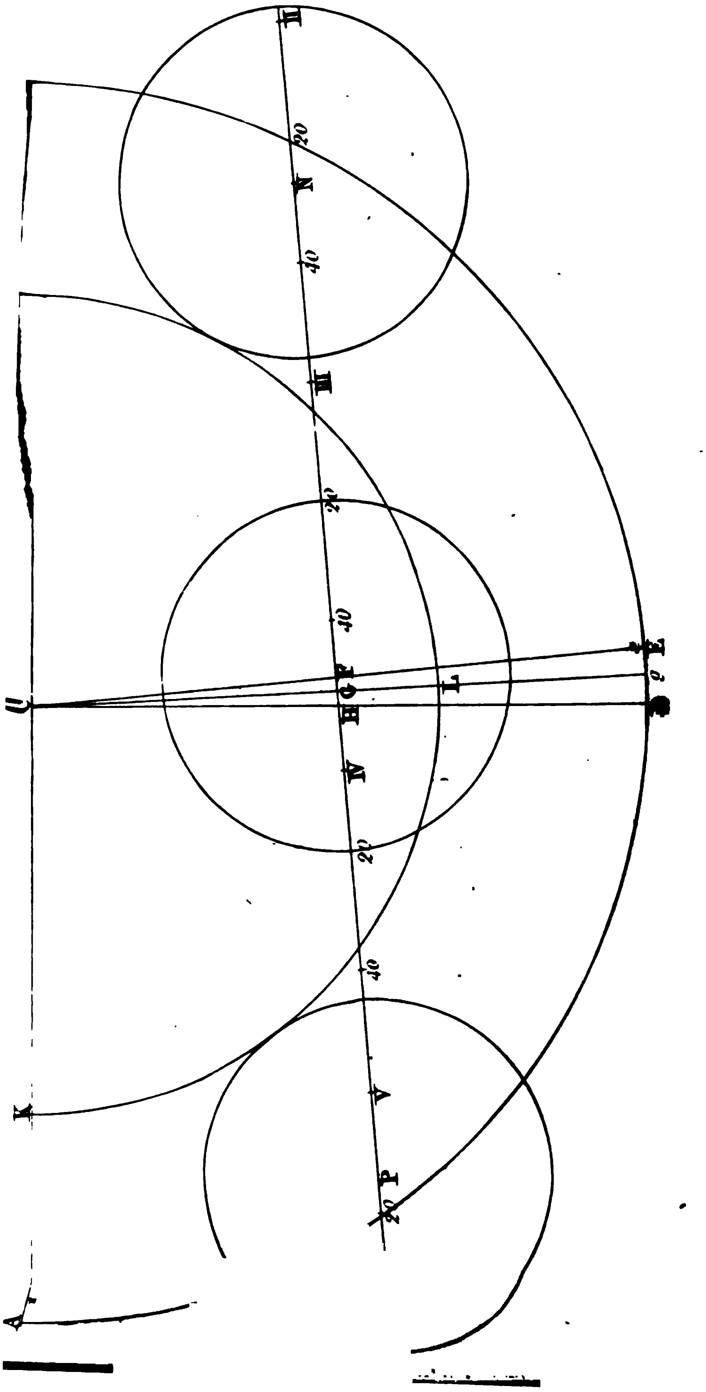
This nutation is additive for a northern, and subtractive for a southern star, if the place of the moon's ascending node be on the right hand of the diameter passing through the A.R. of the star, but the contrary, if on the other side, when the place of the star is held from you.

CONCLUSION.

Now, in the conclusion, when we consider the prodigious magnitude of the space in which the fixed stars are placed, it does not seem probable, that such vast bodies as they must necessarily be, were created for no other purpose, than to afford us a glimmering light in the absence of the sun. If this were the sole intention of their existence, why have the telescopic stars twinkled unseen by any mortal, till the invention of the telescope? or why is their number proportioned to the goodness of the telescope; more being seen continually, as that useful instrument receives a higher improvement? Certainly the supposition is inconsistent with the adequacy of the Infinite Agent to the effect. As there is reason to believe from the discoveries of the telescope, that not one half, nay, it may be, not a thousandth part of their number has ever been seen by any mortal; and that their distances from each other cannot be less, than the distance of some of them from us; this distance must be inconceivably magnified before we can have any conceptions of the dimensions of the creation. The incomprehensible Creator alone can comprehend the unbounded expansion. Here our observations fail, and we are reduced to hypothesis. The most probable design of their existence, is derived from this consideration; that none of them have any parallax, and consequently that if the annual orbit of the earth, which is near 200 millions of miles in diameter, were viewed from the nearest of them, it would not subtend an angle of a single second; and therefore if our sun were removed to the same distance, he would appear no larger than a shining point; and our whole planetary system must become absolutely invisible to such eyes as ours. Hence it is extremely probable, that every one of this countless multitude of stars was ordained for similar purposes with our sun; and that they are the central sun's of innumerable systems, distributing the various influences of light and heat to their attending planets; whose immense distance from us, must for ever conceal them from the human eye.

THE END.

PLATE XVIII.



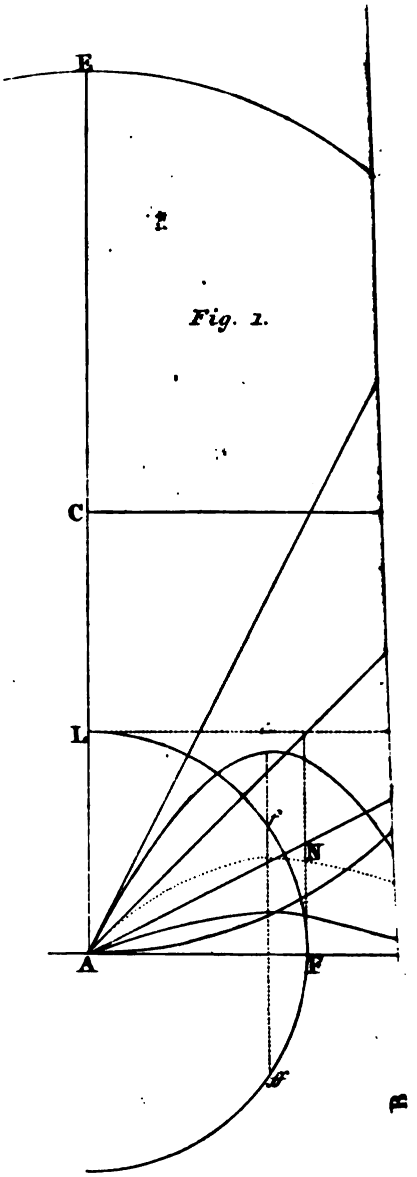
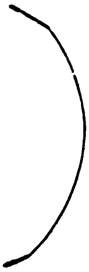
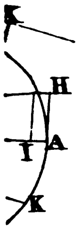
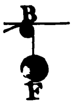


Fig. 1.

Fig. 4.







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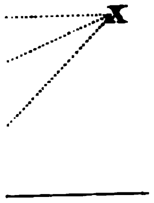
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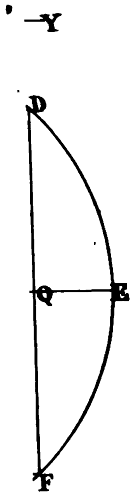
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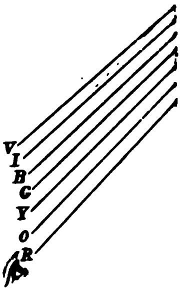
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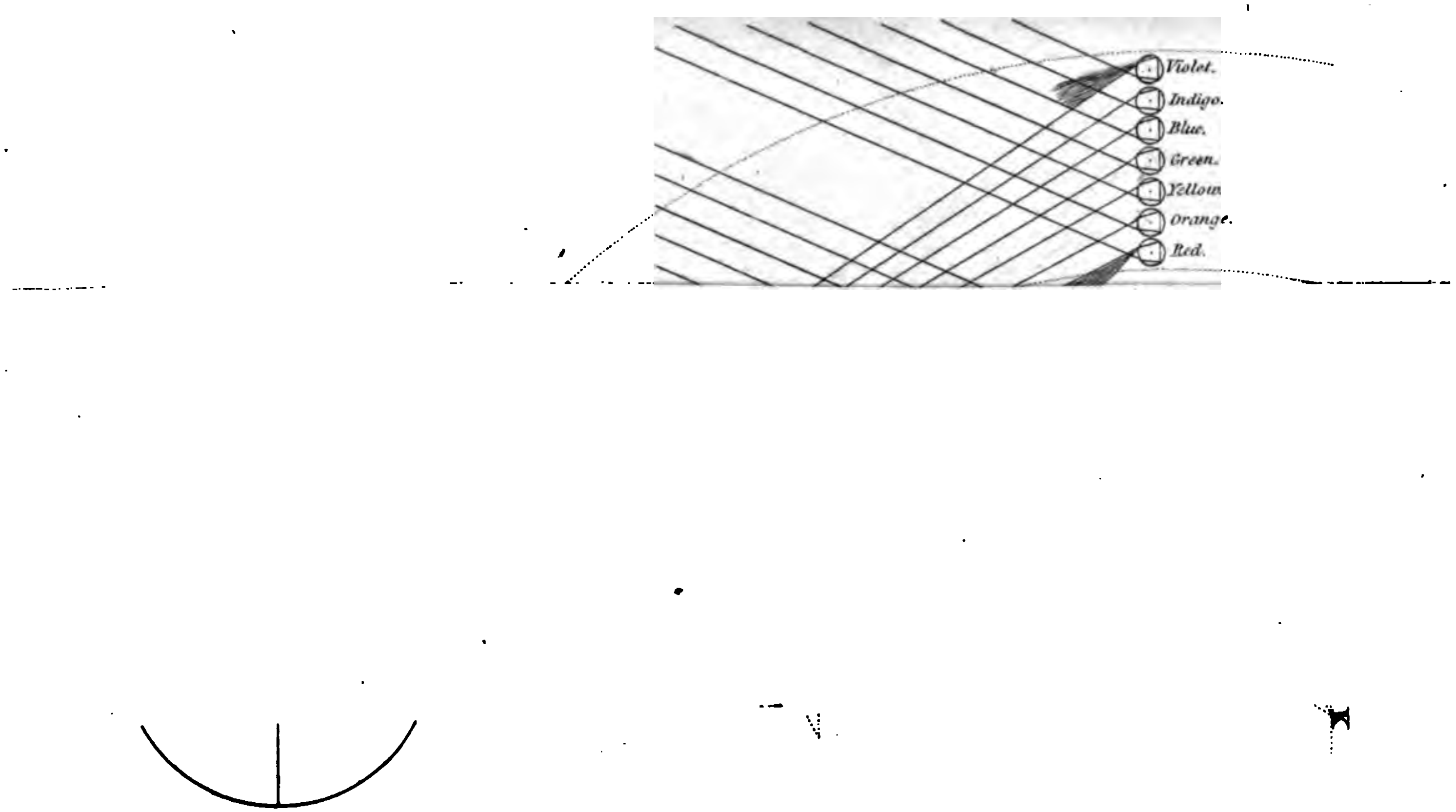
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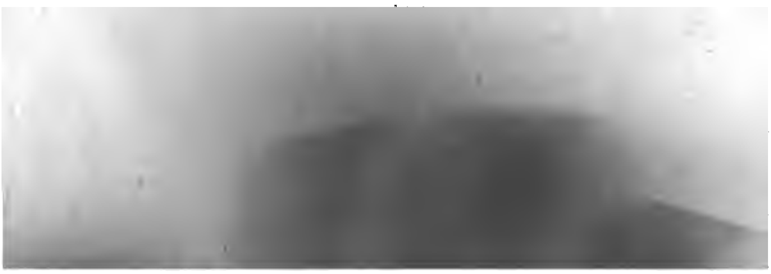

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